Fast-pre slow roll: power spectra and tensor to scalar ratio

L.Lello, D. Boyanovsky

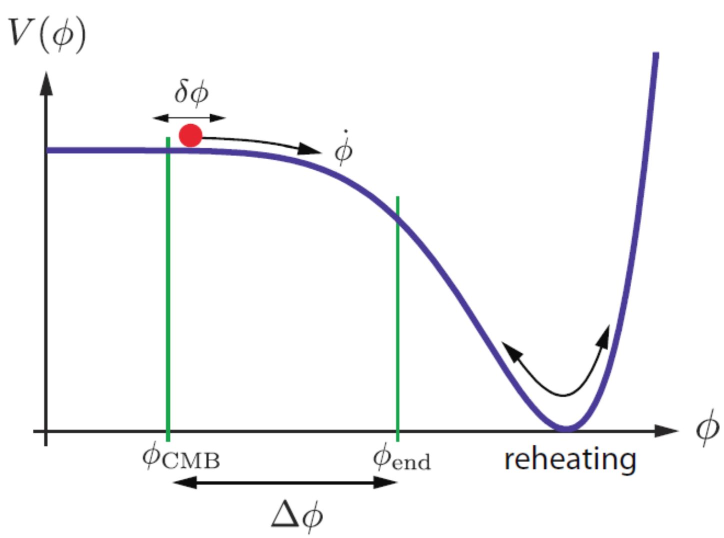
Single field slow roll inflation succesfully explains the CMB dataé .

Basics of slow roll : fairly flat potential

$$H^2 = \frac{1}{3M_{Pl}^2}\left[\cancel{\frac{1}{2}\dot{\Phi}^2} + V(\Phi)\right]$$

$$\cancel{\ddot{\Phi}} + 3H\dot{\Phi} + V'(\Phi) = 0$$

Slow Roll Inflation



Slow roll parameters

$$\epsilon_V = \frac{M_{Pl}^2}{2}\left[\frac{V'_{sr}(\Phi)}{V_{sr}(\Phi)}\right]^2 \qquad \eta_V = M_{Pl}^2 \frac{V''_{sr}(\Phi)}{V_{sr}(\Phi)}$$

Perturbations:

$$\Phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$ds^2 = C^2(\eta)[d\eta^2(1+2\psi(\vec{x},\eta)) - ((1-2\psi(\vec{x},\eta))\delta_{ij} + h_{ij}(\vec{x},\eta))dx^i dx^j]$$

Scalar: gravitational (Newtonian) potential

Tensor: gravitational waves

$$\mathcal{R} = -\psi - \frac{H}{\dot{\phi}}\delta\phi$$ (G.I)

$$u(\vec{x},t) = -z\,\mathcal{R}(\vec{x},t) \qquad z = \frac{\dot{\phi}}{H}a(t)$$

Quantization of Perturbations:

$$u(\vec{x},\eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \left[\alpha_{\mathcal{R}}(k)\, S_{\mathcal{R}}(k;\eta)\, e^{i\vec{k}\cdot\vec{x}} + \alpha^{\dagger}_{\mathcal{R}}(k)\, S^{*}_{\mathcal{R}}(k;\eta)\, e^{-i\vec{k}\cdot\vec{x}} \right]$$

$$h_{ij}(\vec{x},\eta) = \frac{2}{C(\eta)\, M_{Pl}} \sum_{\vec{k}} \sum_{\lambda=\times,+} \underbrace{\epsilon_{ij}(\lambda,\vec{k})}_{\text{transverse-traceless}} \left[\alpha_{\lambda,\vec{k}}\; S_T(k;\eta)\, e^{i\vec{k}\cdot\vec{x}} + \alpha^{\dagger}_{\lambda,\vec{k}}\; S^{*}_T(k;\eta)\, e^{-i\vec{k}\cdot\vec{x}} \right]$$

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_\alpha(\eta) \right] S_\alpha(k;\eta) = 0 \;\; ; \; \alpha = R, T \qquad W_\alpha(\eta) = \begin{cases} z''/z & \text{for curvature perturbations} \\ C''/C & \text{for tensor perturbations} \end{cases}$$

$$\alpha\ ;\alpha^{\text{À}} \rightarrow \text{CCR}$$

In slow roll:

$$W_\alpha(\eta) = \frac{\nu_\alpha^2 - \frac{1}{4}}{\eta^2} \qquad \nu_\alpha = \frac{3}{2} + \begin{cases} 3\epsilon_V - \eta_V & \text{for curvature perturbations} \\ \epsilon_V & \text{for tensor perturbations} \end{cases}$$

General Solution:

$$S_\alpha(k;\eta) = A_{k,\alpha}\, g_{\nu_\alpha}(k;\eta) + B_{k,\alpha}\, g^*_{\nu_\alpha}(k;\eta)$$

$$g_\nu(k,\eta) = \sqrt{\frac{-\pi\eta}{4}}\, H^{(1)}_\nu(-k\eta)$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{k^3}{2\pi^2}\left|\frac{S_\mathcal{R}(k;\eta)}{z(\eta)}\right|^2 \;\; ; \; \mathcal{P}_T(k) = \frac{4k^3}{\pi^2 M_{pl}^2}\left|\frac{S_T(k;\eta)}{C(\eta)}\right|^2$$

Boundary conditions: Bunch Davies: Minkowski-like deep inside the Hubble radius

$$\longrightarrow \qquad \boxed{A_{k,\alpha} = 1 \;\; ; \;\; B_{k,\alpha} = 0}$$

$\boxed{\text{for } -k\eta \ll 1}$ After mode crosses Hubble radius

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{8\pi^2 M_{Pl}^2 \epsilon_V}\left(\frac{k}{k_0}\right)^{n_s-1} \qquad n_s - 1 = -6\epsilon_V + 2\eta_V \longrightarrow \frac{\delta T}{T}$$

$$\mathcal{P}_T(k) = \frac{2H^2}{\pi^2 M_{Pl}^2}\left(\frac{k}{k_0}\right)^{n_T} \qquad n_T = -2\epsilon_V$$

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_{\mathcal{R}}(k)} = 16\epsilon_V(k_0)\left(\frac{k}{k_0}\right)^{4\epsilon_V - 2\eta_V}$$

$k=k_0$

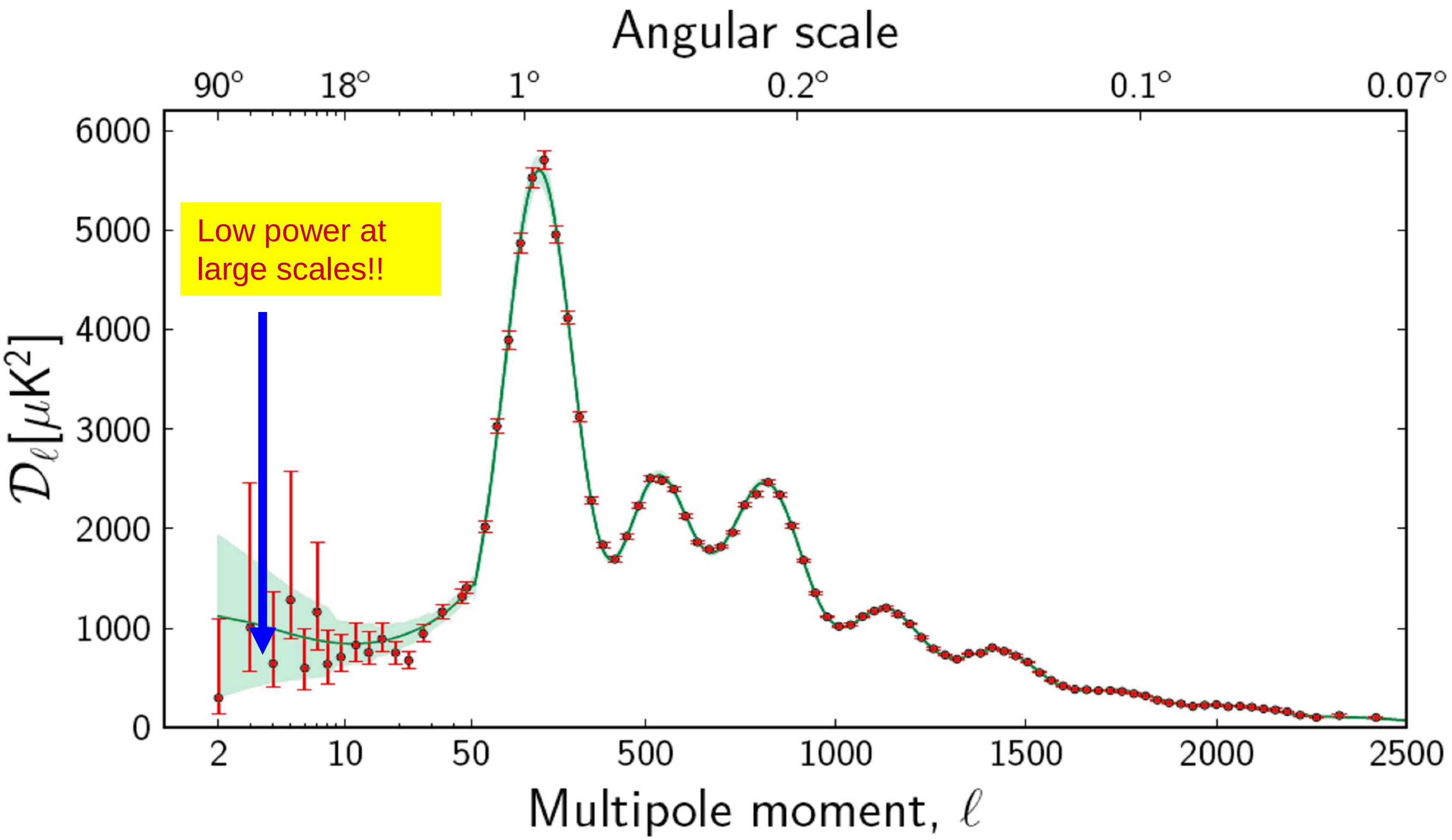
$$\boxed{r(k_0) = -8n_T}$$

``Consistency conditionô single fieldð slow roll

with BUNCH DAVIES B.C!!

$$C_l = \int_0^\infty \frac{dk}{k} \mathcal{P}_{\mathcal{R}}(k) \times \underbrace{\Delta_l(k)}$$

Matter/Radiation transfer function



Summary of slow roll

Å Nearly scale invariant power spectrum, $\boxed{\epsilon_V \ll 1 \ ; \ \eta_V \ll 1}$

modes cross Hubble radius during SLOW ROLL

Å Tensor/Scalar ratio consistency condition $\boxed{r(k_0) = -8 n_T}$

consequence of i) SLOW ROLL+ii) B.D. B.C

Å C_l agree with observation BUT for suppression at l<10

(Low power at large scales!)

Fast Roll: kinetic dominated stage prior to slow roll

A. Linde, JHEP 11, 052 (2001); C. Contaldi, M. Peloso, L. Kofman, A. Linde, JCAP 0307, 002 (2003); D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D74, 123006, 123007 (2006); C. Destri, H. J. de Vega, N. G. Sanchez, Phys.Rev.D81,063520 (2010); C. Destri, H. J. de Vega, N. G. Sanchez, Phys.Rev.D78, 023013 (2008); F. J. Cao, H. J. de Vega, N. G. Sanchez, Phys.Rev.D78, 083508 (2008); W.J. Handley, S.D. Brechet, A.N. Lasenby, and M.P. Hobson, Phys. Rev. D89, 063505 (2014).

Fast roll stage $\longrightarrow$ ***Suppression of low multipoles***

Q1: ***How does it affect TENSOR PERTURBATIONS? (GRAVITATIONAL WAVES)***

Q2: Are ***there observable effects?***

Q3***: Independent information on the scale of inflation?***

Fast Roll: Basics

$$H^2_{sr} \equiv \frac{V_{sr}(\phi)}{3M^2_{Pl}} \longrightarrow H^2 = \frac{1}{3M^2_{Pl}}[\frac{1}{2}\dot{\phi}^2 + V_{sr}(\phi)]$$

same slow roll potential

$$3H\dot{\phi} + V'_{sr}(\phi) = 0 \longleftarrow \ddot{\phi} + 3H\dot{\phi} + V'_{sr}(\phi) = 0 \longrightarrow \ddot{\phi} + 3H\dot{\phi} = 0$$

slow roll ***fast roll***

$$\downarrow \qquad\qquad \downarrow$$

$$\frac{3\dot{\phi}^2_{sr}}{2V_{sr}} = \epsilon_V \qquad\qquad \dot{\phi}(t) = \frac{\dot{\phi}_{sr}}{a^3(t)}$$

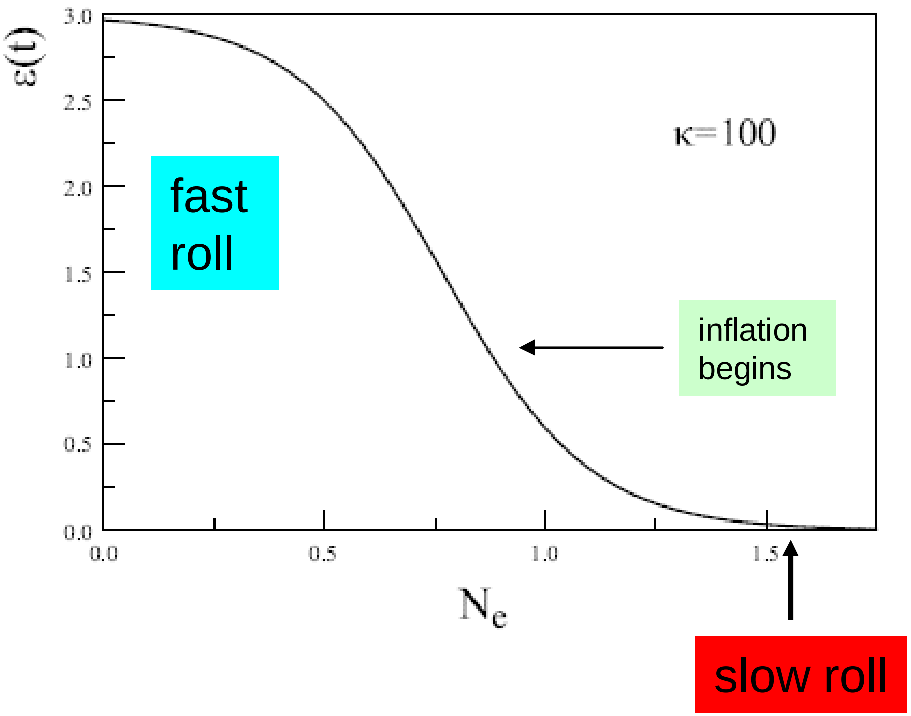
Important parameter:

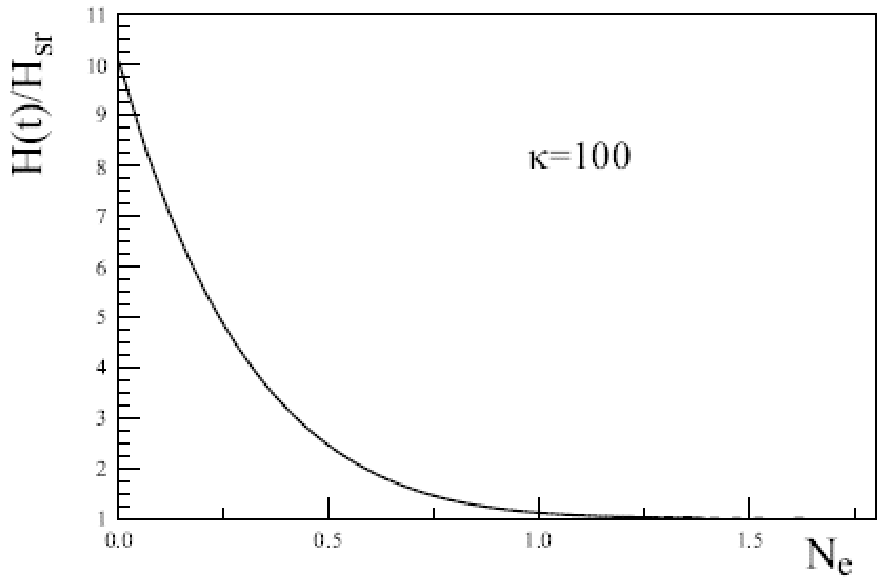
measures ratio of initial

kinetic to potential

$$\boxed{\frac{\dot{\phi}_i^2}{2V_{sr}} = \kappa} \longrightarrow \boxed{a_i = \left[\frac{\epsilon_V}{3\kappa}\right]^{1/6}}$$

$$\varepsilon(t) = -\frac{\dot{H}}{H^2} = \frac{\epsilon_V}{a^6(t) + \frac{\epsilon_V}{3}}$$





Fast roll only lasts a few e-folds !!

$$\boxed{N_e \lesssim 2 \text{ for } \kappa \lesssim 100}$$

Consequences of the fast roll stage:

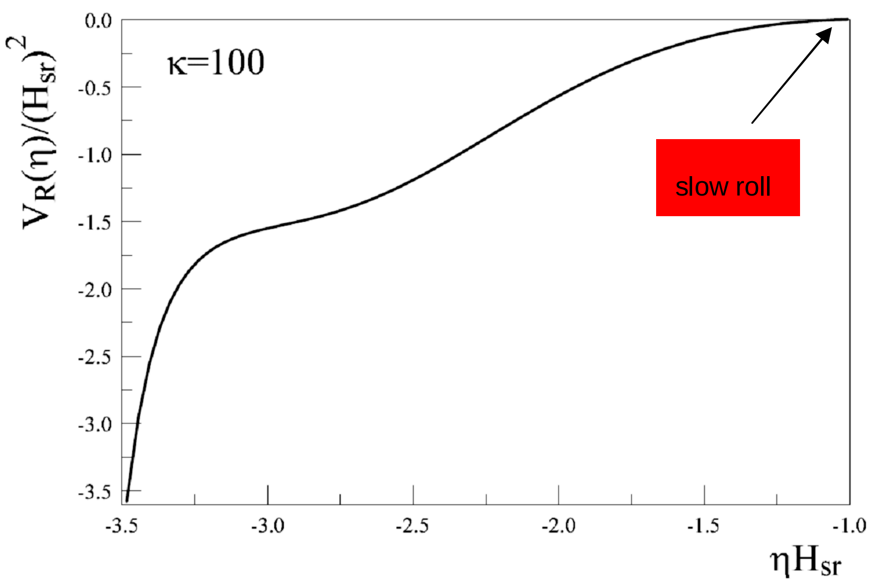
Changes in mode equations for ***SCALAR AND TENSOR*** perturbations

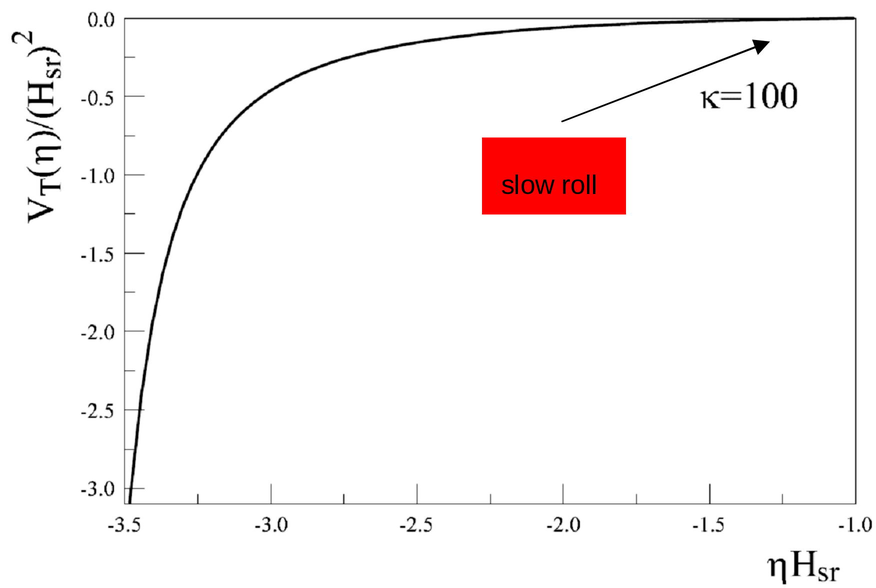
$$\left[\frac{d^2}{d\eta^2}+k^2-W_\alpha(\eta)\right]S_\alpha(k;\eta)=0 \;\; ; \; \alpha=R,T \qquad W_\alpha(\eta)=\begin{cases} z''/z & \text{for curvature perturbations} \\ C''/C & \text{for tensor perturbations}\end{cases}$$

$$W_\alpha(\eta)=V_\alpha(\eta)+\frac{\nu_\alpha^2-1/4}{\eta^2}$$

slow roll

fast roll correction





Fast roll correction = Localized potential:
use methods from scattering theory: Greenôs functionsé

General solution during the slow roll stage:

$$S_\alpha(k;\eta) = A_{k,\alpha}\, g_{\nu_\alpha}(k;\eta) + B_{k,\alpha}\, g^*_{\nu_\alpha}(k;\eta)$$

$$g_\nu(k,\eta) = \sqrt{\frac{-\pi\eta}{4}}\, H^{(1)}_\nu(-k\eta)$$

Can calculate A_k, B_k systematically: for modes deep inside Hubble radius during fast roll Born approximation

$$\boxed{\mathrm{A}_{k,\alpha} \neq 1 \;\; ; \; \mathrm{B}_{k,\alpha} \neq 0} \longrightarrow$$

Non B. D. conditions on mode functions during SLOW ROLL

Consequences on the power spectra:

$$\boxed{\mathcal{P}_{\mathcal{R}}(k) = \underbrace{\frac{H^2}{8\pi^2 M_{Pl}^2 \epsilon_V} \left(\frac{k}{k_0}\right)^{n_s - 1}}_{\text{slow roll}} \left[1 + \underbrace{D_{\mathcal{R}}(k/H_{sr})}_{\text{fast roll}}\right]}$$

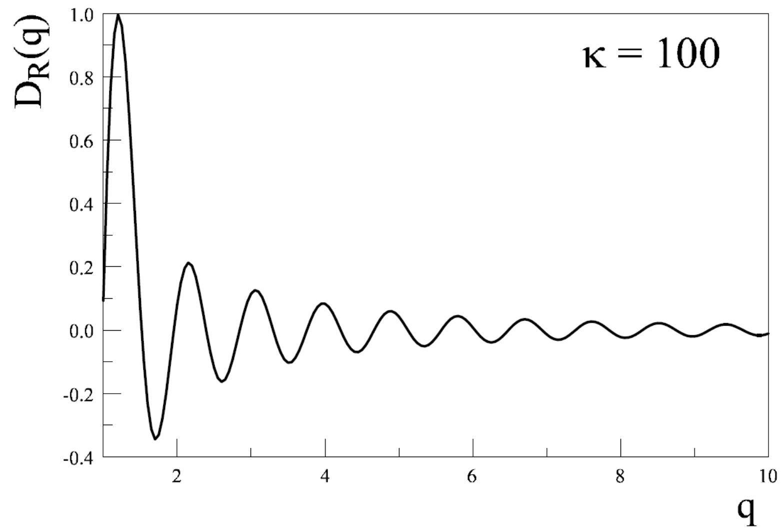
$$\boxed{\mathcal{P}_{\mathcal{T}}(k) = \underbrace{\frac{2H^2}{\pi^2 M_{Pl}^2} \left(\frac{k}{k_0}\right)^{n_T}}_{\text{slow roll}} \left[1 + \underbrace{D_{\mathcal{T}}(k/H_{sr})}_{\text{fast roll}}\right]}$$

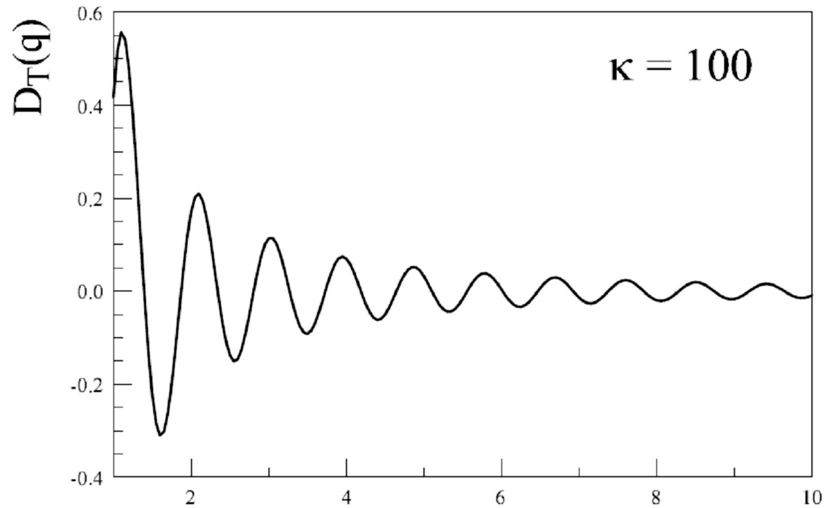
$$\boxed{r(k) = \underbrace{16\epsilon_V(k_0)\left(\frac{k}{k_0}\right)^{4\epsilon_V - 2\eta_V}}_{\text{slow roll}} \left[1 + \underbrace{D_{\mathcal{R}}(k/H_{sr}) - D_{\mathcal{T}}(k/H_{sr})}_{\text{fast roll}}\right]}$$

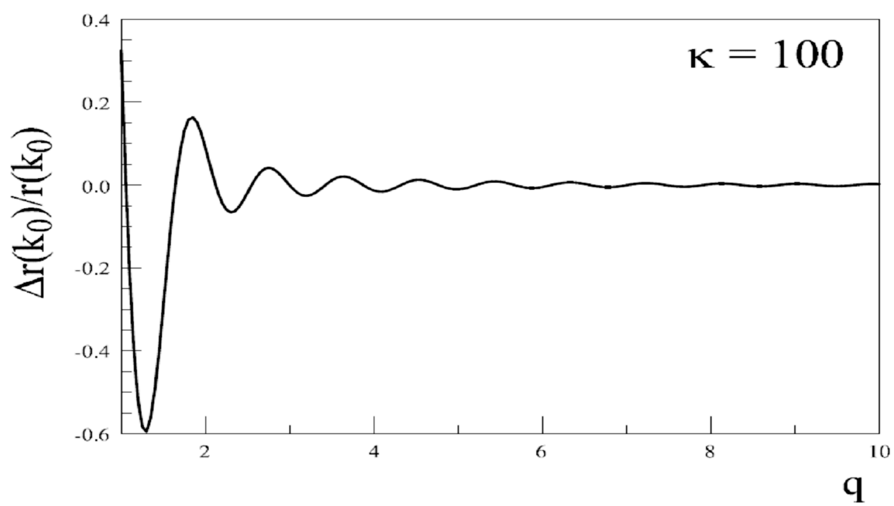
$$\boxed{r(k_0) = \underbrace{-8n_T}_{\text{slow roll}} \left[1 + \underbrace{D_{\mathcal{R}}(k_0/H_{sr}) - D_{\mathcal{T}}(k_0/H_{sr})}_{\text{fast roll}}\right]}$$

Note: fast roll correction $\longrightarrow$ change in ***``consistency condition''***

Fast roll corrections as a function of $q=k/H_{sr}$







$$\frac{\Delta r(k_0)}{r(k_0)} = D_T(q) - D_R(q)$$

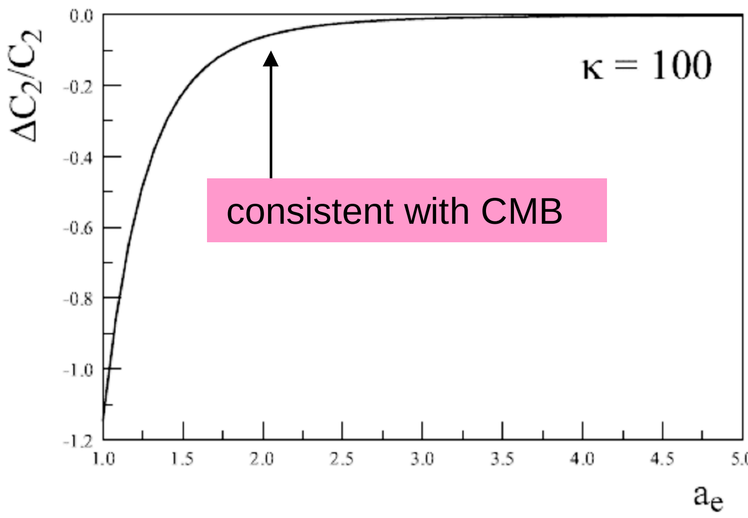
Oscillatory corrections to power spectra

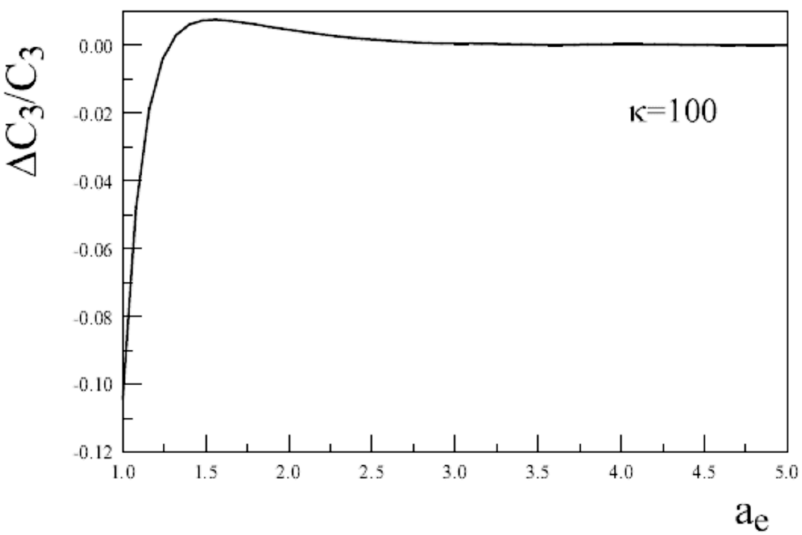
$$\boxed{\mathcal{P}_R(k)=\mathcal{A}_R(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}\left[1+A_R(\kappa)\left(\frac{H_{sr}}{k}\right)^{p(\kappa)}\cos\left[2\pi\omega(\kappa)\frac{k}{H_{sr}}+\varphi_R(\kappa)\right]\right]}$$

$$\boxed{\mathcal{P}_T(k)=\mathcal{A}_T(k_0)\left(\frac{k}{k_0}\right)^{n_T}\left[1+A_T(\kappa)\left(\frac{H_{sr}}{k}\right)^{p(\kappa)}\cos\left[2\pi\omega(\kappa)\frac{k}{H_{sr}}+\varphi_T(\kappa)\right]\right]}$$

$$\boxed{1.5\lesssim p(\kappa)\lesssim 2\ \ ;\ \omega(\kappa)\simeq 1, A_{R,T}\simeq 1 \text{ for } 3\lesssim\kappa\lesssim 100}$$

Large scale power suppression: quadrupole and octupole





5-10% suppression in the quadrupole if wavevector corresponding to horizon today crossed the Hubble radius about 1-e-fold after the beginning of the slow roll stage.

Summary and Conclusions

Fast roll stage prior to slow roll characterized by $\boxed{\frac{\dot{\phi}_i^2}{2V_{sr}} = \kappa}$ lasts 1-2 e-folds and merges smoothly with slow roll.

Suppression of C_l for low l consistent with CMB if modes crossed Hubble radius 1-2 e-folds after beginning of slow roll.

Modifies potentials for mode functions and boundary conditions for modes of cosmological relevance: non Bunch-Davies. Modifications to power spectra.

Oscillatory corrections to power spectra and $r(k_0)$, period of oscillations = Hubble scale during inflation: ***independent information on scale of inflation!!***

``Consistency condition'' is not fulfilled $\boxed{r(k_0) \neq -8n_T}$

Oscillations in $r(k_0)$ as a function of pivot scale, period = H_{sr}, amplitude only depends on κ

Results independent of (smooth) inflaton potential

Details: arXiv:1312.4251