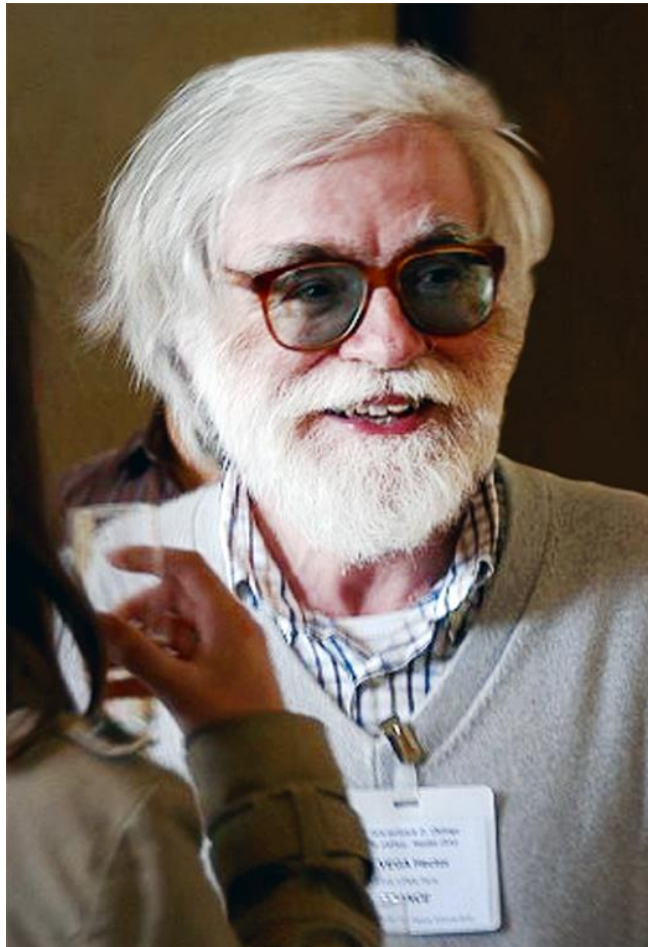


Fermionic Warm Dark Matter and the Thomas-Fermi galaxy structure theory



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Dark Matter in the Universe

81 % of the matter of the universe is **DARK** (DM).

DM is the dominant component of galaxies.

DM interacts through **gravity**.

Further DM interactions **unobserved** so far. Such couplings must be **very weak**: much weaker than weak interactions.

DM is **outside** the standard model of particle physics.

Proposed candidates:

- Cold Dark Matter: CDM, WIMPS, $m \sim 1 - 1000$ GeV.
IN BIG TROUBLE.
- Warm Dark Matter: WDM, sterile neutrinos $m \sim$ keV.
THE ANSWER !

DM particles decouple due to the universe expansion, their distribution function **freezes out** at decoupling.

Early decoupling: $T_d \sim 100$ GeV

Structure Formation in the Universe

Structures in the Universe as galaxies and cluster of galaxies form out of the **small primordial quantum fluctuations** originated by inflation just after the big-bang.

These small linear primordial fluctuations **grow** due to gravitational unstabilities (Jeans) and then classicalize.

Structures form through non-linear gravitational evolution.

Hierarchical formation starts from small scales first.

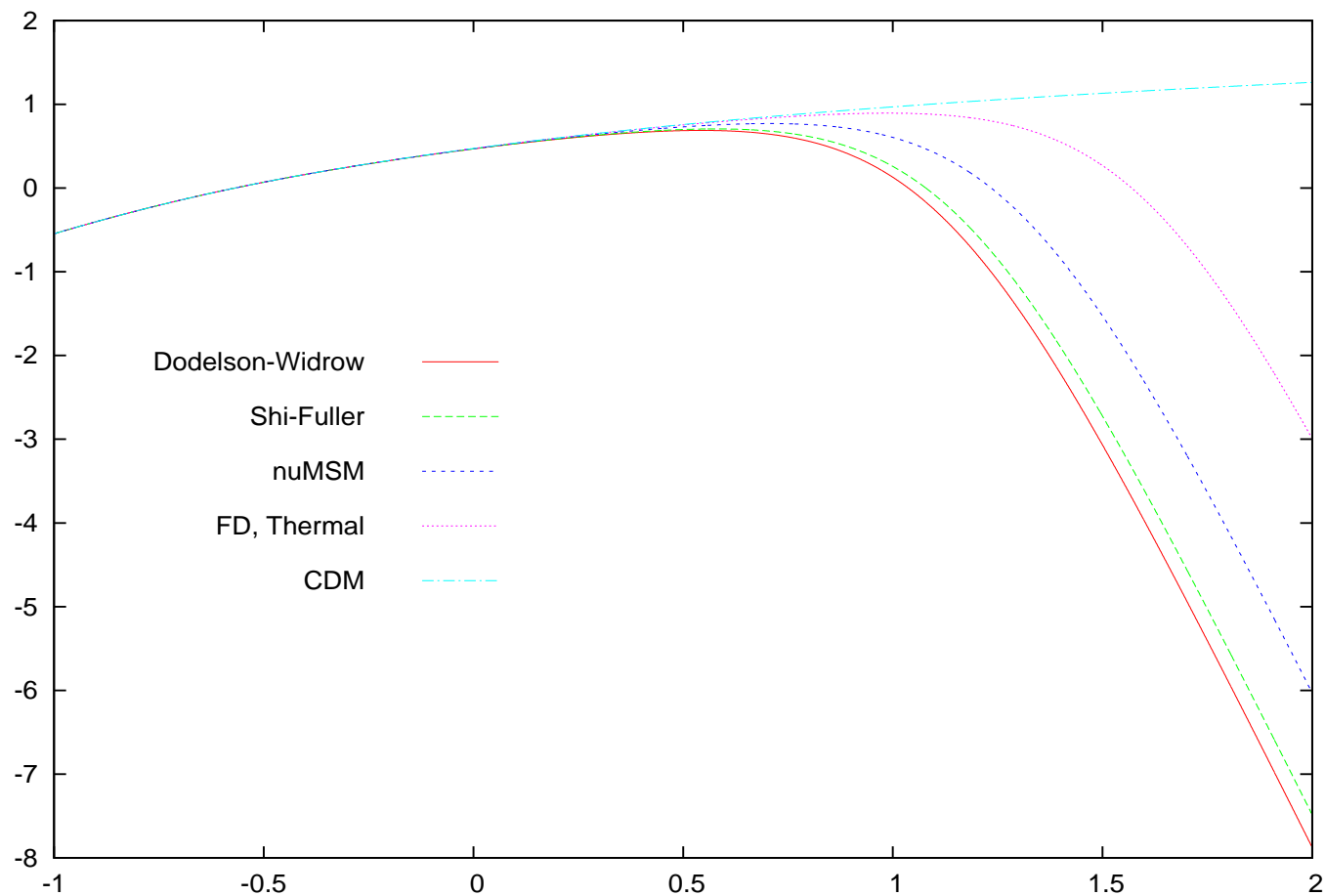
N -body CDM simulations **fail** to produce the observed structures for **small** scales less than some kpc.

Both N -body WDM and CDM simulations yield **identical and correct** structures for scales larger than some kpc.

WDM predicts **correct structures for small scales** (below kpc) when its **quantum** nature is taken into account.

Primordial power $\Delta^2(k)$: first ingredient in galaxy formation.

Linear primordial power spectrum $\Delta^2(k)$ vs. k Mpc / h



$\log_{10} \Delta^2(k)$ vs. $\log_{10}[k \text{ Mpc}/h]$ for a physical mass of 2.5 keV in four different WDM models and in CDM. WDM cuts $\Delta^2(k)$ on small scales. $r \lesssim 73 (\text{keV}/m)^{1.45} \text{ kpc}/h$.
CDM and WDM **are** identical for CMB.

WDM free streaming scale

The scale $l_{1/2}$ is where the WDM power spectrum is one-half of the CDM power spectrum:

$$l_{1/2} = 1/k_{1/2} = 207 \text{ kpc} \left(\text{keV}/m_{FD} \right)^{1.12}$$

This scale reproduces the sizes of the observed DM galaxy cores when the WDM mass is in the **keV scale !!**

$l_{1/2}$ is similar but more precise than the **free streaming scale** (or Jeans' scale):

$$r_{Jeans} = 210 \text{ kpc} \frac{\text{keV}}{m_{FD}} \left(\frac{100}{g_d} \right)^{\frac{1}{3}},$$

g_d = number of UR degrees of freedom at decoupling.

Small structure formation in WDM

DM particles can **freely** propagate over distances of the order of the free streaming scale.

Therefore, structures at scales smaller or of the order of $l_{1/2}$ are **erased** which **agrees** with the observed structures in galaxies !!

WDM sterile neutrinos in different particle models behave primordially just as if their masses were different (FD = thermal fermions):

$$\frac{m_{DW}}{\text{keV}} \simeq 2.85 \left(\frac{m_{FD}}{\text{keV}} \right)^{\frac{4}{3}}, \quad m_{SF} \simeq 2.55 m_{FD}, \quad m_{\nu\text{MSM}} \simeq 1.9 m_{FD}.$$

DW: Dodelson-Widrow model, SF: Shi-Fuller model

H J de Vega, N Sanchez, Warm Dark Matter cosmological fluctuations, Phys. Rev. D85, 043516 and 043517 (2012).

CDM free streaming scale

For CDM particles with $m \sim 100 \text{ GeV} \Rightarrow r_{\text{Jeans}} \sim 0.1 \text{ pc}$.

Hence CDM structures keep forming till scales as small as the solar system.

This is a **robust result** of N -body CDM simulations but **never observed** in the sky. Including baryons do not cure this serious problem. There is **over abundance** of small structures in CDM ('satellite problem') which are **too dense**.

CDM has **many further serious** conflicts with observations:

CDM needs ad-hoc merging and environment to grow gal.

Observations show that galaxy mergers are **rare** ($< 10\%$).

Pure-disk galaxies (bulgeless) are observed whose formation through CDM is **unexplained**.

CDM predicts **cusped** density profiles: $\rho(r) \sim 1/r$ for small r .

Observations show **cored** profiles: $\rho(r)$ bounded for small r .

Adding by hand **strong** enough feedback from baryons **does not** eliminate cusps (F. Marinacci et al., MNRAS 437, 1750 (2014)).

Summary Warm Dark Matter, WDM: $m \sim \text{keV}$

- Large Scales, structures beyond ~ 100 kpc: WDM and CDM yield **identical** results **which agree with observations**
 - Intermediate Scales: WDM simulations give the **correct abundance** of substructures.
 - Inside galaxy cores, below ~ 100 pc: N-body classical physics simulations are **incorrect** for WDM because of **important quantum effects**.
 - Quantum calculations (Thomas-Fermi) give galaxy cores, galaxy masses, velocity dispersions and densities in **agreement with the observations**.
 - Direct Detection of the main WDM candidate: the sterile neutrino. **Beta decay and electron capture**. ${}^3\text{H}$, Re, Ho.
- So far, **not a single valid** objection arose against WDM.
- Baryons ($<16\%$ DM) expected to give a **correction** to WDM

Quantum physics in Galaxies

de Broglie wavelength of DM particles $\lambda_{dB} = \frac{\hbar}{m v}$

d = mean distance between particles, v = mean velocity

$d = \left(\frac{m}{\rho}\right)^{\frac{1}{3}}$, $Q = \rho/v^3$, Q = phase space density.

ratio: $\mathcal{R} = \frac{\lambda_{dB}}{d} = \hbar \left(\frac{Q}{m^4}\right)^{\frac{1}{3}}$

Observed values: $2 \times 10^{-3} \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}} < \mathcal{R} < 1.4 \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}}$

The **larger** \mathcal{R} is for ultracompact dwarfs.

The **smaller** \mathcal{R} is for big spirals.

\mathcal{R} near unity (or above) means a **QUANTUM OBJECT**.

Observations alone show that compact dwarf galaxies are **quantum objects** (for WDM).

No quantum effects in CDM: $m \gtrsim \text{GeV} \Rightarrow \mathcal{R} \lesssim 10^{-8}$

Quantum pressure vs. gravitational pressure

quantum pressure: $P_q = \text{flux of momentum} = n v p$ **repulsive**

$v = \text{mean velocity, momentum} = p \sim \hbar / \Delta x \sim \hbar n^{\frac{1}{3}}$,

particle number density $= n = \frac{M_q}{\frac{4}{3} \pi R_q^3 m}$

galaxy mass $= M_q$, galaxy halo radius $= R_q$

gravitational pressure (**attractive**): $P_G = \frac{G M_q^2}{R_q^2} \times \frac{1}{4 \pi R_q^2}$

Equilibrium: $P_q = P_G \implies$

$$R_q = \frac{3^{\frac{5}{3}}}{(4 \pi)^{\frac{2}{3}}} \frac{\hbar^2}{G m^{\frac{8}{3}} M_q^{\frac{1}{3}}} = 10.6 \text{ pc} \left(\frac{10^6 M_\odot}{M_q} \right)^{\frac{1}{3}} \left(\frac{\text{keV}}{m} \right)^{\frac{8}{3}}$$

$$v = \left(\frac{4 \pi}{81} \right)^{\frac{1}{3}} \frac{G}{\hbar} m^{\frac{4}{3}} M_q^{\frac{2}{3}} = 11.6 \frac{\text{km}}{\text{s}} \left(\frac{\text{keV}}{m} \right)^{\frac{4}{3}} \left(\frac{M_q}{10^6 M_\odot} \right)^{\frac{2}{3}}$$

for WDM the values of M_q , R_q and v are **consistent with the dwarf galaxy observations !!**.

Dwarf galaxies **can be supported** by the fermionic quantum pressure of WDM. Analogous to neutron stars and white dwarfs.

Self-gravitating Fermions in the Thomas-Fermi approach

WDM is non-relativistic in the MD era. A single DM halo in late stages of formation relaxes to a time-independent form especially in the interior.

Chemical potential: $\mu(r) = \mu_0 - m \phi(r)$, $\phi(r) = \text{grav. pot.}$

Poisson's equation: $\frac{d^2 \mu}{dr^2} + \frac{2}{r} \frac{d\mu}{dr} = -4 \pi G m \rho(r)$

$\rho(0) = \text{finite for fermions} \implies \frac{d\mu}{dr}(0) = 0.$

Density $\rho(r)$ and pressure $P(r)$ in terms of the distribution function $f(E)$:

$$\rho(r) = \frac{m}{\pi^2 \hbar^3} \int_0^\infty p^2 dp f\left[\frac{p^2}{2m} - \mu(r)\right]$$

$$P(r) = \frac{1}{3 \pi^2 m \hbar^3} \int_0^\infty p^4 dp f\left[\frac{p^2}{2m} - \mu(r)\right]$$

These are **self-consistent** non-linear Thomas-Fermi equations that determine $\mu(r)$.

Galaxy surface density

The surface density: $\Sigma_0 \equiv r_h \rho_0 \simeq 120 M_\odot/\text{pc}^2$,

takes nearly the **same value** for galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and over different Hubble types.

We take Σ_0 as physical scale to express the galaxy magnitudes in the Thomas-Fermi approach.

Dimensionless variables: ξ , $\nu(\xi)$.

$$r = l_0 \xi \quad , \quad \mu(r) = T_0 \nu(\xi) \quad , \quad \rho_0 \equiv \rho(0).$$

T_0 = effective galaxy temperature, l_0 characteristic length.

From the Thomas-Fermi equations:

$$l_0 \equiv \left(\frac{9\pi}{2^9}\right)^{\frac{1}{5}} \left(\frac{\hbar^6}{G^3 m^8}\right)^{\frac{1}{5}} \left[\frac{\xi_h I_2(\nu_0)}{\Sigma_0}\right]^{\frac{1}{5}} =$$
$$4.2557 [\xi_h I_2(\nu_0)]^{\frac{1}{5}} \left(\frac{2 \text{ keV}}{m}\right)^{\frac{8}{5}} \left(\frac{120 M_\odot}{\Sigma_0 \text{ pc}^2}\right)^{\frac{1}{5}} \text{ pc}$$

$$I_n(\nu) \equiv (n+1) \int_0^\infty y^n dy f(y^2 - \nu) \quad , \quad \nu_0 \equiv \nu(0)$$

WDM Thomas-Fermi equations

Self-consistent dimensionless Thomas-Fermi equation:

$$\frac{d^2\nu}{d\xi^2} + \frac{2}{\xi} \frac{d\nu}{d\xi} + I_2(\nu) = 0 \quad , \quad \nu'(0) = 0$$

Core size r_h of the halo defined as for Burkert profile:

$$\frac{\rho(r_h)}{\rho_0} = \frac{1}{4} \quad , \quad r_h = l_0 \xi_h$$

Fermi-Dirac Phase-Space distribution function $f(E/T_0)$:

Contrasting the theoretical Thomas-Fermi solution with **galaxy data**, T_0 turns to be $10^{-3} \text{ }^\circ\text{K} < T_0 < 20 \text{ }^\circ\text{K}$

colder = **ultracompact**, warmer = **large spirals**.

$$T_0 \sim m \langle v^2 \rangle_{\text{observed}} \quad \text{for} \quad m \sim 2 \text{ keV.}$$

All results are **independent** of any WDM particle physics model, they only follow from the **gravitational** interaction of the WDM particles and their **fermionic** nature.

Lower bound on the particle mass m

In the **degenerate** quantum limit $\nu_0 \rightarrow +\infty$, $T_0 \rightarrow 0$ the galaxy mass and halo radius take their **minimum** values

$$r_h^{min} = 11.3794 \left(\frac{2 \text{ keV}}{m} \right)^{\frac{8}{5}} \left(\frac{120 M_\odot}{\Sigma_0 \text{ pc}^2} \right)^{\frac{1}{5}} \text{ pc}$$

$$M_h^{min} = 30998.7 \left(\frac{2 \text{ keV}}{m} \right)^{\frac{16}{5}} \left(\frac{\Sigma_0 \text{ pc}^2}{120 M_\odot} \right)^{\frac{3}{5}} M_\odot$$

Observed halo masses must be **larger or equal** than M_h^{min}

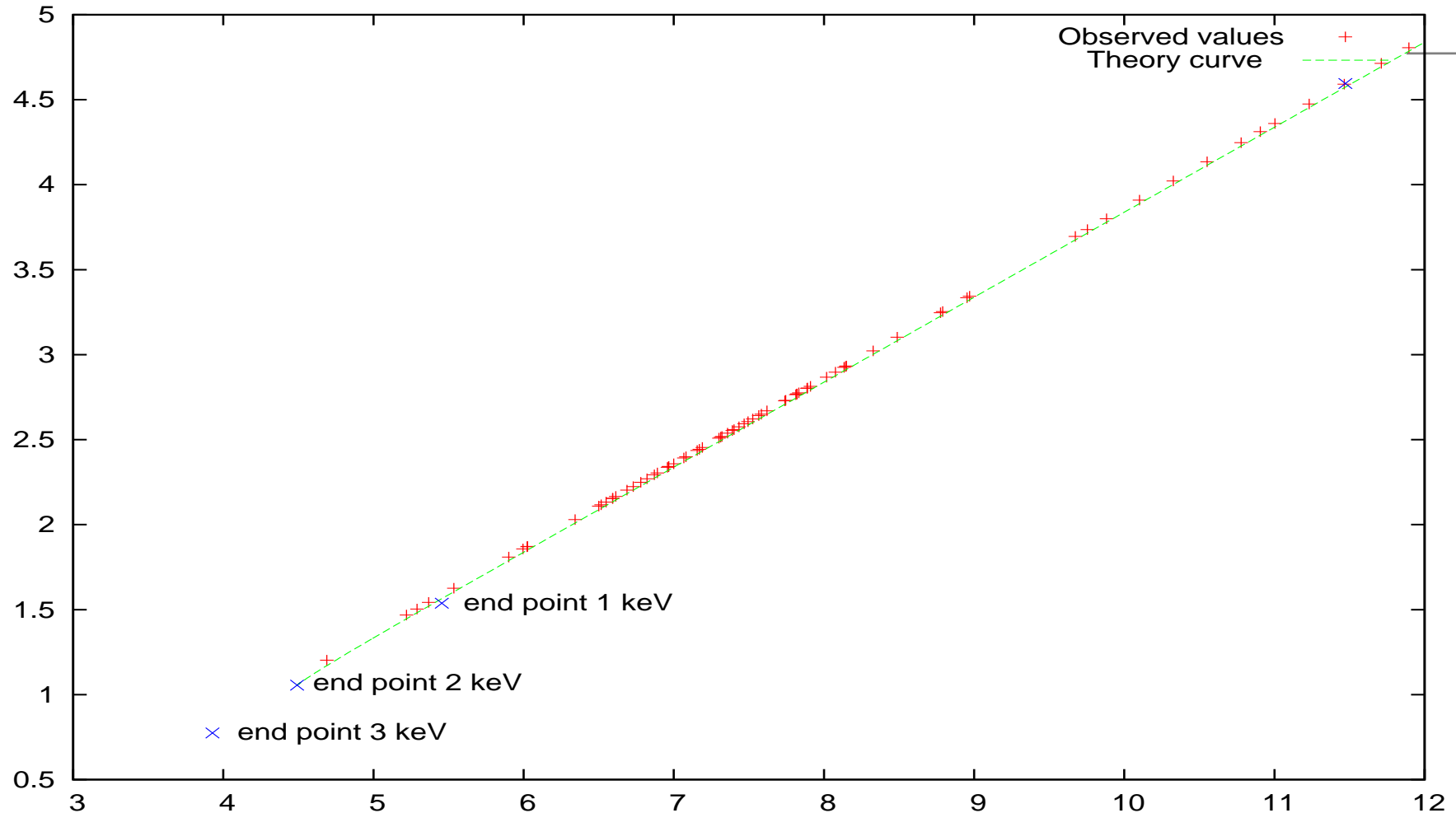
From the minimum observed value of the halo mass M_h^{min} a **lower bound** for the WDM particle mass m follows

$$m \geq m_{min} \equiv 1.387 \text{ keV} \left(\frac{10^5 M_\odot}{M_h^{min}} \right)^{\frac{5}{16}} \left(\frac{\Sigma_0 \text{ pc}^2}{120 M_\odot} \right)^{\frac{3}{16}}$$

The minimal known halo mass is for Willman I:

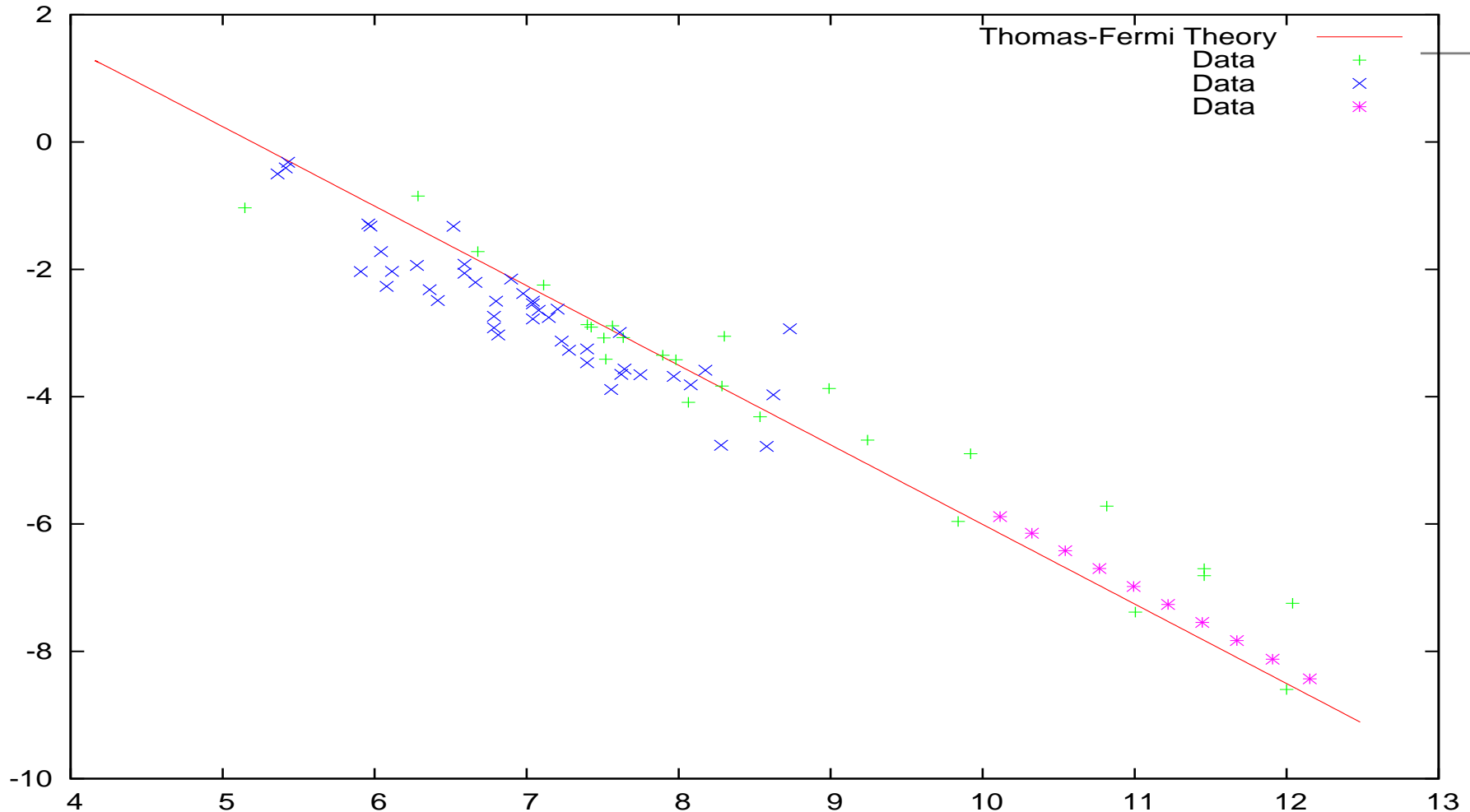
$$M_{Willman I} = 3.9 \cdot 10^4 M_\odot \text{ which implies } m \geq 1.86 \text{ keV}$$

Galaxy halo radius vs. Galaxy halo Mass



$\hat{r}_h = r_h \left(\Sigma_0 \text{ pc}^2 / [120 M_\odot] \right)^{\frac{1}{5}}$ vs. $\hat{M}_h = M_h \left(120 M_\odot / [\Sigma_0 \text{ pc}^2] \right)^{\frac{3}{5}}$.
 r_h follows **with precision** the square-root of M_h and the amplitude factor as predicted theoretically.

Galaxy Phase-space density Q vs. Galaxy halo Mass



$\log_{10} Q$ vs. $\log_{10} \hat{M}_h$ theory and data.

$Q \equiv \rho(0)/\sigma^3(0)$. Theoretical curve Q obtained from the Thomas-Fermi expression.

Diluted regime of Galaxies

In the diluted regime of Galaxies

$$M_h \gtrsim 10^6 M_\odot, \quad \nu_0 \lesssim -5, \quad T_0 \gtrsim 0.017 \text{ K} = 17 \text{ mK.}$$

r_h , M_h and $Q(0)$ **scale** as functions of each other.

$$M_h = 1.75572 \Sigma_0 r_h^2, \quad r_h = 68.894 \sqrt{\frac{M_h}{10^6 M_\odot}} \sqrt{\frac{120 M_\odot}{\Sigma_0 \text{ pc}^2}} \text{ pc}$$

$$Q(0) = 1.2319 \left(\frac{10^5 M_\odot}{M_h} \right)^{\frac{5}{4}} \left(\frac{\Sigma_0 \text{ pc}^2}{120 M_\odot} \right)^{\frac{3}{4}} \text{ keV}^4$$

These scaling behaviours are **very accurate** except near the degenerate limit.

C. Destri, H. J. de Vega, N. G. Sanchez, *New Astronomy* **22**, 39 (2013) and *Astroparticle Physics*, **46**, 14 (2013).

H. J. de Vega, P. Salucci, N. G. Sanchez, arXiv:1309.2290, to appear in *MNRAS*.

H. J. de Vega, N. G. Sanchez, arXiv:1310.6355.

Classical and Quantum regimes of WDM Galaxies

I. Diluted and classical regime:

$$\hat{M}_h \gtrsim 10^6 M_\odot, \quad \nu_0 \lesssim -5, \quad T_0 \gtrsim 0.017 \text{ K}.$$

The density and the velocity profiles are **universal**.

Exact scaling laws for r_h , M_h and $Q(0)$.

II. Quantum compact regime:

$$10^6 M_\odot \gtrsim \hat{M}_h \gtrsim \hat{M}_{h,min} = 3.1 \cdot 10^4 M_\odot,$$

$$\nu_0 \gtrsim -5, \quad 0 \leq T_0 \lesssim 0.017 \text{ K}.$$

The density and the velocity profiles are **non-universal**: the profiles **depend** on the galaxy mass M_h .

Small deviations from the scaling laws for r_h , M_h and $Q(0)$.

III. Degenerate limit

$$\hat{M}_h = \hat{M}_{h,min} = 3.1 \cdot 10^4 M_\odot, \quad \nu_0 = +\infty, \quad T_0 = 0$$

Circular Velocities and Density Profiles

The circular velocity $v_c(r)$ follows from the virial theorem

$$v_c(r) = \sqrt{\frac{G M(r)}{r}} = \sqrt{-\frac{r}{m} \frac{d\mu}{dr}}$$

The circular velocity normalized at the core radius r_h

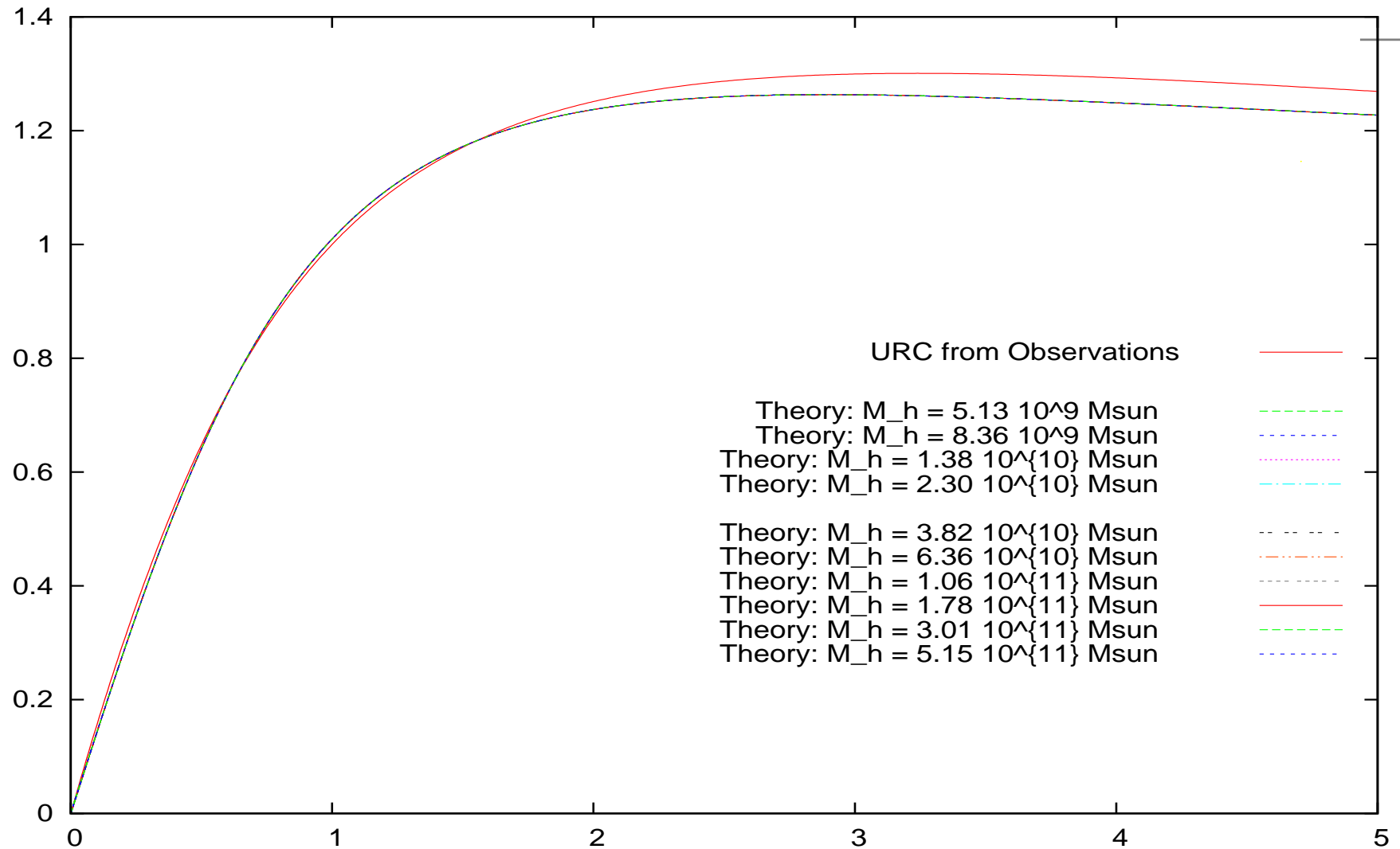
$$U(x) \equiv \frac{v_c(r)}{v_c(r_h)} \quad , \quad x = \frac{r}{r_h}$$

Solving the Thomas-Fermi equations we find:

- $U(x) = v_c(r)/v_c(r_h)$ is **only** function of $x = r/r_h$.
- $U(x)$ takes the **same** values for all galaxy halo masses in the range $5.1 \cdot 10^9 M_\odot$ till $5.1 \cdot 10^{11} M_\odot$.
- $U(x)$ turns to be an **universal** function.
- The observational universal curves and the theoretical Thomas-Fermi curves **coincide** for $r \lesssim 2 r_h$, $x \lesssim 2$.

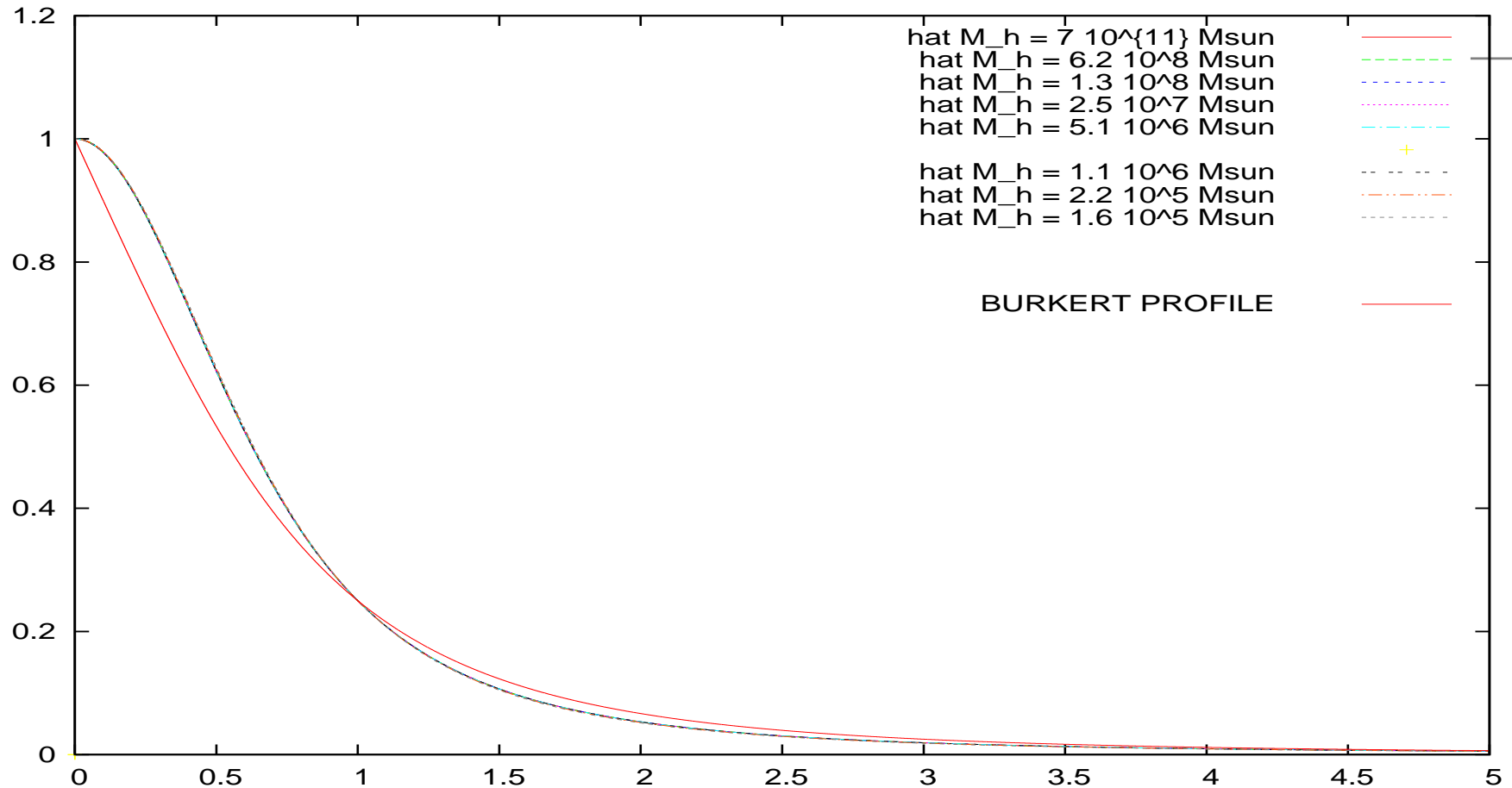
These are **remarkable** results !!

Normalized circular velocities



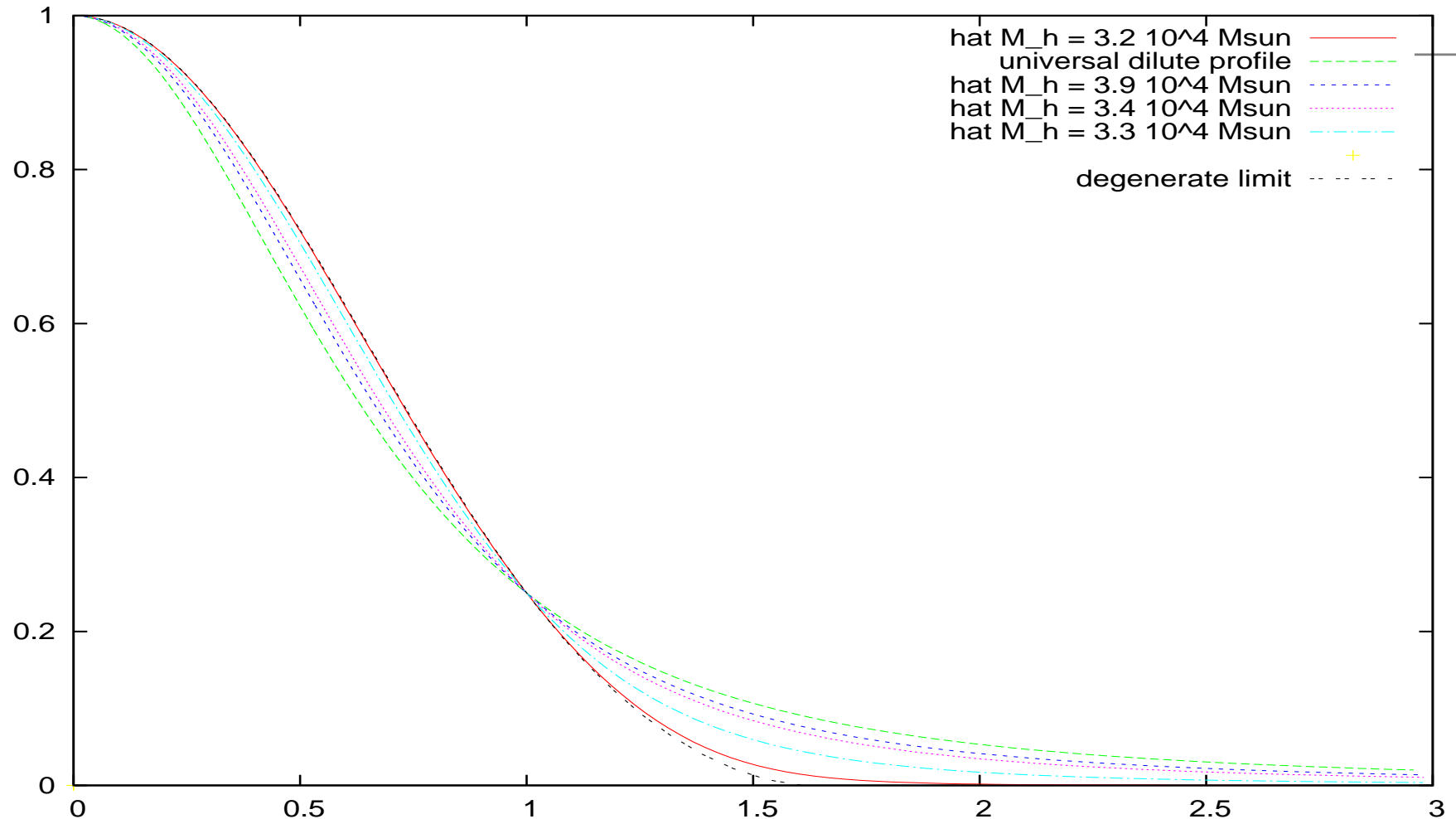
$$U(x) = v_c(r)/v_c(r_h) \text{ VS. } x = r/r_h.$$

Theoretical vs. observational density profiles



$\rho(r)/\rho(0)$ as functions of r/r_h . **ALL** the theoretical profiles in the diluted regime: $1.4 \cdot 10^5 M_{\odot} < \hat{M}_h < 7.5 \cdot 10^{11} M_{\odot}$ **fall** into the **same and universal** density profile in very good agreement with the **empirical** Burkert profile.

Density profiles in the Quantum regime



$\rho(r)/\rho(0)$ as functions of r/r_h : **Non-Universal.**

Galaxy halo masses $M_h^{\min} = 3.1 \cdot 10^4 M_{\odot} \leq \hat{M}_h < 3.9 \cdot 10^4 M_{\odot}$
in the **quantum** regime exhibit **shrinking** density profiles for
 $r > r_h$.

The local equation of state of WDM Galaxies

The pressure $P(r)$ as a function of the density $\rho(r)$

$$\rho = \frac{m^{\frac{5}{2}}}{3\pi^2 \hbar^3} (2T_0)^{\frac{3}{2}} I_2(\nu) \quad , \quad P = \frac{m^{\frac{3}{2}}}{15\pi^2 \hbar^3} (2T_0)^{\frac{5}{2}} I_4(\nu).$$

through the potential ν from the **Thomas-Fermi** equation.

$$P = \frac{T_0}{m} \rho \quad , \quad \nu \ll -1, \text{ WDM diluted galaxies.}$$

$$P = \frac{\hbar^2}{5} \left(\frac{3\pi^2}{m^4} \right)^{\frac{2}{3}} \rho^{\frac{5}{3}} \quad , \quad \nu \gg 1, \text{ WDM degenerate quantum limit.}$$

Simple formula accurately representing the exact equation of state obtained by solving the Thomas-Fermi equation:

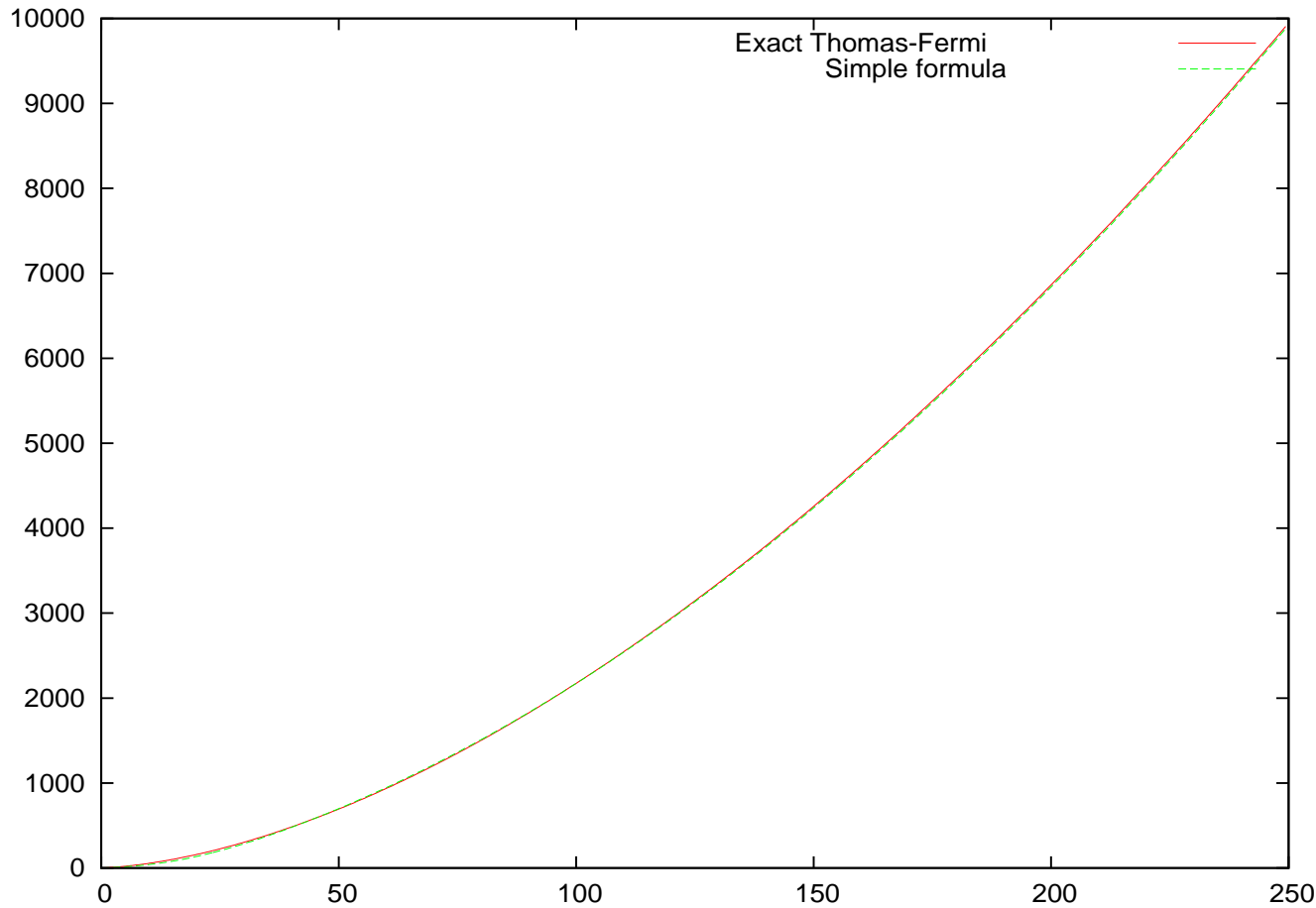
$$P = \frac{m^{\frac{3}{2}} (2T_0)^{\frac{5}{2}}}{15\pi^2 \hbar^3} \left(1 + \frac{3}{2} e^{-\beta_1 \tilde{\rho}} \right) \tilde{\rho}^{\frac{1}{3}} \left(5 - 2 e^{-\beta_2 \tilde{\rho}} \right),$$

$$\tilde{\rho} \equiv \frac{3\pi^2 \hbar^3}{m^{\frac{5}{2}} (2T_0)^{\frac{3}{2}}} \rho = I_2(\nu),$$

best fit to the Thomas-Fermi equation of state for:

$$\beta_1 = 0.047098 \quad , \quad \beta_2 = 0.064492$$

The equation of state of Galaxies: exact T-F and simple formula



The equation of state \tilde{P} vs. $\tilde{\rho}$ obtained by solving the Thomas-Fermi equation and the simple formula.

$$\tilde{P} = \frac{15 \pi^2 \hbar^3}{m^{\frac{3}{2}} (2 T_0)^{\frac{5}{2}}} P = I_4(\nu) , \quad \tilde{\rho} \equiv \frac{3 \pi^2 \hbar^3}{m^{\frac{5}{2}} (2 T_0)^{\frac{3}{2}}} \rho = I_2(\nu)$$

The Eddington equation for Dark Matter in Galaxies

$f(E)$ DM distribution function, $E = p^2 / (2m) - \mu$,
 m DM particle mass, μ the chemical potential.

Equilibrium condition: $\mu(r) = \mu_0 - m \phi(r)$,

$\phi(r)$ = gravitational potential.

The Poisson equation takes the **self-consistent** form:

$$\frac{d^2 \mu}{dr^2} + \frac{2}{r} \frac{d\mu}{dr} = -4\pi G m \rho(r) = -\frac{4 G m^2}{\pi \hbar^3} \int_0^\infty dp p^2 f \left[\frac{p^2}{2m} - \mu(r) \right]$$

Dimensionless variables: q , $\nu(q)$:

$$r = r_h q \quad , \quad \mu(r) = T_0 \nu(q) \quad , \quad f(E) = \Psi(E/T_0)$$

T_0 plays the role of the temperature and depends on the galaxy mass. The density profile is known from the observations:

$$\rho(r) = \rho_0 F \left(\frac{r}{r_h} \right) = \rho_0 F(q) \quad , \quad \rho_0 \equiv \rho(0) \quad , \quad F(1) = 1/4.$$

To be **determined**: the DM distribution function $\Psi(E/T_0)$.

Abel's equation and its solution

Dimensionless Poisson's equation:

$$\frac{d^2\nu}{dq^2} + \frac{2}{q} \frac{d\nu}{dq} = -b_0 F(q), \quad b_0 \equiv 4 \pi G \rho_0 r_h^2 \frac{m}{T_0}$$

$$\nu(q) = \nu(0) + b_0 \varepsilon(q), \quad \varepsilon(q) = \int_0^q \left(1 - \frac{q'}{q}\right) q' F(q') dq'$$

Self-consistent Poisson equation in dimensionless variables:

$$\rho(r) = \frac{\sqrt{2}}{\pi^2} m^{\frac{5}{2}} T_0^{\frac{3}{2}} \int_{\nu(\infty)}^{\nu} d\nu' \sqrt{\nu - \nu'} \Psi(-\nu'), \quad \nu' \equiv \nu - \frac{p^2}{2mT_0}$$

and in terms of the density profile $F(q)$

$$F(\nu) = \frac{\sqrt{2}}{\pi^2} \frac{m^{\frac{5}{2}} T_0^{\frac{3}{2}}}{\rho_0} \int_{\nu(\infty)}^{\nu} d\nu' \sqrt{\nu - \nu'} \Psi(-\nu')$$

This is an **Abel integral** equation and its solution, the **Eddington formula**:

$$\Psi(-\nu) = \sqrt{2} \pi \frac{\rho_0}{m^{\frac{5}{2}} T_0^{\frac{3}{2}}} \int_{\nu(\infty)}^{\nu} \frac{d\nu'}{\sqrt{\nu - \nu'}} \frac{d^2 F}{d\nu'^2}$$

Boundary condition: Ψ and $d\Psi/d\nu$ vanish at infinite distance.

The Distribution Function in terms of the Density Profile

We explicitly find the distribution function $\Psi(q)$ in terms of the density profile $F(q)$ in H. J. de Vega, N. G. Sanchez, arXiv:1401.0726.

$$\Psi(q) = \frac{1}{G^{\frac{3}{2}} r_h^3 m^4 \sqrt{\rho_0}} \mathcal{D}(q), \quad \mathcal{D}(q) \equiv \frac{1}{\sqrt{32} \pi} \int_q^\infty \frac{\mathcal{J}(q') dq'}{\sqrt{\varepsilon(q) - \varepsilon(q')}}}$$

$$\mathcal{J}(q) \equiv \frac{1}{\begin{pmatrix} \frac{d\varepsilon}{dq} \\ -\frac{d\varepsilon}{dq} \end{pmatrix}} \left[\begin{array}{c} \frac{d^2\varepsilon}{dq^2} \\ \frac{d^2F}{dq^2} - \frac{\frac{d^2\varepsilon}{dq^2}}{\frac{d\varepsilon}{dq}} \frac{dF}{dq} \end{array} \right]. \quad \text{Notice that } \left(-\frac{d\varepsilon}{dq} > 0 \right).$$

We explicitly find the **velocity dispersion** and the **pressure** in terms of the density profile $F(q)$:

$$v^2(r) = 6 \pi G \rho_0 r_h^2 \frac{1}{F(q)} \int_q^\infty dq' [\varepsilon(q) - \varepsilon(q')]^2 \mathcal{J}(q')$$

$$P(r) = 2 \pi G \Sigma_0^2 \int_q^\infty dq' [\varepsilon(q) - \varepsilon(q')]^2 \mathcal{J}(q')$$

Physical results from the Distribution Function

Cored density profiles behaving quadratically for small distances $\rho(r) \stackrel{r \rightarrow 0}{\approx} \rho(0) - K r^2$ produce **finite and positive distribution functions** at the halo center while **cusped** density profiles always produce **divergent** distribution functions at the center.

We explicitly compute the phase–space distribution function and the equation of state for the **family** of α -density profiles

$$\rho(r) = \frac{\rho_0}{\left[1 + \left(\frac{r}{r_h}\right)^2\right]^\alpha}, \quad 1 \leq \alpha \leq 2.5$$

This cored density profile generalizes the pseudo-thermal profile and with $\alpha \sim 1.5$, it is **perfectly appropriate** to fit galaxy observations.

For $\alpha = 5/2$ this is the **Plummer** profile describing the density of stars in globular clusters.

Halo Thermalization from the Distribution Function

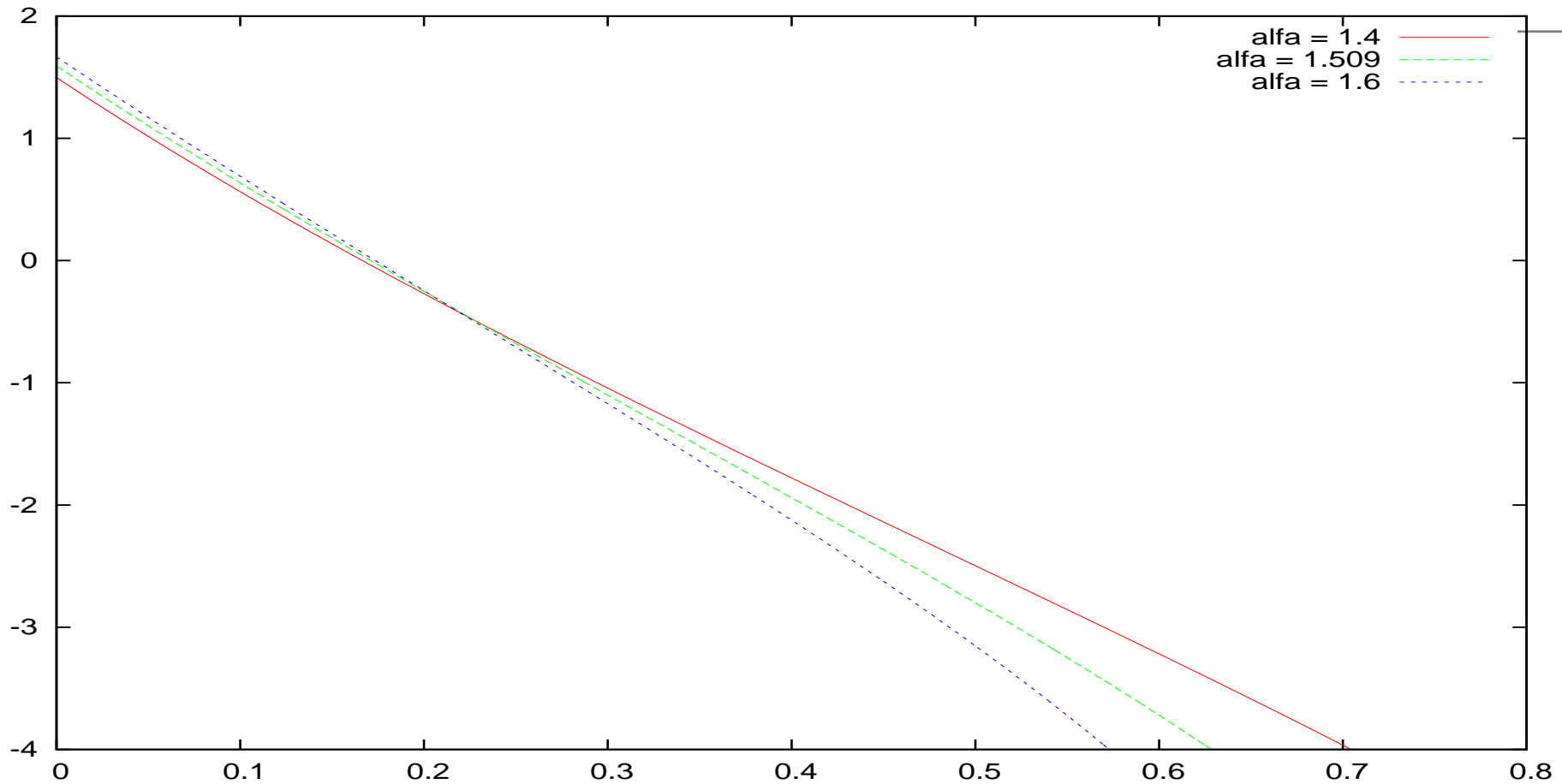
The obtained distribution function $\Psi(q)$ is **positive** for all values of q in the whole range $1 \leq \alpha \leq 2.5$. Therefore, the α -profiles are **physically meaningful**. [In general, there is no guarantee that $\Psi(q)$ from the Eddington formula will be nowhere negative.]

$\ln \mathcal{D}(-\varepsilon)$ is approximately a **linear** function of the energy $-\varepsilon$ for $\alpha \sim 1.5$ and $0 < -\varepsilon \lesssim 0.6$ which corresponds to $0 < r \lesssim 7 r_h$.

Therefore, the distribution function corresponding to α -profiles for $\alpha \sim 1.5$ is approximately a **thermal Boltzman** distribution in this interval. These are **realistic** halo galaxy density profiles.

The galaxy halos are therefore **thermalized, supporting and confirming** the Thomas-Fermi WDM approach.

Halo Thermalization



The distribution function $\ln \mathcal{D}(-\varepsilon)$ vs. the energy $-\varepsilon$.

This **linear** behaviour of $\ln \mathcal{D}(-\varepsilon)$ indicates a **Boltzman** distribution function for $0 \leq -\varepsilon \lesssim 0.7$ and $0 < r \lesssim 7 r_h$. No assumption about the DM particle nature is made here.

The Halo Dark Matter equation of state from the density profile

From the density profile we obtained the **pressure** and therefore the DM equation of state

$$\frac{P(r)}{\rho(r)} = \frac{1}{3} v^2(r) = G \Sigma_0 r_h \frac{\Pi(q)}{F(q)}$$

The local temperature $T(r)$ is given by $T(r) = \frac{1}{3} m v^2(r)$.

Hence, the dark matter obeys **locally an ideal gas** equation of state

$$P(r) = \frac{T(r)}{m} \rho(r) , T(r) \equiv m G \Sigma_0 r_h t(q) , t(q) \equiv \frac{\Pi(q)}{F(q)}$$

The temperature $T(r)$ turns to be **approximately constant** inside the halo radius $r \lesssim r_h : t(q) \simeq 1.419$.

$$T(r) = 8.238 t(q) \frac{m}{2 \text{ keV}} \sqrt{\frac{\Sigma_0 \text{ pc}^2}{120 M_\odot} \frac{M_h}{10^6 M_\odot}} \text{ m } ^\circ\text{K}$$

The temperature **grows** as the square root of the galaxy halo mass.

Circular velocity and circular temperature

The circular velocity and the circular temperature are defined by the **virial theorem**:

$$v_c^2(r) \equiv \frac{G M(r)}{r} , \quad T_c(r) \equiv \frac{1}{3} m v_c^2(r) = \frac{G m M(r)}{3 r}$$

$$T_c(r) = m G \rho_0 r_h^2 t_c(q)$$

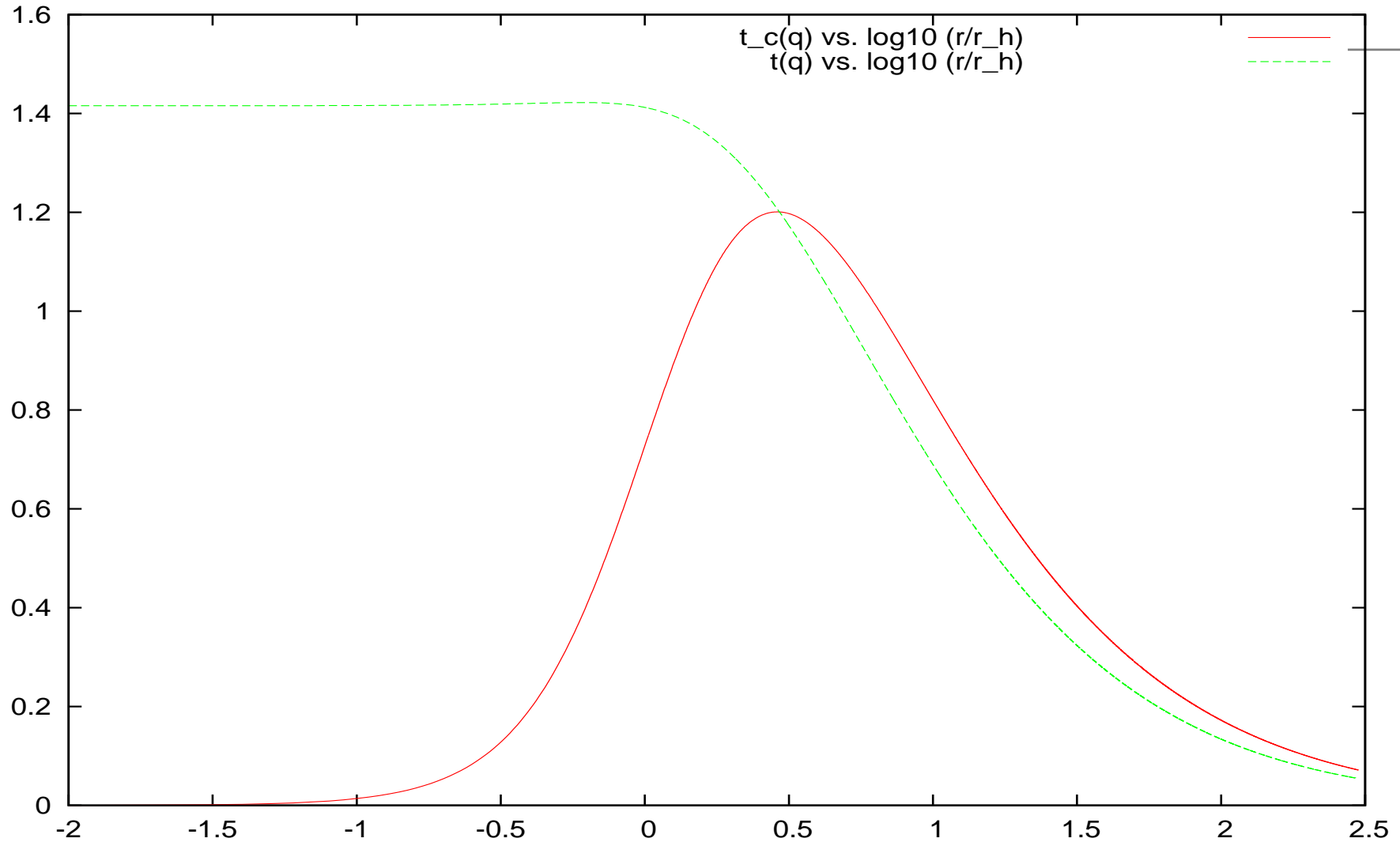
The local temperature $t(q)$ **turns to follow the decrease** of the circular temperature $t_c(q)$ for $r \gtrsim r_h$.

Conclusion:

- Halo thermalization for $r < r_h$.
- Halo virialization for $r > r_h$.

H. J. de Vega, N. G. Sanchez, arXiv:1401.0726

Thermalization and Virialization



The normalized temperature $t(r/r_h)$ and the circular temperature $t_c(r/r_h)$ vs. $\log_{10}(r/r_h)$ for $\alpha = 1.509$.

For $r \gtrsim r_h$, the local temperature **decreases slowly** with r .

Axions are ruled out as dark matter

Hot Dark Matter (eV particles or lighter) are ruled out because their free streaming length is **too large** \gtrsim Mpc and hence galaxies are not formed.

A Bose-Einstein condensate of light scalar particles **evades** this argument because of the quantum nature of the BE condensate. $r_{Jeans} \sim 5$ kpc implies $m_{axion} \sim 10^{-22}$ eV.

The phase-space density $Q = \rho/\sigma^3$ **decreases** during structure formation: $Q_{today} < Q_{primordial} \propto m^4$.

Computing $Q_{primordial}$ for a DM BE condensate we derived **lower bounds** on the DM particle mass m using the data for Q_{today} in dwarf galaxies:

$$\text{TE: } m \geq 0.155 \text{ MeV} \left(\frac{25}{g_d}\right)^{5/3}. \quad \text{Out of TE: } m \geq 14 \text{ eV} \left(\frac{25}{g_d}\right)^{5/3}$$

Axions with $m \sim 10^{-22}$ eV **are ruled out as DM candidates.**

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, PRD 77, 043518 (08). H. de Vega, N. Sanchez, arXiv:1401.1214

Sterile Neutrinos $\nu_s \simeq \nu_R + \theta \nu_L$

Sterile neutrinos ν_s : named by Bruno Pontecorvo (1968).

Singlets under all SM symmetries.

Do not interact weak, neither EM, nor strongly.

WDM ν_s can be produced from active neutrinos by mixing.

Mixing angles: $\theta \sim 10^{-3} - 10^{-4}$ (depending on the model) are appropriate **to produce enough** ν_s accounting for the observed total DM.

Smallness of θ makes sterile neutrinos **difficult** to detect.

Sterile neutrinos **can be detected** in beta decay and in electron capture (EC) when a ν_s with mass in the keV scale is produced **instead** of an active ν_e .

Beta decay: the electron spectrum is slightly modified at energies around the mass (\sim keV) of the ν_s .



The electron energy spectrum is observed.

Electron Capture and Sterile Neutrinos

Electron capture: $^{163}\text{Ho} + e^- \implies ^{163}\text{Dy}^* + \nu_e$

The nonradiative de-excitation of the Dy^* is observed and is different for ν_s in the keV range than for active ν_e .

Available energies:

$Q(^{187}\text{Re}) = 2.47 \text{ keV}$, $Q(^3\text{H}_1) = 18.6 \text{ keV}$, $Q(^{163}\text{Ho}) \simeq 2.5 \text{ keV}$.

Theoretical analysis of ν_s detection in Rhenium and Tritium beta decay: H J de V, O. Moreno, E. Moya, M. Ramón Medrano, N. Sánchez, Nucl. Phys. B866, 177 (2013).

Present experiments searching the small active neutrino mass also look for sterile neutrinos in the keV scale:

MARE (Milan, Italy), Rhenium beta decay and Holmium EC.

KATRIN (Karlsruhe, Germany), Tritium beta decay.

ECHo (Heidelberg, Germany), Holmium EC.

Project 8, (Seattle, USA) Tritium beta decay (still in project).

X-ray detection of DM sterile neutrinos

Sterile neutrinos ν_s decay into active neutrinos ν_e plus **X-rays** with a lifetime $\sim 10^{11} \times$ age of the universe.

These X-rays **may be seen** in the sky looking to galaxies !
recent review: C. R. Watson et al. JCAP, (2012).

Future observations:

- Satellite projects: Xenia (NASA), ASTRO-H (Japan).
- **CMB**: WDM decay distorts the blackbody CMB spectrum. The projected PIXIE satellite mission (A. Kogut et al.) can measure WDM sterile neutrino mass.
- PTOLEMY experiment: Princeton Tritium Observatory. Aims to detect the cosmic neutrino background and WDM (keV scale) sterile neutrinos through the electron spectrum of the Tritium beta decay induced by the **capture** of a cosmic neutrino or a WDM sterile neutrino.
- HOLMES electron capture in ^{163}Ho calorimeter G Sasso

Detection of a 3.56 keV X-ray line in galaxy clusters

E. Bulbul et al. ApJ 789, 23 (2014) reported the detection of a new X-ray line in galaxy clusters that may be originated by the decay of a 7.1 keV sterile neutrino.

Sterile neutrinos remain out of thermal equilibrium today.

From the conversion formulas, a 7.1 keV DW sterile neutrino behaves as a 1.99 keV thermal relic and a 7.1 keV SF sterile neutrino behaves as a 2.8 keV thermal relic.

Besides, a 7.1 keV SF sterile neutrino with lepton asymmetry yields similar results (K Abazajian, 2014).

WDM thermal relics with thermal mass near 2 keV provide correct small scale structure formation.

Therefore, a 7.1 keV sterile neutrino may be the dark matter particle.

Confirmation of the detection and identification of the 3.56 keV X-ray line from Astro-H is awaited for 2015 !

Direct searches of CDM particles

All direct searches of wimps look for $m \gtrsim 1$ GeV.

Past, present and future reports of signals in such wimp experiments **cannot be due to DM detection** because the DM particle mass is in the keV scale.

The inconclusive signals in such experiments should be originated by **other kinds of phenomena**.

Contradictions between supposed detection signals in DAMA, CDMS-II, CoGeNT, CRESST, XENON100.

CONCLUSION: These signals are **unrelated to DM**.

e^+ and \bar{p} excess in cosmic rays reported by Pamela and Fermi may be explained by astrophysics: P. L. Biermann et al. PRL (2009), P. Blasi, P. D. Serpico PRL (2009).

AMS02: precise measure of the positron fraction in Galactic cosmic rays (CR) (PRL 2013) that increases with energy.

Blum, Katz & Waxman (arXiv:1305.1324) show that this is consistent with positron production by the collision of high energy primary CR with the interstellar medium.

Summary: keV scale DM particles

- The phase-space density evolution since DM decoupling till today (observed in galaxies) **implies** keV scale DM particles (de Vega, Sanchez, MNRAS 2010).
- The Thomas-Fermi approach gives physical galaxy magnitudes: mass, halo radius, phase-space density and velocity dispersion **fully compatible** with observations from the largest spiral galaxies till the ultracompact dwarf galaxies for a WDM particle mass **around 2 - 3 keV**. Compact dwarf galaxies are close to a degenerate WDM Fermi gas while large galaxies are classical WDM Boltzmann gases.
- The galaxy surface density $\Sigma_0 \equiv \rho_0 r_0$ value $\Sigma_0 \simeq 120 M_\odot/pc^2 \simeq (18 \text{ MeV})^3$ is reproduced by WDM (de Vega, Salucci, Sanchez, New Astronomy, 2012). CDM simulations give 1000 times the observed value of μ_0 (Hoffman et al. ApJ 2007).

Summary: keV scale DM particles

- Alleviate the CDM **satellite** problem (Avila-Reese et al. 2000, Götz & Sommer-Larsen 2002, Markovic et al. JCAP 2011) and the CDM **voids** problem (Tikhonov et al. MNRAS 2009).
- Velocity widths in galaxies from 21cm HI surveys. ALFALFA survey clearly favours WDM over CDM. Papastergis et al. ApJ 2011, Zavala et al. ApJ 2009
- Highlights and Conclusions of the **Chalonge Meudon Workshop 2011**: Warm dark matter in the galaxies, arXiv:1109.3187, the **15th Paris Cosmology Colloquium 2011**, arXiv:1203.3562, HJdeV, NGS, and the **Chalonge Meudon Workshop 2012**, arXiv:1305.7452, PLB, HJdeV, NGS. **16th Paris Cosmology Colloquium 2012**, arXiv:1307.1847 HJdeV, M.C. Falvella, NGS.

Future Perspectives

WDM particle models must explain the baryon asymmetry of the universe. An appealing **mass** neutrino hierarchy appears:

- Active neutrino: \sim mili eV
- Light sterile neutrino: \sim eV
- Dark Matter: \sim keV
- Unstable sterile neutrino: \sim MeV....

Need WDM simulations showing substructures, galaxy formation and evolution including **quantum** dynamical evolution. **Quantum** pressure must be included !

WDM simulations should be performed matching semiclassical Hartree-Fock (Thomas-Fermi) dynamics in regions where $Q/m^4 > 0.1$ with classical evolution in regions where $Q/m^4 \ll 1$. Not easy but unavoidable!

Future Perspectives: Detection!

Sterile neutrino detection depends **upon** the particle physics model. There are sterile neutrino models where the keV sterile is **stable** and thus hard to detect.

Astronomical observation of steriles:
X-ray data from galaxy halos.

Direct detection of steriles in Lab:

Bounds on mixing angles from
Mare, Katrin, ECHo, Project 8 and PTOLEMY are expected.

For a **particle detection** a **dedicated** beta decay or electron capture experiment looks **necessary** to search sterile neutrinos with mass around 2 keV.

Calorimetric techniques seem **well suited**.

Best nuclei for study:

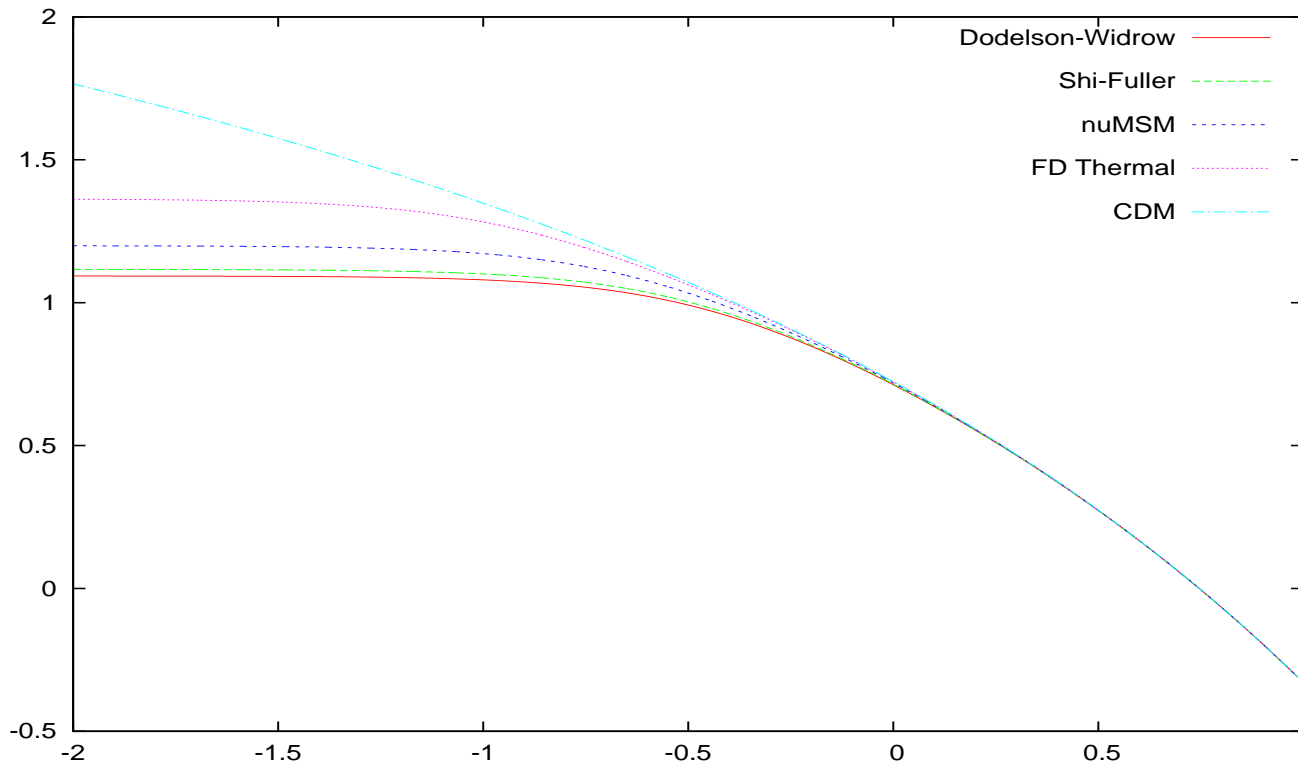
Electron capture in ^{163}Ho , beta decay in ^{187}Re and Tritium.

THANK YOU VERY MUCH
FOR YOUR ATTENTION!!

The expected overdensity

The expected overdensity within a comoving radius R in the linear regime

$$\sigma^2(R) = \int_0^\infty \frac{dk}{k} \Delta^2(k) W^2(kR) \quad , \quad W(kR) : \text{window function.}$$



$\log_{10} \sigma^2(R, z = 0)$ vs. $\log_{10}[R h/\text{Mpc}]$ for $m = 2.5$ keV in four different WDM models and in CDM.

WDM flattens and reduces $\sigma(R)$ for small scales.

Redshift dependence and Relative overdensity $D(R)$

$\sigma(M, z) = \frac{g(z)}{z+1} \sigma(M, 0)$ during the MD/ Λ dominated era.

$g(z)$: the effect of the cosmological constant. $g(z)$ is a hypergeometric function ${}_2F_1$, $g(0) = 0.76$, $g(\infty) = 1$

We introduce the **relative overdensity**: $D(R) \equiv \frac{\sigma_{WDM}^2(R, z)}{\sigma_{CDM}^2(R, z)}$
(z dependence cancels out).

Characteristic scale below which structures are suppressed in WDM compared with CDM: $R_{1/2}$ where $D(R_{1/2}) = 1/2$,

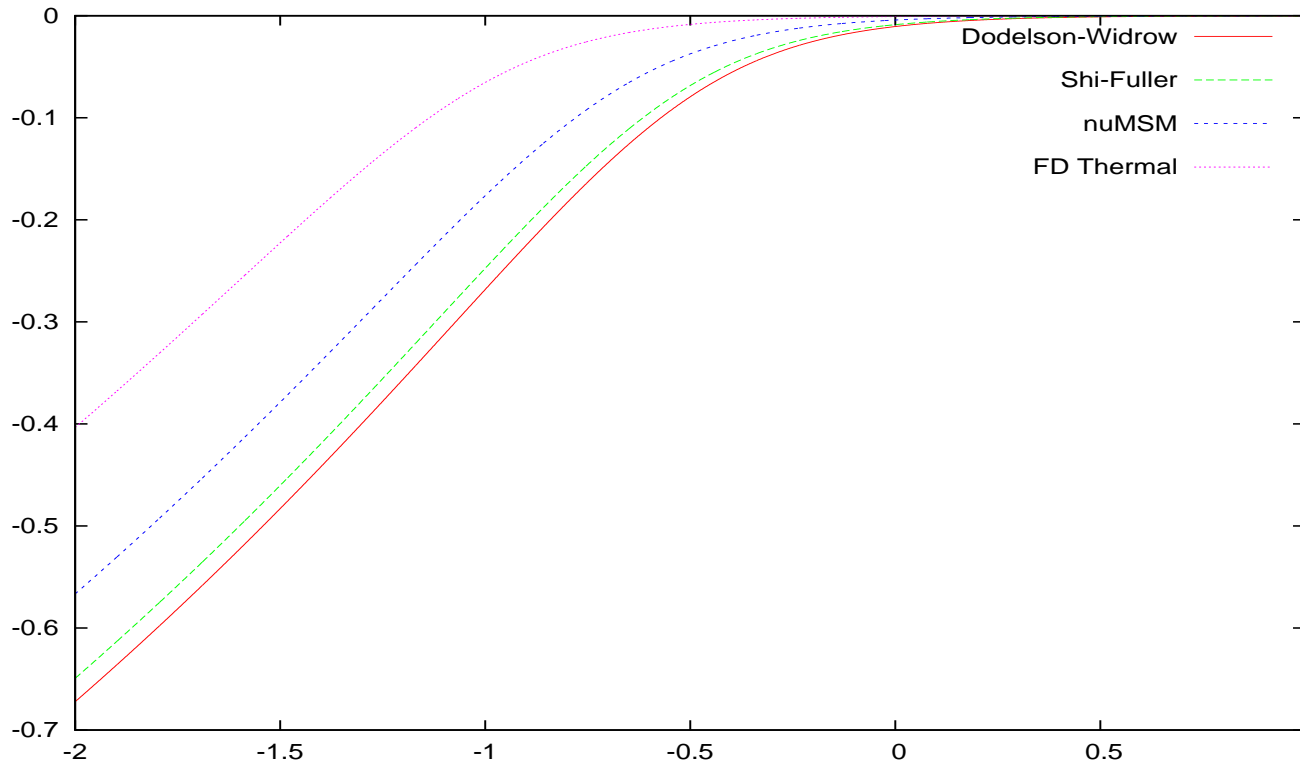
$$R_{1/2} = 73.1 \frac{\text{kpc}}{h} \left(\frac{\text{keV}}{m_{FD}} \right)^{1.45}$$

$D(R)$ can be reproduced by the simple formula:

$$D(R) = \left[1 + \left(2^{1/\beta} - 1 \right) \left(\frac{R_{1/2}}{R} \right)^\alpha \right]^{-\beta}$$

$$\alpha \simeq 2.2 \quad , \quad \beta \simeq 0.17 \quad , \quad 2^{1/\beta} - 1 \simeq 58$$

The Relative Overdensity



$\log_{10} D(R)$ vs. $\log_{10}[R h/\text{Mpc}]$.

The scales where practically **all** the CDM structures are suppressed in WDM and the scales where both CDM and WDM give the **same** structures are separated by a factor ~ 500 . This **slow** transition is due to the smallness of the exponent $\alpha\beta \simeq 0.37$.

Relative overdensity $D(R)$ and Press-Schechter approach

The number of isolated bounded structures with mass between M and $M + dM$: (Press-Schechter)

$$\frac{dN}{dM} = - \frac{2 \delta_c}{\sqrt{2 \pi} \sigma^2(M, z)} \frac{\rho_M(z)}{M^2} \frac{d\sigma(M, z)}{d \ln M} e^{-\delta_c^2 / [2 \sigma^2(M, z)]}, \quad \delta_c = 1.686 \dots$$

$\sigma(M, z)$ is **constant** for WDM for small scales: small objects formation is **suppressed** in WDM in comparison with CDM.

Computing dN/dM in WDM shows that small scale structure **suppression** with respect to CDM **increases** with z .
H. J. de Vega, N. G. Sanchez, arXiv:1308.1109, Phys.Rev.D

It is therefore **important** to compare the observations at $z > 1$ with the theoretical predictions:

Menci et al. ApJ 2013, Nierenberg et al. ApJ 2013,

Conclusion: WDM **reproduces** the observed small scale structures **better** than CDM for redshifts up to eight where observations are available.

Effective Theory of Inflation (ETI) confirmed by Planck

Quantity	ETI Prediction	Planck 2013
Spectral index $1 - n_s$	order $1/N = 0.02$	0.04
Running $dn_s/d\ln k$	order $1/N^2 = 0.0004$	< 0.01
Non-Gaussianity f_{NL}	order $1/N = 0.02$	< 6
	ETI + WMAP+LSS	
tensor/scalar ratio r	$r > 0.02$	< 0.11 see BICEP
inflaton potential curvature $V''(0)$	$V''(0) < 0$	$V''(0) < 0$

ETI + WMAP+LSS means the MCMC analysis combining the ETI with WMAP and LSS data. Such analysis calls for an inflaton potential with negative curvature at horizon exit. **The double well potential** is favoured (new inflation).
D. Boyanovsky, C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0901.0549, IJMPA 24, 3669-3864 (2009).

Sterile neutrino models

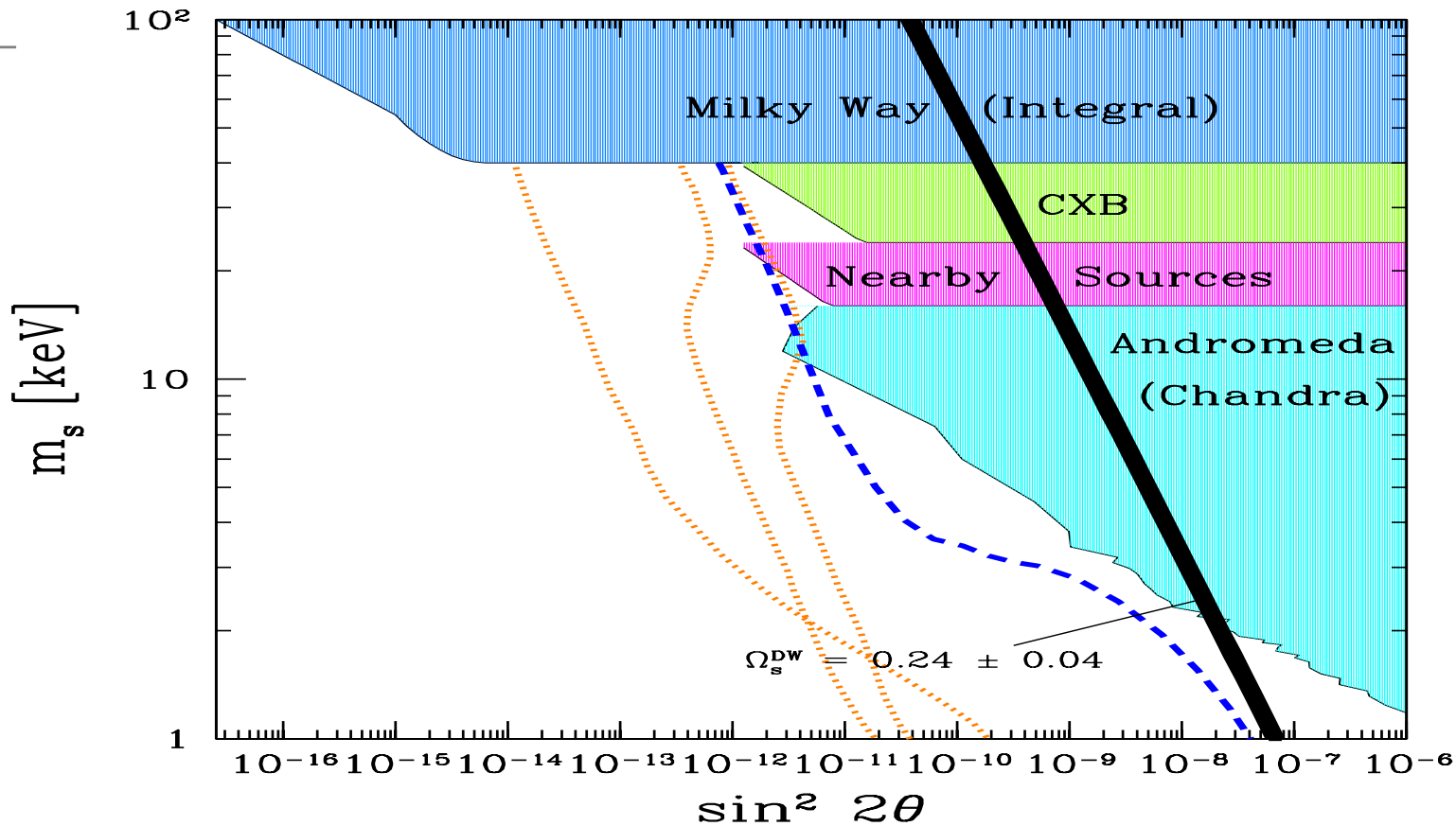
- DW: Dodelson-Widrow model (1994) sterile neutrinos produced by non-resonant mixing from active neutrinos.
- Shi-Fuller model (1998) sterile neutrinos produced by resonant mixing from active neutrinos.
- ν MSM model (2005) sterile neutrinos produced by a Yukawa coupling from a real scalar χ .
- Models based on: Froggatt-Nielsen mechanism, flavor symmetries, see-saw mechanisms and several variations of it, left-right symmetries and others. Review by A Merle (2013).

WDM particles in the first 3 models behave primordially just as if their masses were different (FD = thermal fermions):

$$\frac{m_{DW}}{\text{keV}} \simeq 2.85 \left(\frac{m_{FD}}{\text{keV}}\right)^{\frac{4}{3}}, \quad m_{SF} \simeq 2.55 m_{FD}, \quad m_{\nu\text{MSM}} \simeq 1.9 m_{FD}.$$

H J de Vega, N Sanchez, Warm Dark Matter cosmological fluctuations, Phys. Rev. D85, 043516 and 043517 (2012).

Constraints on the sterile neutrino mass and mixing angle



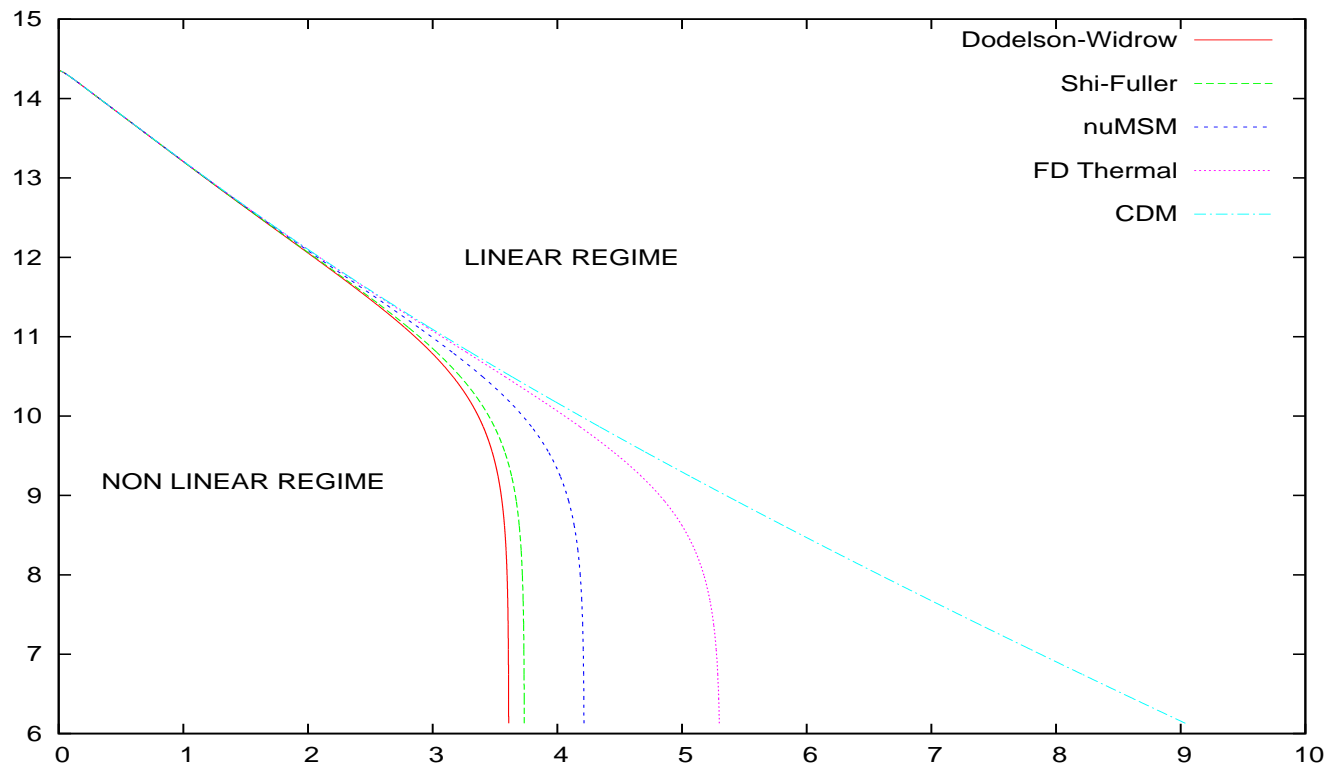
Dashed = Shi-Fuller model. Dotted = Dodelson-Widrow for fermion asymmetry $L = 0.1, 0.01$ and 0.003 .

Allowed sterile neutrino region in the right lower corner.
Main difficulty: to distinguish the sterile neutrino decay X-ray from narrow X-ray lines emitted by hot ions as Fe.

Linear and non-linear regimes in z and R

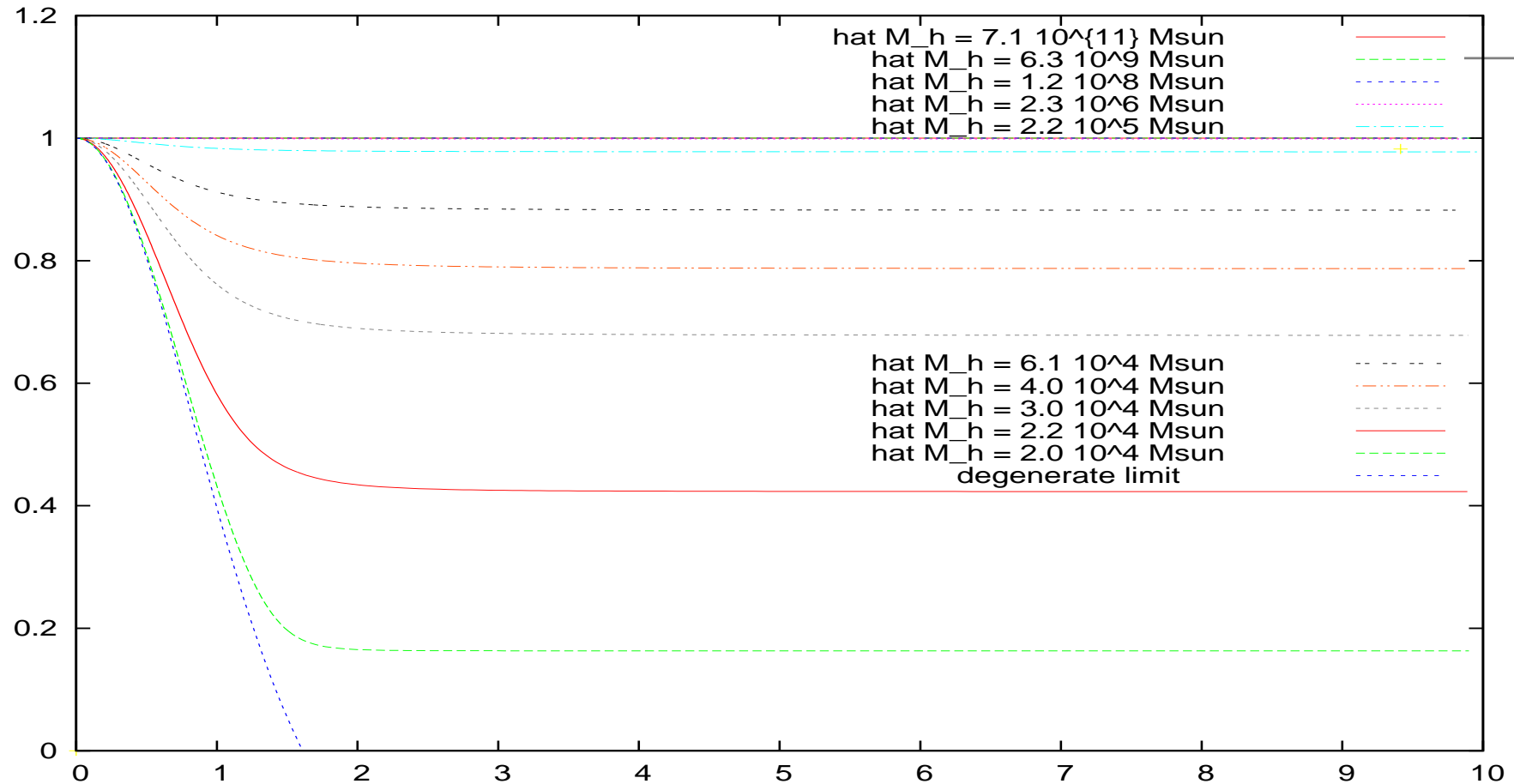
$\sigma^2(R, z) \sim 1$: borderline between linear and non-linear regimes. Objects (galaxies) of scale R and mass $\sim R^3$ start to form when this scale becomes non-linear.

Smaller objects can form **earlier**.



$\sigma^2(M, z) = 1$ in the $z, \log[h M/M_\odot]$ plane for $m = 2.5$ keV in four different WDM models and in CDM.

Velocity dispersion profiles $\sigma^2(r)/\sigma^2(0)$ vs. $x = r/r_h$



ALL velocity profiles in the **classical diluted regime**

$\hat{M}_h > 2.3 \cdot 10^6 M_\odot$ fall into a **constant universal** value.

In the **quantum regime**: $1.6 \cdot 10^6 M_\odot > \hat{M}_h > \hat{M}_{h,min}$ the profiles **are not universal** and do depend on \hat{M}_h and x .

WDM Primordial Power Spectrum

The WDM Primordial Power Spectrum is obtained solving the linear Boltzmann-Vlasov equations.

We define the transfer function ratio $T^2(k) \equiv \frac{\Delta_{wdm}^2(k)}{\Delta_{cdm}^2(k)}$

Reproduced by the analytic formula $T^2(k) = \left[1 + \left(\frac{k}{\kappa}\right)^a\right]^{-b}$

a and b are independent of the WDM particle mass m .

κ scales with m . In our best fit:

$$a = 2.304, \quad b = 4.478, \quad \kappa = 14.6 (m_{FD}/\text{keV})^{1.12} h/\text{Mpc}$$

At the wavenumber $k_{1/2}$: $T^2(k_{1/2}) = 1/2$ and

$$k_{1/2} = 6.72 (m_{FD}/\text{keV})^{1.12} h/\text{Mpc}$$

The characteristic length scale is

$$l_{1/2} = 1/k_{1/2} = 207 \text{ kpc} (\text{keV}/m_{FD})^{1.12}$$

This scale reproduces the sizes of the DM galaxy cores when the WDM mass is in the **keV scale** !!