Structure Formation and Evolution in Warm Dark Matter Cosmologies - Numerical Simulations -

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In memoriam Hector de Vega

Motivation

- $\Lambda$CDM fails to explain observed properties of galaxies
- Missing Satellites Problem
- Cores vs Cusps
- Mergers vs No Mergers
- Pure Disk Galaxies
- Where is the WIMP?
Simulating the Warm Dark Matter - The Challenges

- How to treat the particles?
- How to cut the power spectrum?
- What about velocities? The impact of velocity dispersion.
- How to compare WDM sims with CDM sims and observations
- Resolution and softening
- Fragmentation
- Structure formation and its hidden treasures
- Halo internal structure
- The trustworthy factor and the catch 22
Speed Matters

Bode, Ostriker & Turok 2001

Assumptions: entropy production & negligible chemical potential

\[ \frac{n_X}{n_\gamma} = \left( \frac{43/4}{g_{\text{dec}}} \right) \left( \frac{4}{11} \right) \frac{g_X}{2} \]

\[ \Omega_X h^2 \approx \frac{115}{g_{\text{dec}}} \frac{g_X}{1.5 \text{ keV}} \frac{m_X}{g_{\text{dec}}} \]

\[ v_0(z) = 0.012 \left( \frac{\Omega_X}{0.3} \right)^{1/3} \left( \frac{h}{0.65} \right)^{2/3} \left( \frac{1.5}{g_X} \right)^{1/3} \left( \frac{\text{keV}}{m_X} \right)^{4/3} \text{ km s}^{-1} \]

\[ (\exp(v/v_0) + 1)^{-1} \]

\[ v_0(z) \approx 0.12 \left( \frac{1}{g_{\text{dec}}} \right)^{1/3} \frac{\text{keV}}{m_X} \text{ km s}^{-1} \]

\[ g_{\text{dec}} = 1000 \left( \frac{g_X}{1.5} \right)^{1/3} \]
Different Assumptions:
• number conservation
• non-entropy production
• quantum pressure
• thermalization caused by an exchange potential

\[ f(p) = \frac{g}{(2\pi\hbar)^3} \frac{1}{\exp((\epsilon - \mu)/kT) \pm 1} \]

\[ n(T, \mu) = \frac{4\pi g}{\hbar^3} \int_0^\infty \frac{p^2 \exp((\epsilon - \mu)/kT) \pm 1}{\exp((\epsilon - \mu)/kT) \pm 1} \, dp \]

\[ e(T, \mu) = \frac{4\pi g}{\hbar^3} \int_0^\infty \frac{p^2 \epsilon \exp((\epsilon - \mu)/kT) \pm 1}{\exp((\epsilon - \mu)/kT) \pm 1} \, dp \]

\[ P(T, \mu) = \frac{4\pi g}{\hbar^3} \int_0^\infty \frac{p^2 \exp((\epsilon - \mu)/kT) \pm 1}{\exp((\epsilon - \mu)/kT) \pm 1} \frac{1}{3 \epsilon + mc^2} \, dp \]

\[ v = 0.2226 \left( \frac{n}{115 \text{ cm}^{-3} \cdot g} \right)^{1/3} \left( \frac{mc^2}{\text{keV}} \right)^{-1} \text{ km s}^{-1} \]
CAVEAT EMPTOR

\[ \frac{v_0(z)}{1 + z} = 0.012 \left( \frac{\Omega_X}{0.3} \right)^{\frac{1}{3}} \left( \frac{h}{0.65} \right)^{\frac{2}{3}} \left( \frac{1.5}{g_X} \right)^{\frac{1}{3}} \left( \frac{keV}{m_X} \right)^{\frac{4}{3}} \text{ km s}^{-1} \]

\[ \Omega_X h^2 \approx \frac{115}{g_{dec}} \frac{g_X m_X}{1.5 \text{ keV}} \]

‘valid’ only for the case in which the full dark matter content is made up by only one type of particles

\[ v = 0.2226 \left( \frac{n}{115 \text{ cm}^{-3}} \right)^{1/3} \left( \frac{mc^2}{\text{keV}} \right)^{-1} \text{ km s}^{-1} \]

<table>
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<th>Pierpaoli et al.</th>
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Bode et al. 2001

Paduroiu et al. 2015
Simulating the WDM…

\[ \frac{v_0(z)}{1+z} = 0.012 \left( \frac{\Omega_X}{0.3} \right)^{1/3} \left( \frac{h}{0.65} \right)^{2/3} \left( \frac{1.5}{g_X} \right)^{1/3} \left( \frac{keV}{m_X} \right)^{1/3} \text{ km s}^{-1} \]

\[ T^2(k) = \frac{P^{WDM}_{\perp}}{P_{\perp CDM}} = \left[ 1 + (\alpha k)^{2\nu} \right]^{-10/\nu} \]

\[ \alpha = 0.049 \cdot \left( \frac{m_X}{1\text{ keV}} \right)^{-1.11} \cdot \left( \frac{\Omega_\nu}{0.25} \right)^{0.11} \cdot \left( \frac{h}{0.7} \right)^{1.22} h^{-1}\text{Mpc}. \]

\[ k_s \approx \left( \frac{0.3}{\Omega_X} \right)^{0.15} \left( \frac{m_X}{keV} \right)^{1.15} \text{Mpc}^{-1} \]

Bode, Turok, and Ostriker (2001)

Viel et al.2005

Smith & Markovic 2011
Structure formation in warm dark matter cosmologies
Top-Bottom Upside-Down

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24 June 2015

ABSTRACT
The damping on the fluctuation spectrum and the presence of thermal velocities as properties of warm dark matter particles like sterile neutrinos imprint a distinct signature found from the structure formation mechanisms to the internal structures of halos. Using warm dark matter simulations we explore these effects on the structure formation for different particle energies and we find that the formation of structure is more complex than originally assumed, a combination of top-down collapse and hierarchical (bottom-up) clustering on multiple scales. The degree on which one scenario is more prominent with respect to the other depends globally on the energy of the particle and locally on the morphology and architecture of the analyzed region. The presence of shells and caustics in warm dark matter halos is another important effect seen in simulations. Furthermore we discuss the impact of thermal velocities on the structure formation from theoretical considerations as well as from the analysis of the simulations. We re-examine the assumptions considered when estimating the velocity dispersion for warm dark matter particles that have been adopted in previous works for more than a decade and we give an independent estimation for the velocities. We identify some inconsistencies in previous published results. The relation between the warm dark matter particle mass and its corresponding velocity dispersion is strongly model dependent, hence the constraints on particle mass from simulation results are weak. Finally, we review the technical difficulties that arise in warm dark matter simulations along with possible improvements of the methods.
Mildly non-linear regions at \( z=3 \) in CDM and WDM
i.e. overdensities between 1 and 5 w.r.t. mean
Virialised regions at $z=3$ in CDM and WDM
i.e. overdensities higher than 100 w.r.t. mean
Thermal Velocities

No Thermal Velocities
Liang Gao & Tom Theuns, Science sept 2007
Simulations details: \(2.72 \times 10^5\) \(M_\odot\) / particle
355 pc spline gravitational softening
WMAP7 cosmological parameters
\(z=100\) initial redshift

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The movies...

https://www.youtube.com/playlist?list=PLnGS4wkStJ1aqi3M9hTDaUzuZ-vs-Qg6i

or on demand ...
Hybrid mechanism of structure formation

• During the early stages one sees the formation of well contoured filaments.

• In the higher density regions, usually situated at the intersection of such filaments, the first halos are formed through gravitational collapse. These halos continue growing into larger ones by accreting particles from the disrupted filaments.

• In medium density regions, haloes show a hierarchical formation trend. Small haloes collapse first and then merge into bigger haloes.

• In less dense regions, the ones isolated by voids and have a very slow evolution, we have observed filaments that collapse very late. The top down formed halo survives without any mergers until redshift zero.

• Finally there is the more complex scenario in which we observe large haloes formed earlier which merge into clusters.
Assumptions in determining the core radius:

- Isothermal spheres
- Liouville - Phase space density (PSD) is conserved
- Pauli exclusion principle
- PSD constant as mixing occurs
- Velocity dispersion in central halo = constant
- Density profile in central halo = constant
Constraints on the core radius of Fornax as a function of the central phase-space density and maximum circular velocity derived from the velocity dispersion profile.

Strigari et al 2006
Cores in warm dark matter haloes: a *Catch 22* problem

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11 May 2012

ABSTRACT

The free streaming of warm dark matter particles dampens the fluctuation spectrum, flattens the mass function of haloes and sets a fine-grained phase density limit for dark matter structures. The phase space density limit is expected to imprint a constant density core at the halo center on the contrary to what happens for cold dark matter. We explore these effects using high resolution simulations of structure formation in different warm dark matter scenarios. We find that the size of the core we obtain in simulated haloes is in good agreement with theoretical expectations based on Liouville’s theorem. However, our simulations show that in order to create a significant core, \((r_c \sim 1 \text{ kpc})\), in a dwarf galaxy \((M \sim 10^{10} M_{\odot})\), a thermal candidate with a mass as low as 0.1 keV is required. This would fully prevent the formation of the dwarf galaxy in the first place. For candidates satisfying large-scale structure constraints \((m_\psi \text{ larger than } \approx 1 - 2 \text{ keV})\) the expected size of the core is of the order of 10 (20) pc for a dark matter halo with a mass of \(10^{10} \text{ (}10^{9}) M_{\odot}\). We conclude that “standard” warm dark matter is not viable solution for explaining the presence of cored density profiles in low mass galaxies.

Key words:
Dark matter: N-body simulations – galaxies, haloes.
Figure 4. Phase space density profile for the CDM, WDM5 and WDM3 models at z = 0.

\[ Q = \rho / \sigma^3 \left( M_\odot / pc^3 \right) (km/s)^{-3} \]

<table>
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<tr>
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<td>rvir (kpc)</td>
<td>0.82</td>
<td>0.82</td>
<td>0.82</td>
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Figure 5. The spherical average density profiles for CDM.

\[ \rho / \rho_{cr} \]
How to cook a big core...

Simulations of CDM+gas+AGN feedback+star formation fine-tuned

If one tunes it for solving the missing satellites problem loses the core and vice-versa

Another Catch 22

Unfortunately, proper KeV simulations+baryons haven’t been performed yet.

The formation of disc galaxies in high resolution moving-mesh cosmological simulations

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<<There is no evidence for any dark matter core formation in our simulations, even so they include repeated baryonic outflows by supernova-driven winds and black hole quasar feedback >>
Abstract

We derive the main physical galaxy properties: mass, halo radius, phase space density and velocity dispersion from a semiclassical gravitational approach in which fermionic WDM is treated quantum mechanically. They turn out to be fully compatible with observations. The Pauli Principle implies for the fermionic DM phase-space density \( Q(\vec{r}) = \rho(\vec{r})/\sigma^2(\vec{r}) \) the quantum bound \( Q(\vec{r}) \leq K m^4/h^3 \), where \( m \) is the DM particle mass, \( \sigma(\vec{r}) \) is the DM velocity dispersion and \( K \) is a pure number of order one which we estimate. Cusped profiles from N-body galaxy simulations produce a divergent \( Q(r) \) at \( r = 0 \) violating this quantum bound. The combination of this quantum bound with the behaviour of \( Q(r) \) from simulations, the virial theorem and galaxy observational data on \( Q \) implies lower bounds on the halo radius and a minimal distance \( r_{\text{min}} \) from the centre at which classical galaxy dynamics for DM fermions breaks down. For WDM, \( r_{\text{min}} \) turns to be in the parsec scale. For cold dark matter (CDM), \( r_{\text{min}} \) is between dozens of kilometers and a few meters, astronomically compatible with zero. For hot dark matter (HDM), \( r_{\text{min}} \) is from the kpc to the Mpc. In summary, this quantum bound rules out the presence of galaxy cusps for fermionic WDM, in agreement with astronomical observations, which show that the DM halos are cored. We show that compact dwarf galaxies are natural quantum macroscopic objects supported against gravity by the fermionic WDM quantum pressure (quantum degenerate fermions) with a minimal galaxy mass and minimal velocity dispersion. Quantum mechanical calculations which fulfil the Pauli principle become necessary to compute galaxy structures at kpc scales and below. Classical N-body simulations are not valid at scales below \( r_{\text{min}} \). We apply the Thomas-Fermi semiclassical approach to fermionic WDM galaxies, we resolve it numerically and find the physical galaxy magnitudes: mass, halo radius, phase-space density, velocity...
“Low” resolution

“High” resolution

Large Softening
CONCLUSIONS and COMPLICATIONS

- Formation of haloes in WDM models differs from CDM.
  Hybrid mechanism - Top-Down & Hierarchical; long distance & nearest neighbours
  Looking at high redshift galaxies for T-D memory.

- The exact recipe for structure formation seems to depend only on the morphology and architecture of the environment

- ☼ Quantum Pressure; Baryons and their physics

- Warm dark matter haloes contain visible caustics and shells.

- The finite initial fine grained PSD is also a maximum of coarse grained PSD.
  The turn over in PSD results in constant density core with characteristic size.

- Spurious fragmentation below the free streaming scale hard to overcome – in case of infinite resolution a filament collapses into a two dimensional line
  $$\text{Adaptive softening?}$$

- The velocity dispersion is crucial in describing warm dark matter particles!
  There is no universal one to one correspondence between mass and thermal velocity
  Mass constraints on the particle’s mass, are not accurate