

A simulation of the cosmic web, showing a complex network of blue filaments and nodes against a dark background, representing the large-scale structure of the universe.

Warm Dark Matter Cosmologies in Theory and Simulations

From Collapse to Caustics and Cores

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20th Paris Cosmology Colloquium 2016

A wide-angle photograph of a majestic mountain range covered in snow, with a clear sky and some light clouds, suggesting a high-altitude or alpine environment.

CDM vs WDM – Battle Galactica

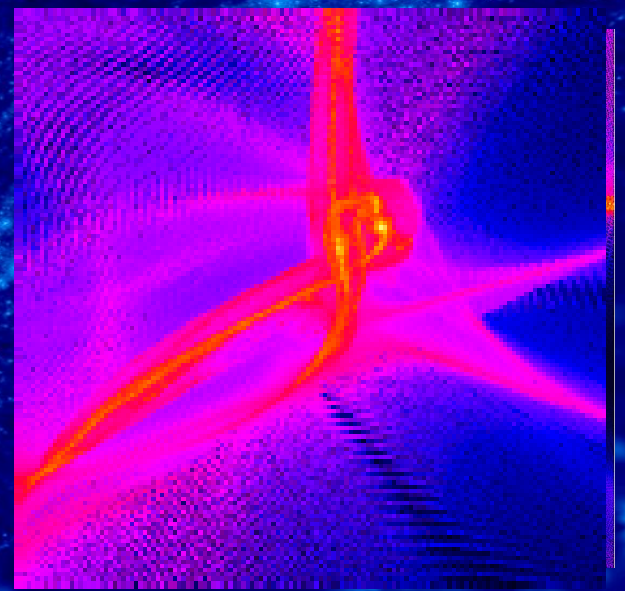
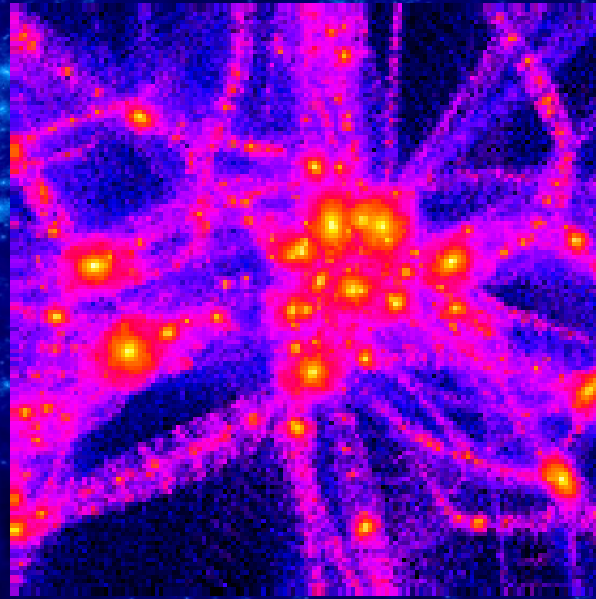
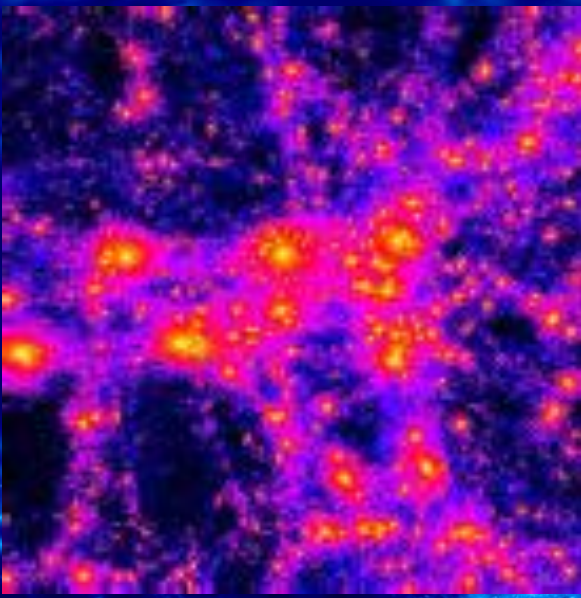
- Λ CDM fails to explain observed properties of galaxies
 - Missing Satellites Problem
 - Cores vs Cusps
 - Pure Disk Galaxies
 - Mergers vs No Mergers
 - Dwarf population in voids
 - Where is the WIMP?
- Connection?

CDM

Structure Forming Region

WDM - 200 eV

HDM - 50 eV



Simulating the Warm Dark Matter - The Challenges

- How to treat the particles?
- How to cut the power spectrum?
- What about velocities ? The impact of velocity dispersion.
- How to compare WDM sims with CDM sims and observations
- Resolution and softening
- Fragmentation / Segregation
- Structure formation and its hidden treasures
- Halo internal structure
- The trustworthy factor and the catch 22

Simulating the WDM...

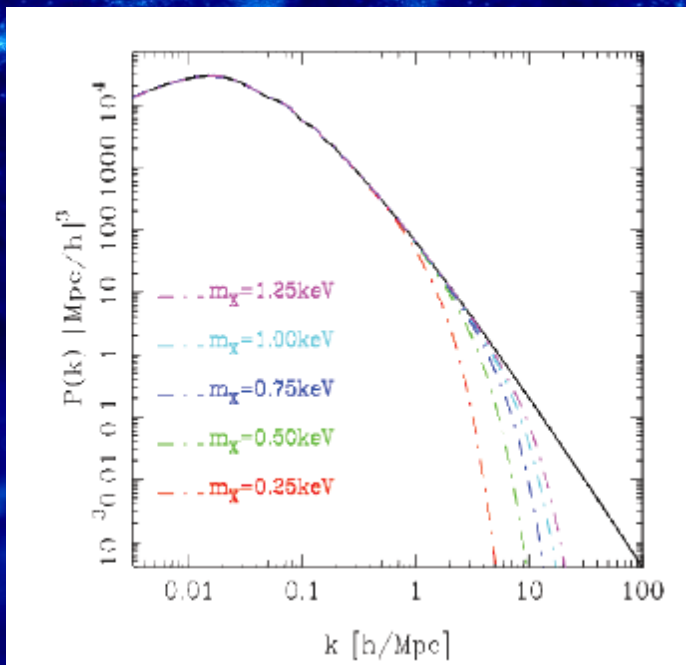
$$\frac{v_0(z)}{1+z} = .012 \left(\frac{\Omega_X}{0.3} \right)^{\frac{1}{3}} \left(\frac{h}{0.65} \right)^{\frac{2}{3}} \left(\frac{1.5}{g_X} \right)^{\frac{1}{3}} \left(\frac{\text{keV}}{m_X} \right)^{\frac{4}{3}} \text{ km s}^{-1}$$

Bode, Turok, and Ostriker 2001

$$T^2(k) = \frac{P^{WDM}}{P^{CDM}} = [1 + (\alpha k)^{2\nu}]^{-10/\nu}$$

$$\alpha = 0.049 \cdot \left(\frac{m_x}{1\text{keV}} \right)^{-1.11} \cdot \left(\frac{\Omega_\nu}{0.25} \right)^{0.11} \cdot \left(\frac{h}{0.7} \right)^{1.22} h^{-1} \text{ Mpc.}$$

Viel et al. 2005



$$k_S \approx \left(\frac{0.3}{\Omega_X} \right)^{0.15} \left(\frac{m_X}{\text{keV}} \right)^{1.15} \text{ Mpc}^{-1}$$

Smith & Markovic 2011

Speed Matters

Bode, Ostriker & Turok 2001

Assumptions: entropy production & negligible chemical potential

$$\frac{n_X}{n_\gamma} = \left(\frac{43/4}{g_{\text{dec}}} \right) \left(\frac{4}{11} \right) \frac{g_X}{2}$$

$$\Omega_X h^2 \approx \frac{115}{g_{\text{dec}}} \frac{g_X}{1.5} \frac{m_X}{\text{keV}}$$

$$\frac{v_0(z)}{1+z} = .012 \left(\frac{\Omega_X}{0.3} \right)^{1/3} \left(\frac{h}{0.65} \right)^{2/3} \left(\frac{1.5}{g_X} \right)^{1/3} \left(\frac{\text{keV}}{m_X} \right)^{4/3} \text{ km s}^{-1}$$

$$(\exp(v/v_0) + 1)^{-1}$$

$$\frac{v_0(z)}{1+z} \approx 0.12 \left(\frac{1}{g_{\text{dec}}} \right)^{1/3} \frac{\text{keV}}{m_X} \text{ km s}^{-1}$$

$$g_{\text{dec}} = 1000 (g_X/1.5)^{1/3}$$

For 1 keV !

Different Assumptions:

- number conservation
- non-entropy production
- quantum pressure
- thermalization caused by an exchange potential

Paduroiu et al. 2015

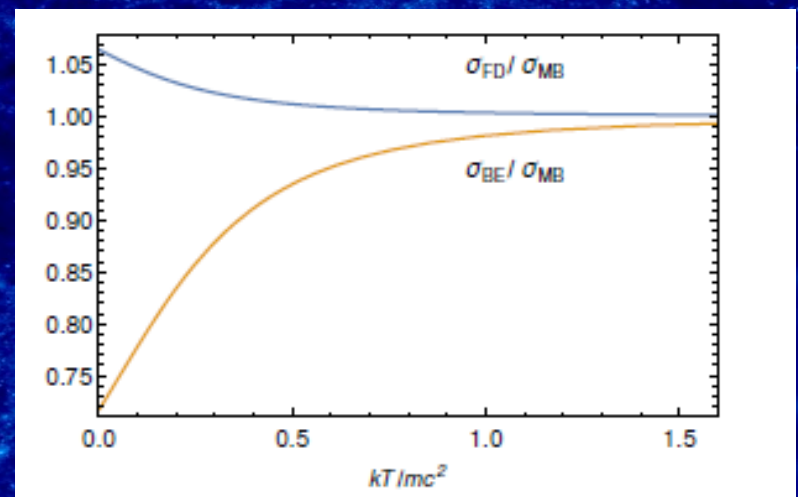
$$f(\mathbf{p}) = \frac{g}{(2\pi\hbar)^3} \frac{1}{\exp((\epsilon - \mu)/kT) \pm 1}$$

$$n(T, \mu) = \frac{4\pi g}{h^3} \int_0^\infty \frac{p^2}{\exp((\epsilon - \mu)/kT) \pm 1} dp$$

$$e(T, \mu) = \frac{4\pi g}{h^3} \int_0^\infty \frac{p^2 \epsilon}{\exp((\epsilon - \mu)/kT) \pm 1} dp$$

$$P(T, \mu) = \frac{4\pi g}{h^3} \int_0^\infty \frac{p^2}{\exp((\epsilon - \mu)/kT) \pm 1} \frac{1}{3} \frac{c^2 p^2}{\epsilon + mc^2} dp$$

$$v = 0.2226 \left(\frac{n}{115 \text{ cm}^{-3}} \frac{1}{g} \right)^{1/3} \left(\frac{mc^2}{\text{keV}} \right)^{-1} \text{ km s}^{-1}$$



CAVEAT EMPTOR

$$\frac{v_0(z)}{1+z} = .012 \left(\frac{\Omega_X}{0.3} \right)^{\frac{1}{3}} \left(\frac{h}{0.65} \right)^{\frac{2}{3}} \left(\frac{1.5}{g_X} \right)^{\frac{1}{3}} \left(\frac{\text{keV}}{m_X} \right)^{\frac{4}{3}} \text{ km s}^{-1}$$

Bode et al. 2001

‘valid’ only for the case in which the total dark matter content is made up by only one type of particles

$$\Omega_X h^2 \approx \frac{115}{g_{\text{dec}}} \frac{g_X}{1.5 \text{ keV}} \frac{m_X}{\text{keV}}$$

$$v = 0.2226 \left(\frac{n}{115 \text{ cm}^{-3}} \frac{1}{g} \right)^{1/3} \left(\frac{mc^2}{\text{keV}} \right)^{-1} \text{ km s}^{-1}$$

Paduroiu et al. 2015

Mass	Bode et al. $v_0 \times 3.571$	Pierpaoli et al.	Paduroiu et al.	Boyarsky et al. TR	Boyarsky et al. NRP
keV/c ²	km/s	km/s	km/s	km/s	km/s
0.2	0.366	0.4032	1.113	0.29	0.785
1.0	0.0429	0.0225	0.223	0.034	0.157
3.5	0.00806	0.0230	0.0636	0.0064	0.00448

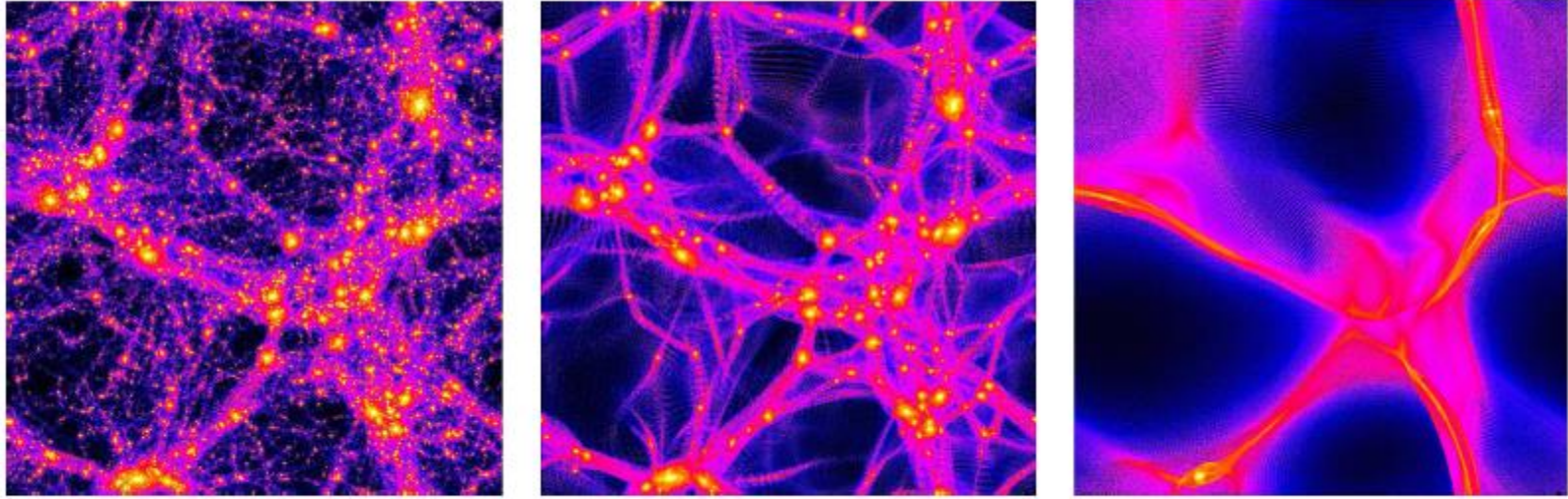
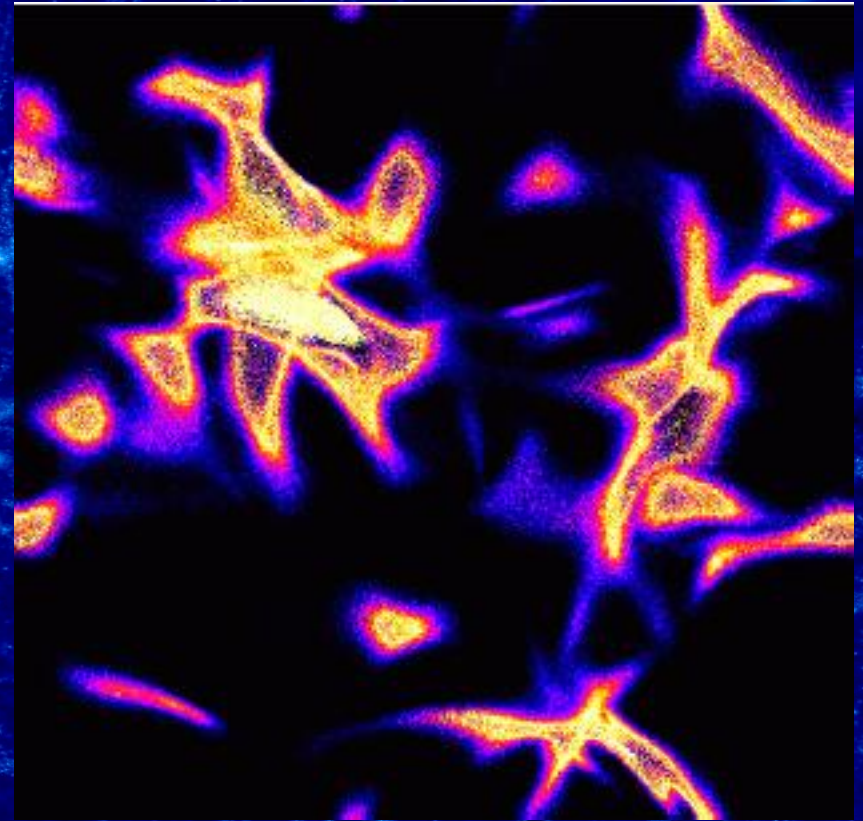
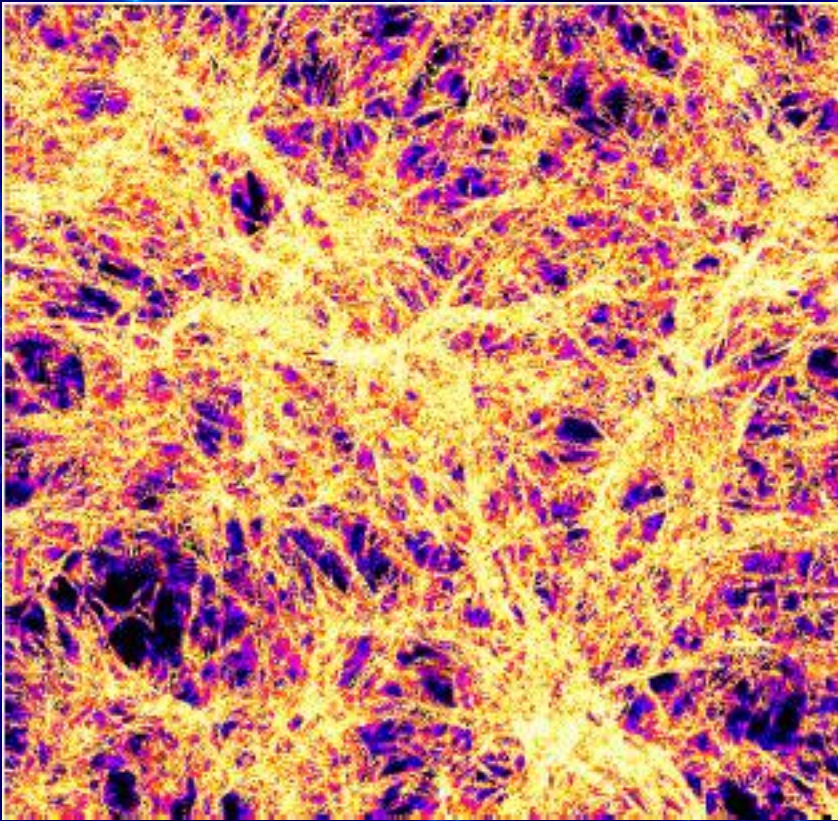


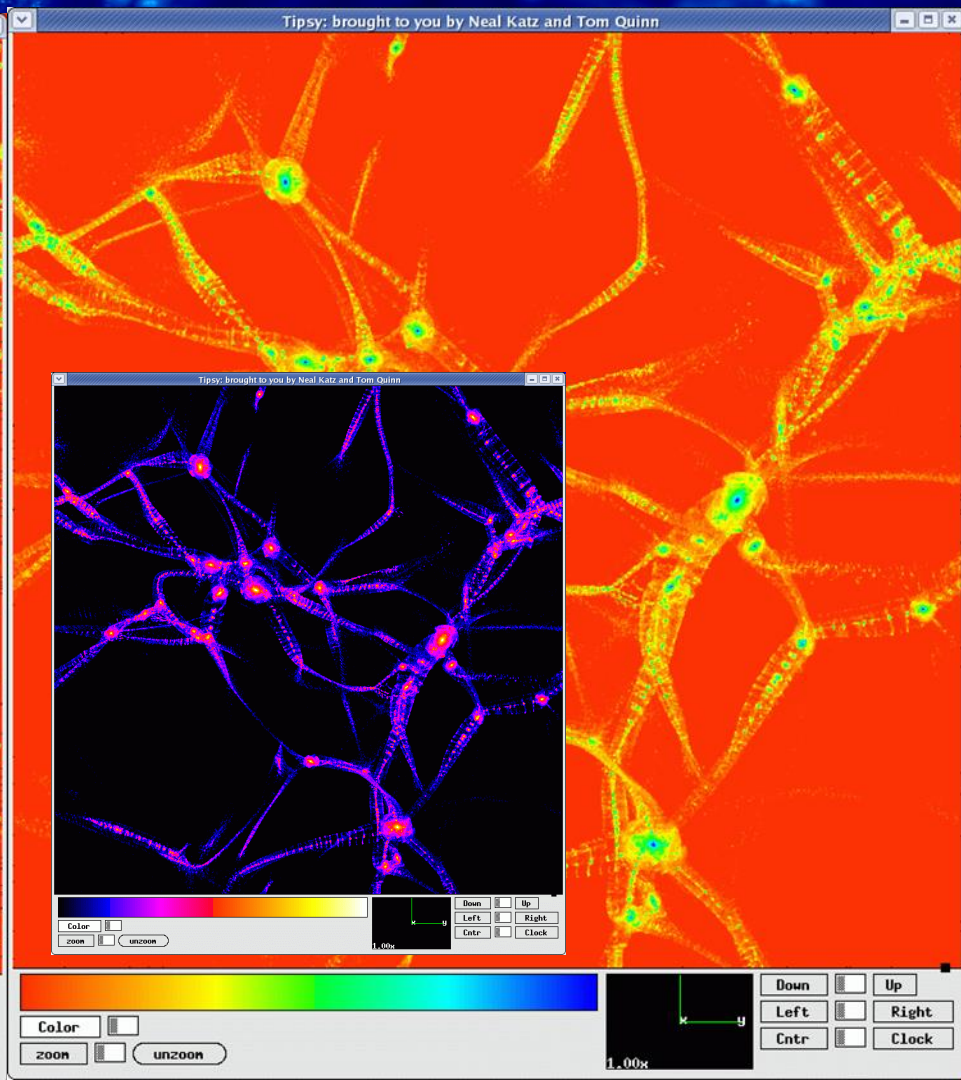
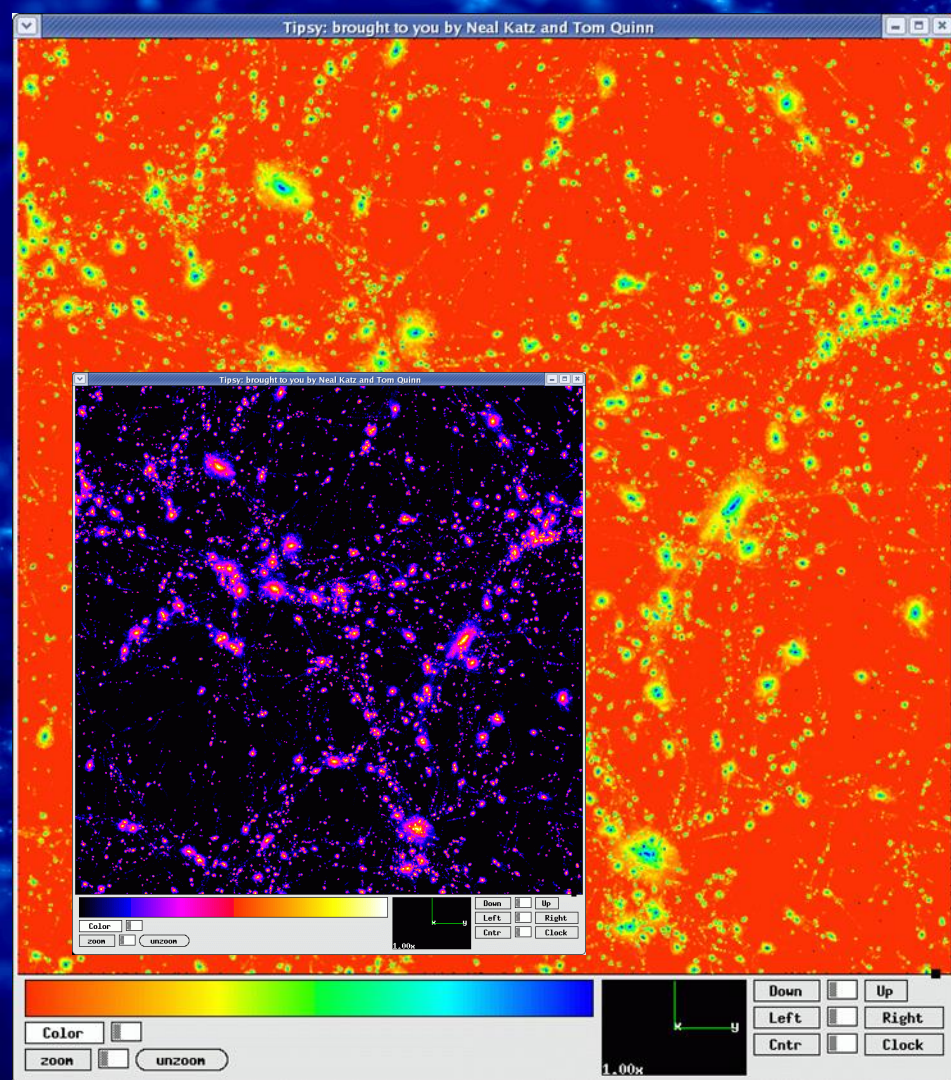
Figure 1. Three snapshots of different simulations at redshift $z = 0$. CDM, WDM3 and WDM4 are shown from left to right.

Table 1. Details of the simulations

Label	particle mass	velocities	box size	no.of particles	softening (r_{200})	halo mass	r_{200} (kpc)	$N(< r_{200})$
CDM	-	no	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	160	3.6×10^6
WDM1	200 eV	no	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	140	2.7×10^6
WDM2	200 eV	100 eV	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	140	1.7×10^6
WDM3	200 eV	20 eV	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	132	2.7×10^6
WDM4	50 eV	no	40 Mpc	160^3	2.6×10^{-3}	-	-	-
WDM5	200 eV	no	42.51 Mpc	300^3	0.66×10^{-3}	10^{13}	425	18.67×10^6
WDM6	200 eV	200 eV	42.51 Mpc	300^3	0.66×10^{-3}	10^{13}	425	18.66×10^6



Mildly non-linear regions at $z=3$ in CDM and WDM
i.e. overdensities between 1 and 5 w.r.t. mean

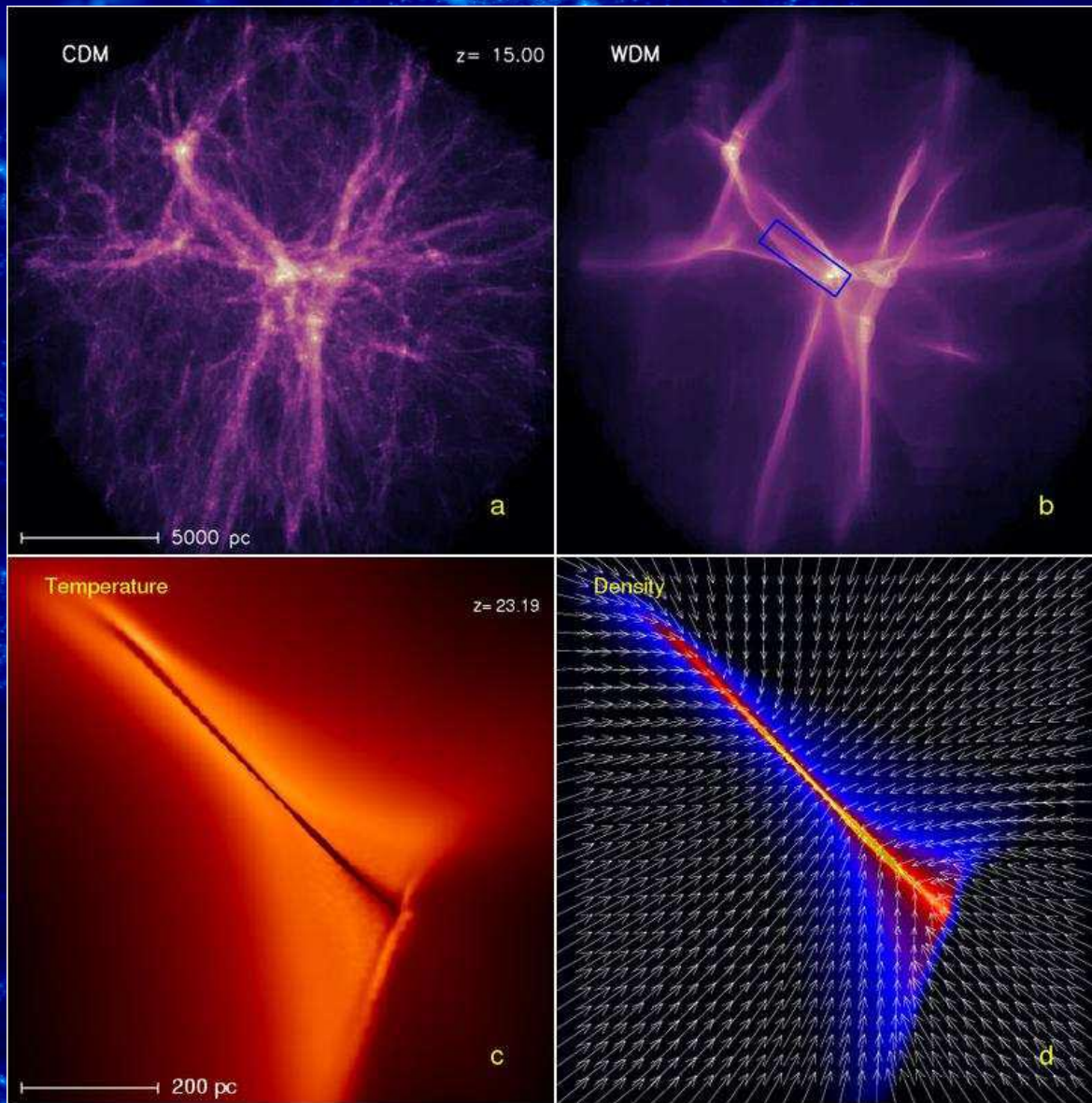


Virialised regions at $z=3$ in CDM and WDM
i.e. overdensities higher than 100 w.r.t. mean

Thermal Velocities

No Thermal Velocities





Liang Gao & Tom Theuns, Science 2007

Label	velocities z_i km/s	cutoff keV	box size Mpc/h	N	softening kpc
CDM	no	-	40	300^3	1
WDM1	no	0.2	40	300^3	1
WDM2	36.6	0.2	40	300^3	1
WDM3	no	1	40	300^3	1
WDM4	4.6	1	40	300^3	1
WDM5	36.6	0.2	30	256^3	2.5

Simulations details: $2.72 \times 10^5 M_\odot$ / particle – refined
 355 pc – 2.5 kpc spline gravitational softening
 WMAP7 cosmological parameters
 $z=100$ initial redshift

Label	size box	first collapse z	average density critical	highest density critical
lu.avi	1/4	10.88	0.264	477
ld.avi	1/4	10.18	0.258	481
ru.avi	1/4	10.18	0.268	480
rd.avi	1/4	10.64	0.258	474

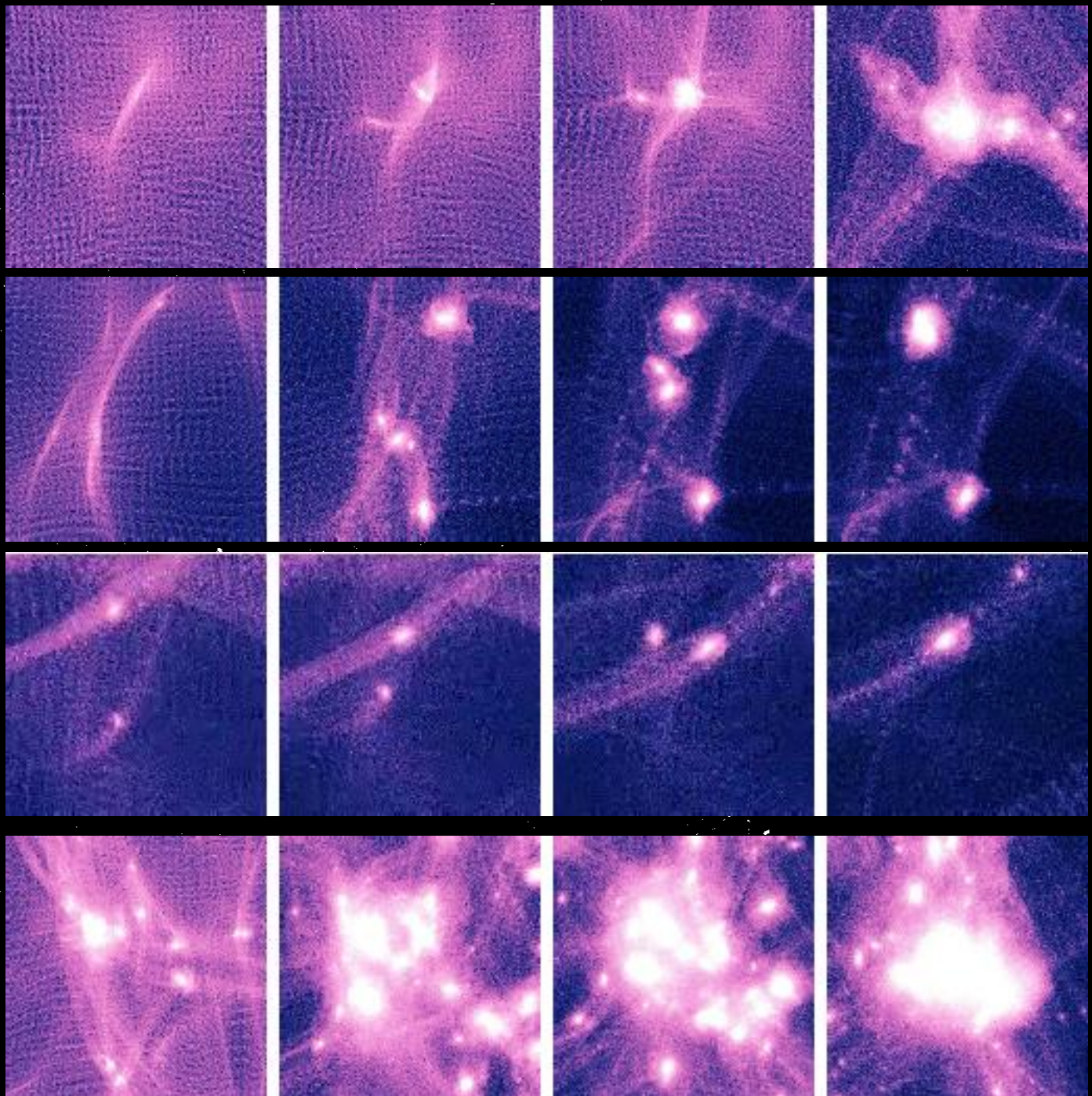
Movie Time...

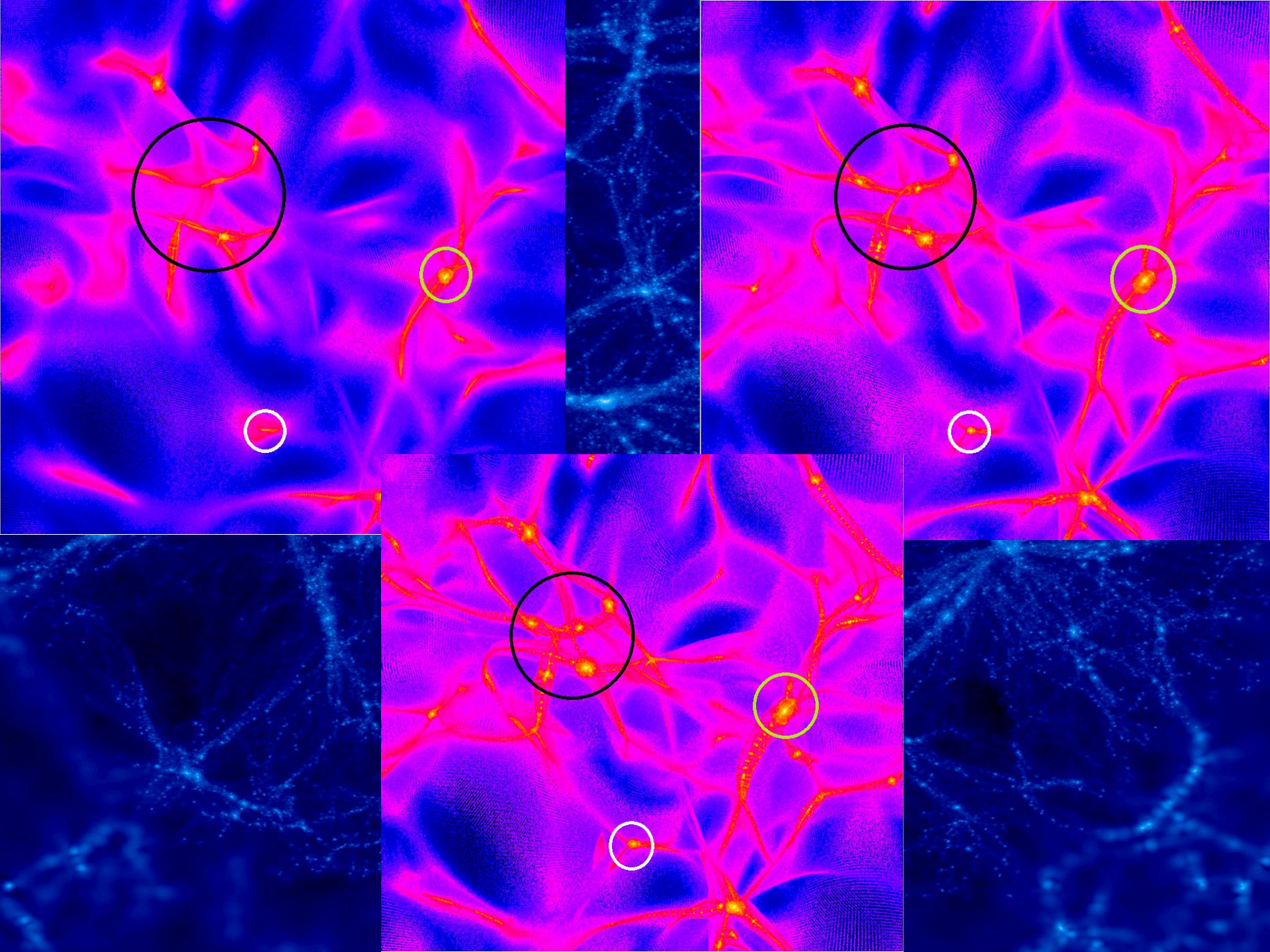
<https://www.youtube.com/playlist?list=PLnGS4wkStJ1aqi3M9hTDaUzuZ-vs-Qg6i>

or on demand ...

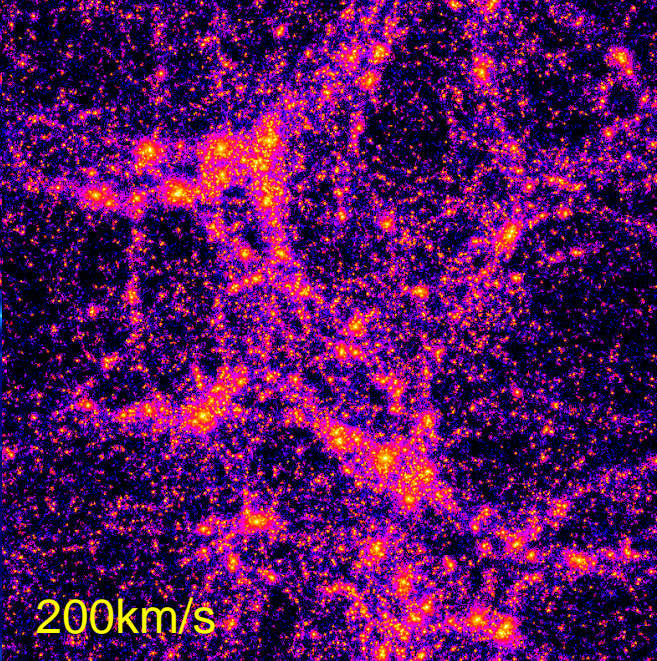
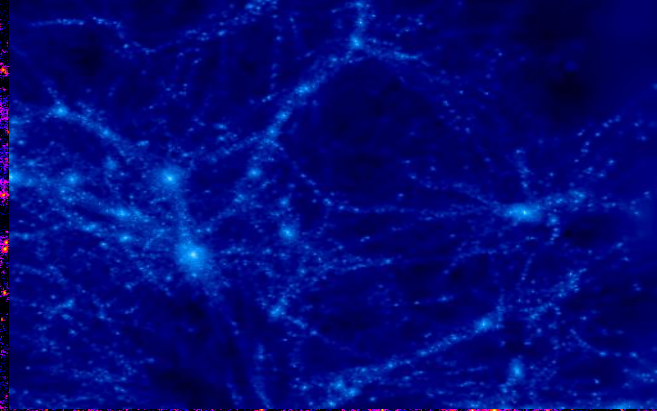
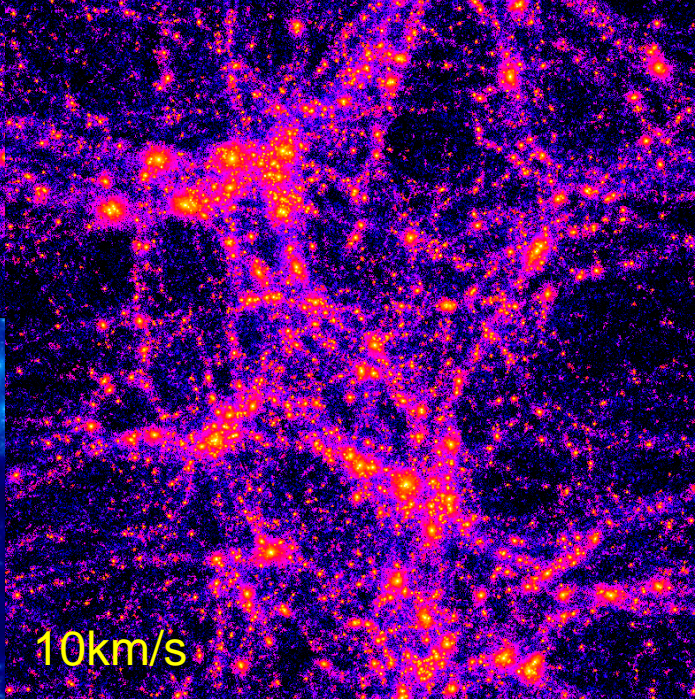
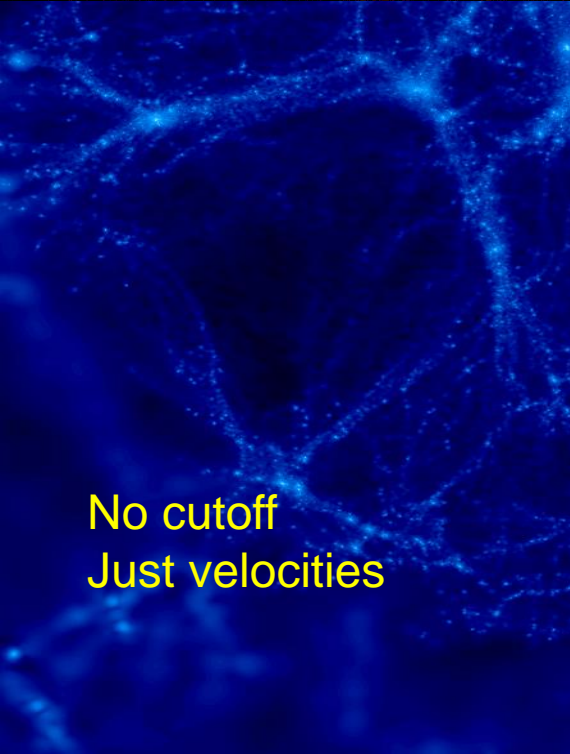
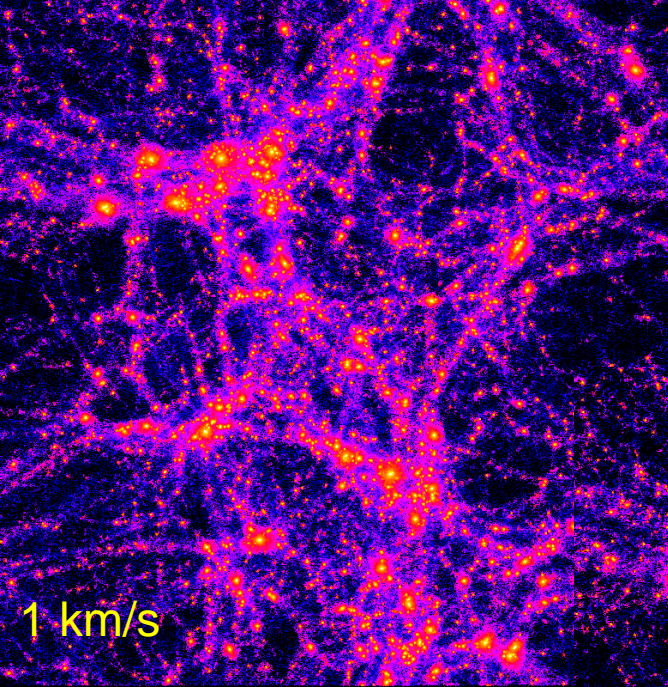
Hybrid Mechanism of Structure Formation

- During the early stages one sees the formation of well contoured **filaments**.
- In the higher density regions, usually situated at the intersection of such filaments, the first halos are formed through **gravitational collapse**. These halos continue growing into larger ones by **accreting** particles from the disrupted filaments.
- In medium density regions, haloes show a **hierarchical** formation trend. Small haloes collapse first and then merge into bigger haloes.
- In less dense regions, the ones isolated by voids and have a very slow evolution, we have observed filaments that collapse very late. The **top-down** formed halo ***survives without any mergers*** until redshift zero.
- Finally there is the more complex scenario in which we observe large haloes formed earlier which **merge** into clusters.





For illustration purposes only...

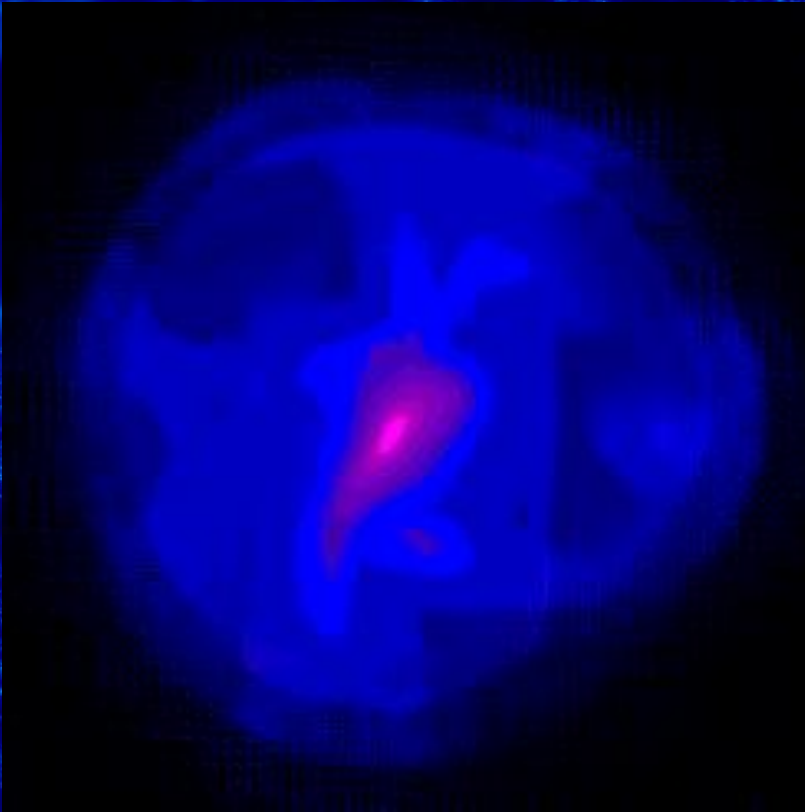


10km/s

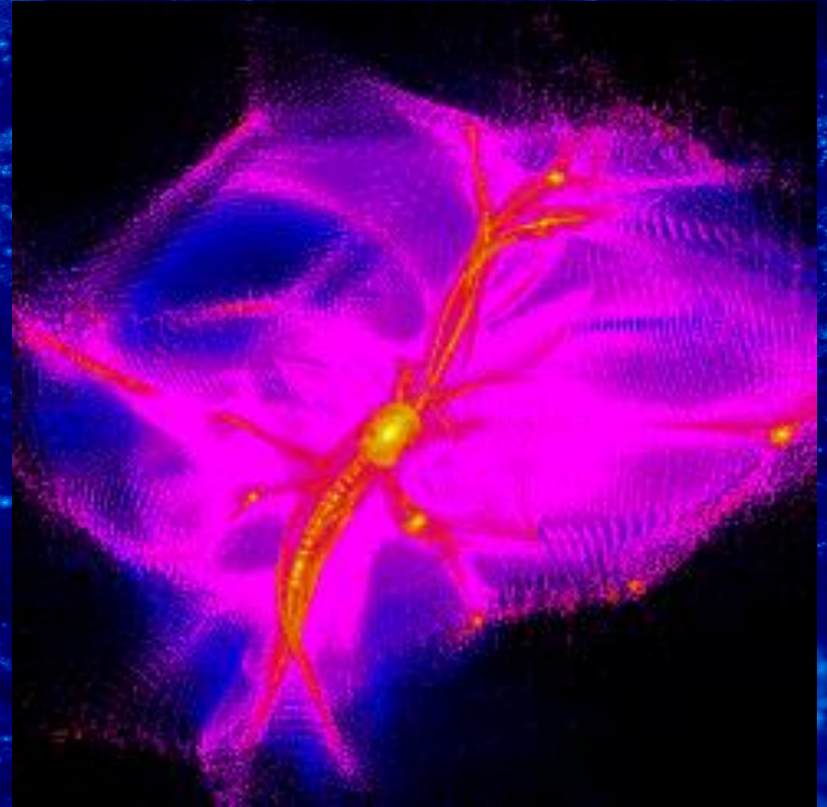
No cutoff
Just velocities

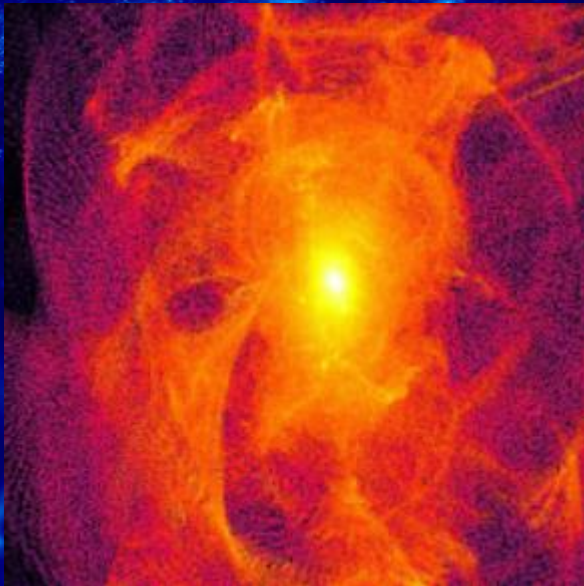
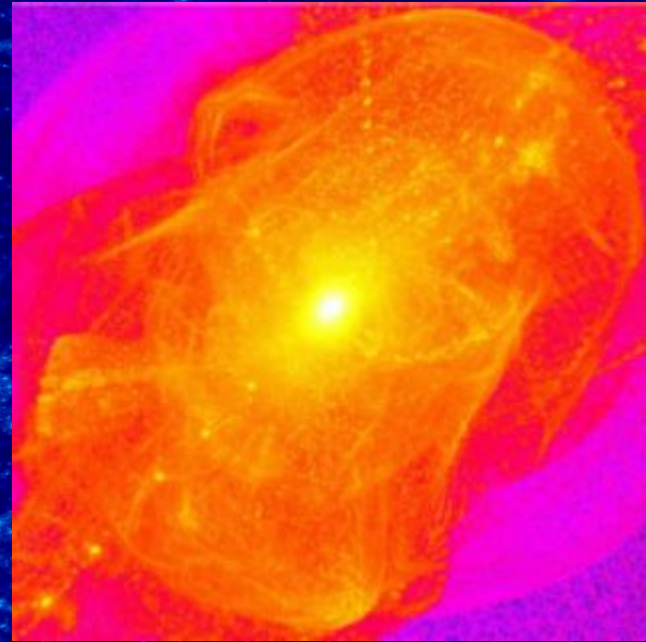
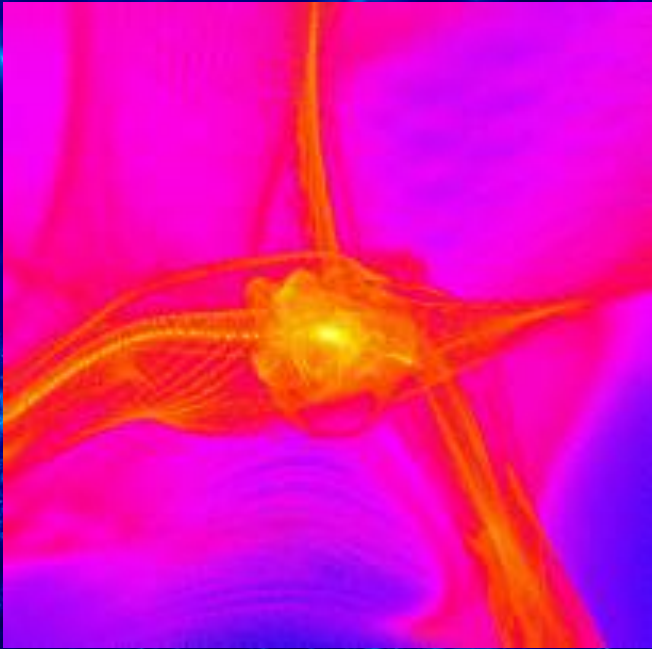
200km/s

Refined region ~ 1/3 of the box

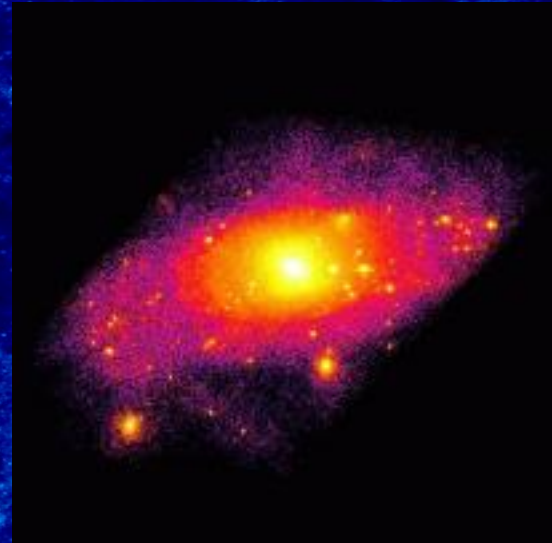


Free streaming → larger region to be refined

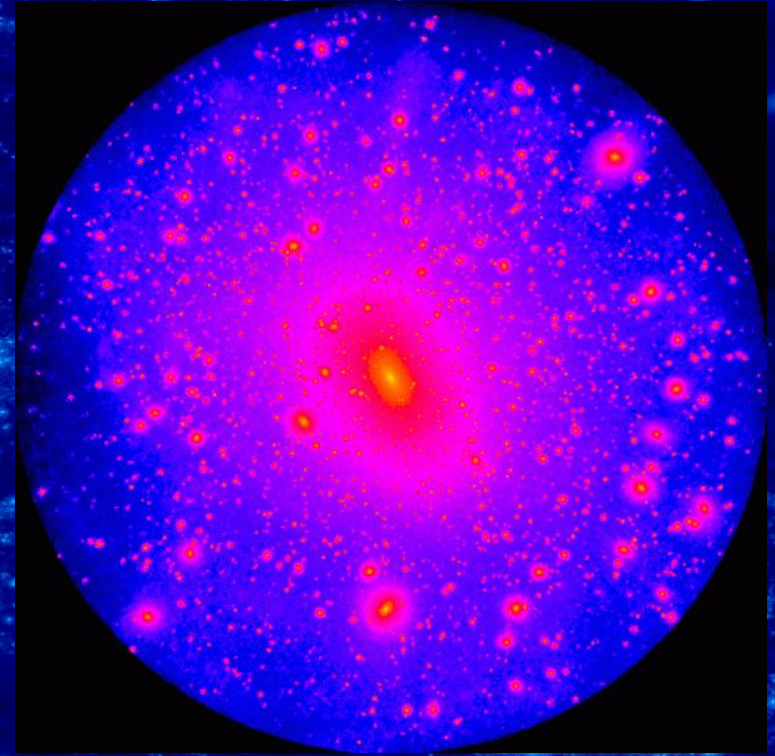
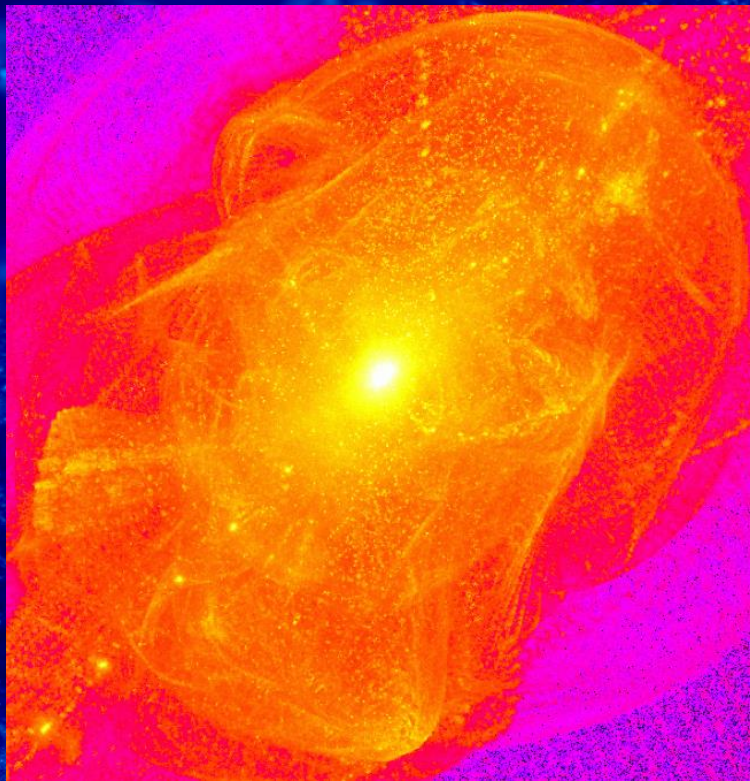




WDM

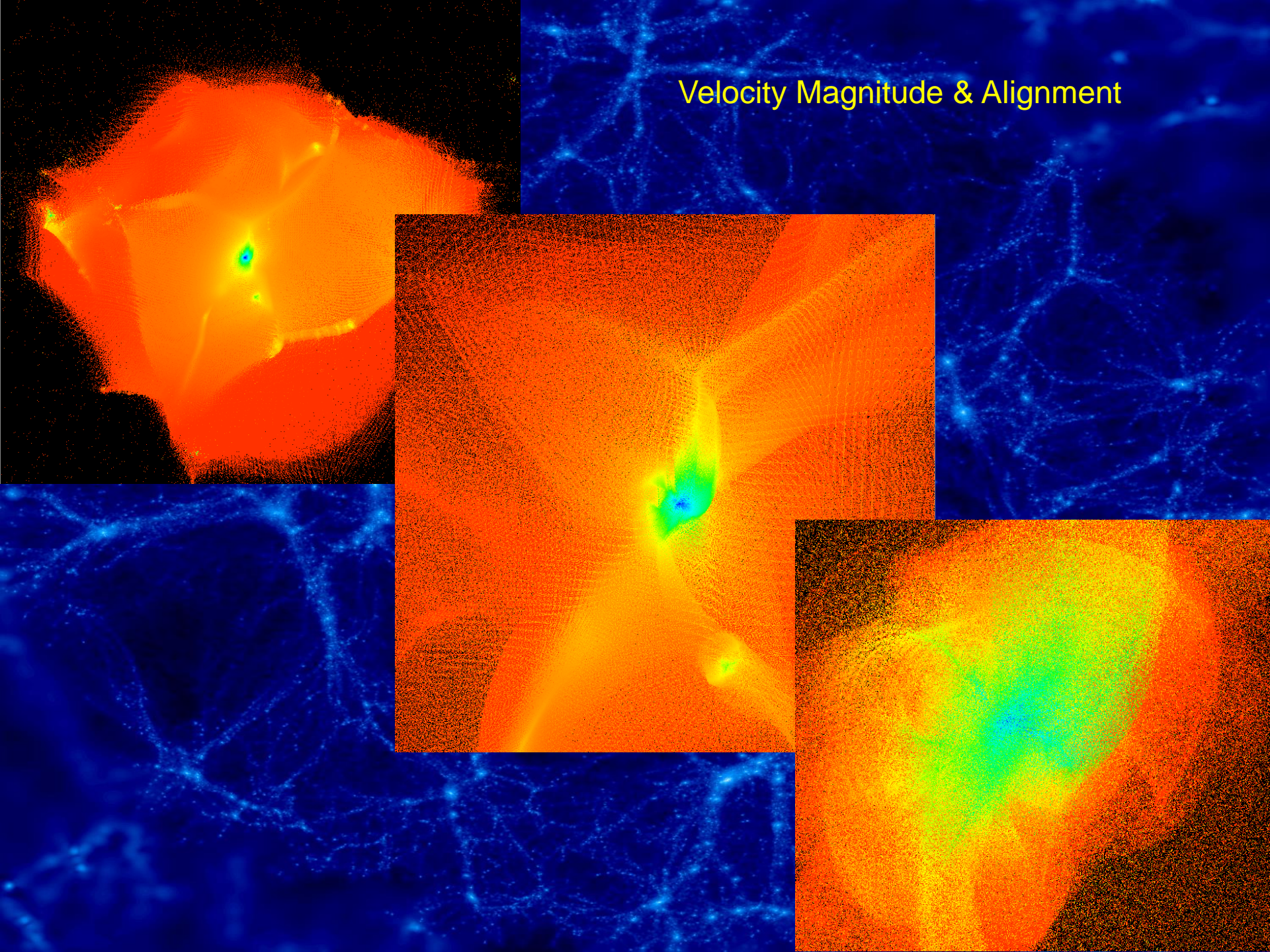


CDM



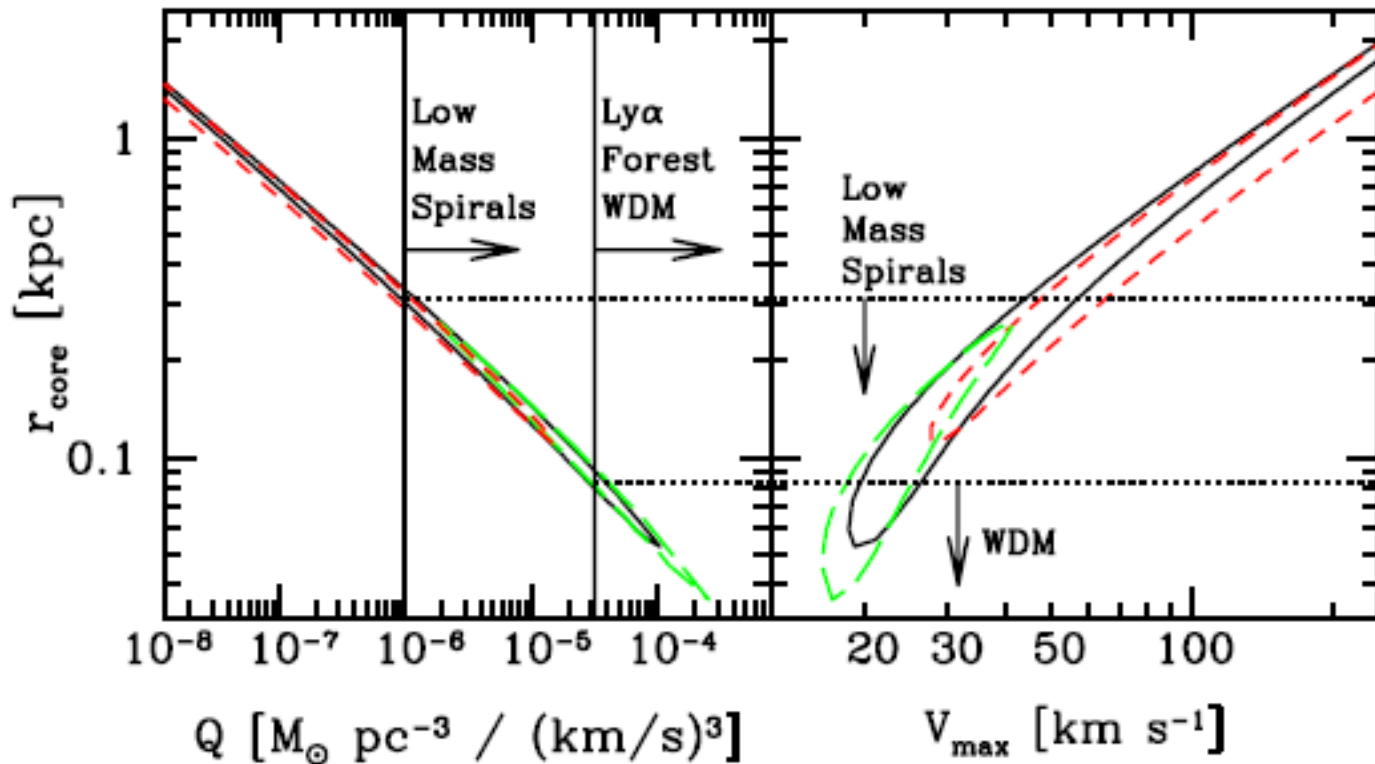
	Number of part.	r_{vir} [kpc]	mass p. part. [M_{\odot}]	boxsize [Mpc]
$m < 200\text{eV}$	18 mil.	630	10^5	42.5
$m > 2\text{keV}$	50 mil.	200	10^7	40

Velocity Magnitude & Alignment



Assumptions in determining the core radius:

- ⌘ Isothermal spheres
- ⌘ Liouville - Phase space density (PSD) is conserved
- ⌘ Pauli exclusion principle
- ⌘ PSD constant as mixing occurs
- ⌘ Velocity dispersion in central halo = constant
- ⌘ Density profile in central halo = constant

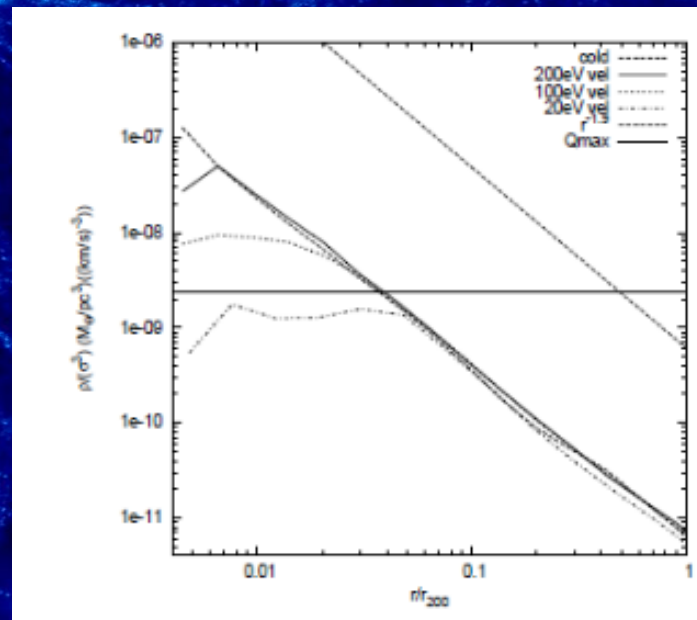
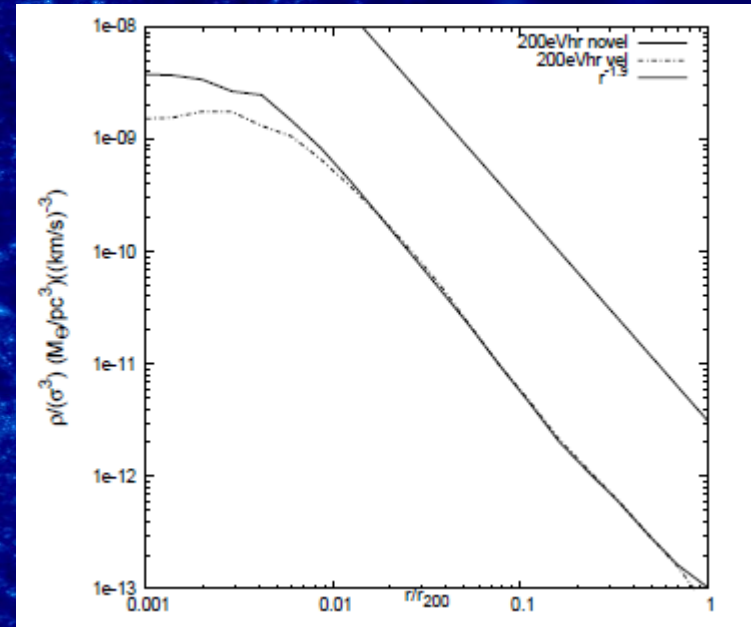
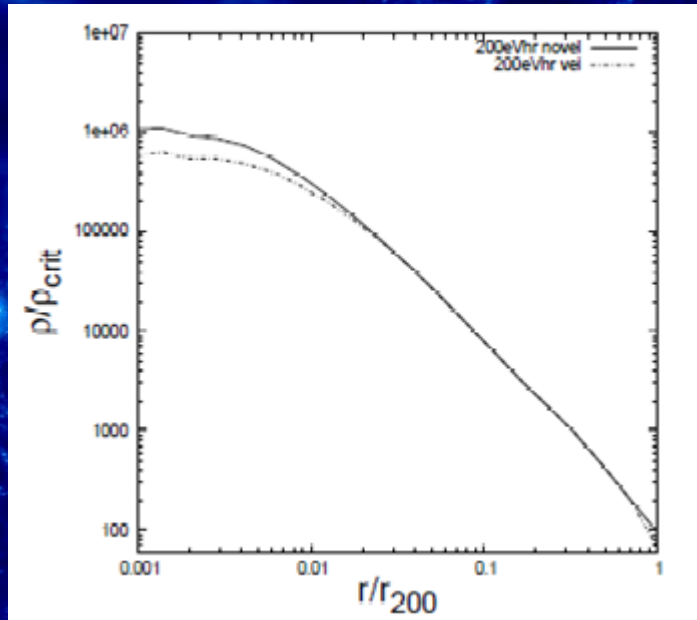


Constraints on the core radius of Fornax as a function of the central phase-space density and maximum circular velocity derived from the velocity dispersion profile

Strigari et al 2006

$$\rho(r) = \frac{\rho_0}{[1 + (r/r_0)^\alpha]^{3/\alpha}}$$

Density and PSD profiles in simulations



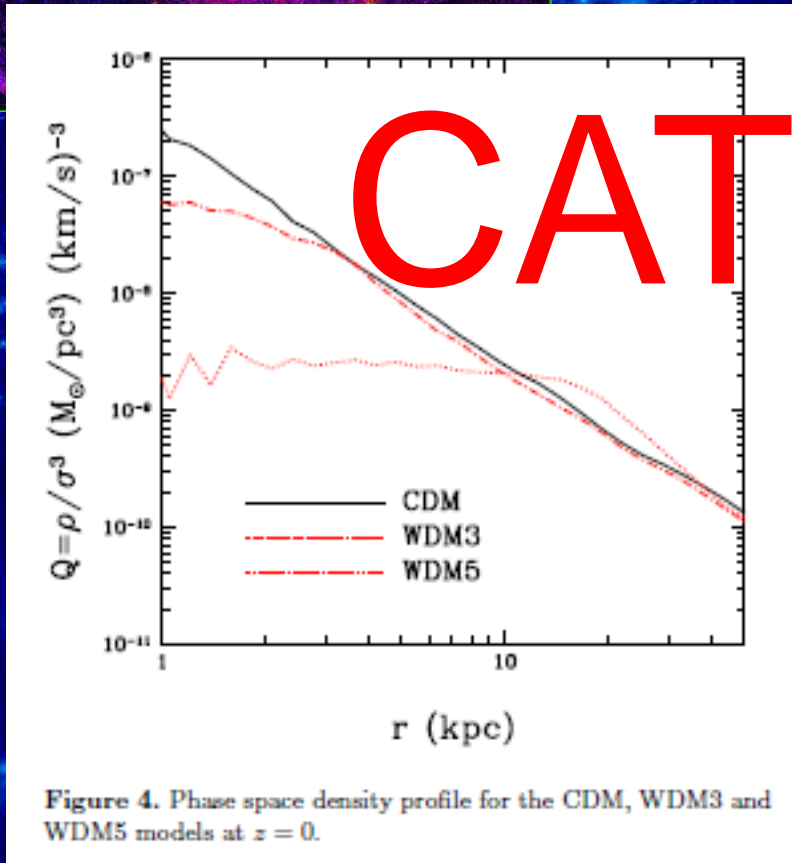
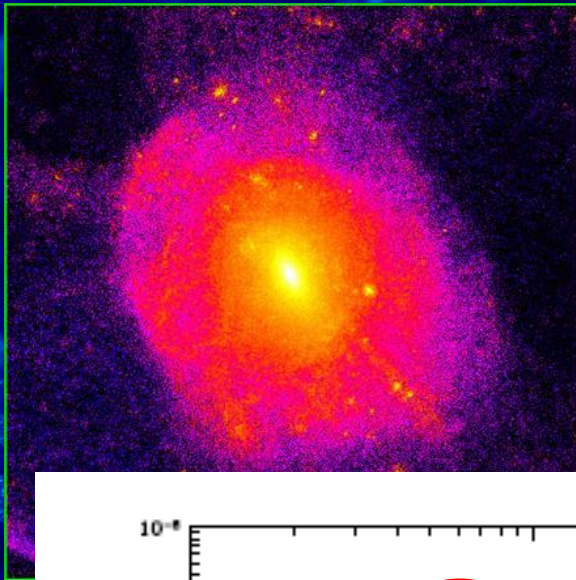


Figure 4. Phase space density profile for the CDM, WDM3 and WDM5 models at $z = 0$.

CATCH 22

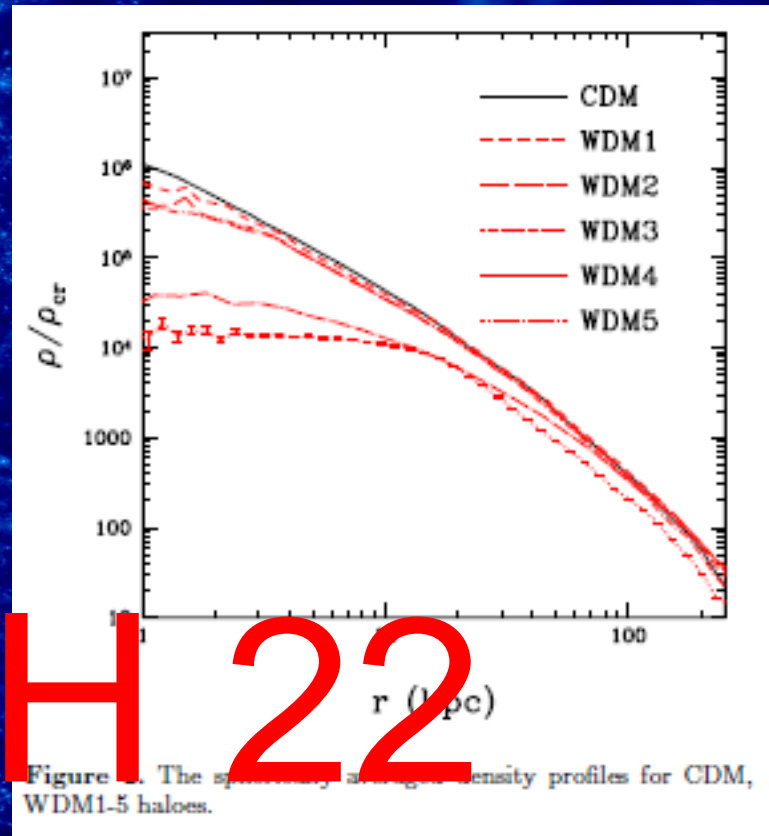


Figure 5. The spherically averaged density profiles for CDM, WDM1-5 haloes.

Label	m_ν (keV)	$m_{\nu, \text{vel}}$ (keV)	N_{utr} (10^6)	M_{utr} ($10^{12} M_\odot$)
CDM	∞	–	10.2	1.42
WDM1	2.0	1.32	8.6	1.22
WDM2	2.0	0.33	8.4	1.20
WDM3	2.0	0.13	8.5	1.21
WDM4	2.0	0.15	6.7	0.93
WDM5-N	2.0	0.05	4.9	0.71
WDM5	2.0	0.03	5.1	0.82

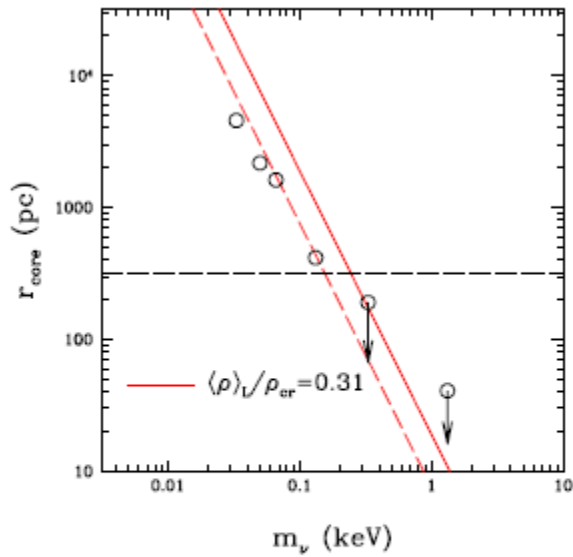


Figure 1. Comparison between core size in simulations (open symbols) and the theoretical expectation for a $M = 10^{12} M_{\odot}$ halo (solid line). The dashed horizontal line is the gravitational softening of our simulations. All points below this line should be considered as upper limits on the core size. The red dashed line is a linear fit to the simulation results.

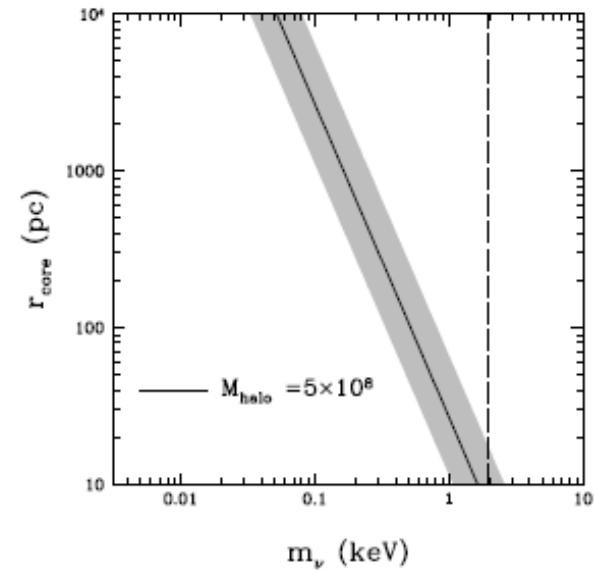
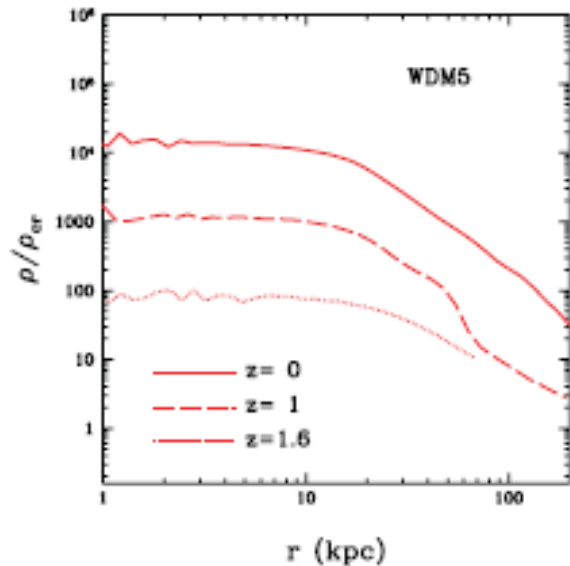
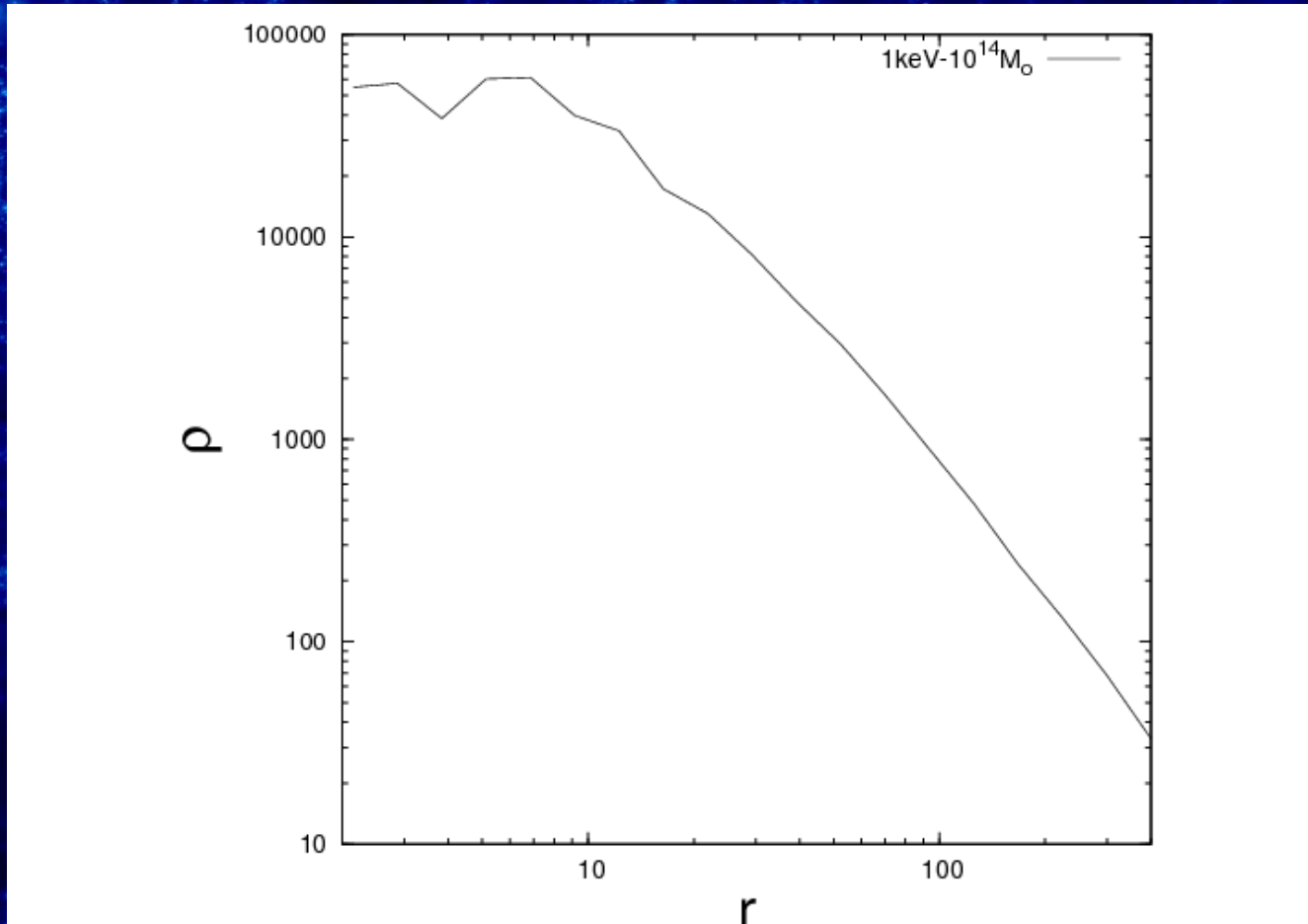


Figure 2. Expected core size for the typical dark matter mass of Milky Way satellites as a function of the WDM mass m_{ν} . The shaded area takes into account possible different values of the local density parameter $0.15 < \Omega_m < 0.6$. The vertical dashed line shows the current limits on the WDM mass from large scale structure observations.



Label	$r_{\text{core},a}$ (kpc)	$r_{\text{core},Q}$ (kpc)	$r_{\text{core},t}$ (kpc)
CDM	< 0.4	< 0.4	∞
WDM1	< 0.4	< 0.4	0.005
WDM2	< 0.4	< 0.4	0.075
WDM3	0.42	< 1.1	0.48
WDM4	1.63	1.80	1.91
WDM5	4.56	4.85	6.98

Clusters of galaxies



M87 ~ 150 kpc central core – Recent observations

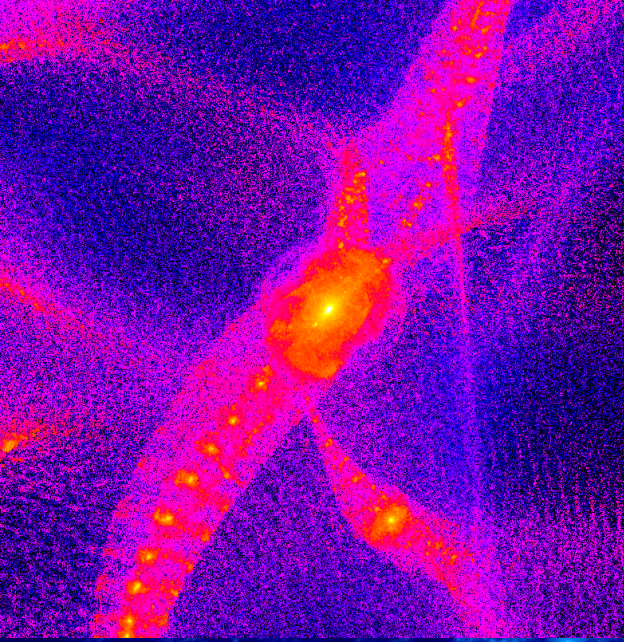
WDM – only ~ 1 keV similar mass halo - ~15 kpc

How to cook a big core...

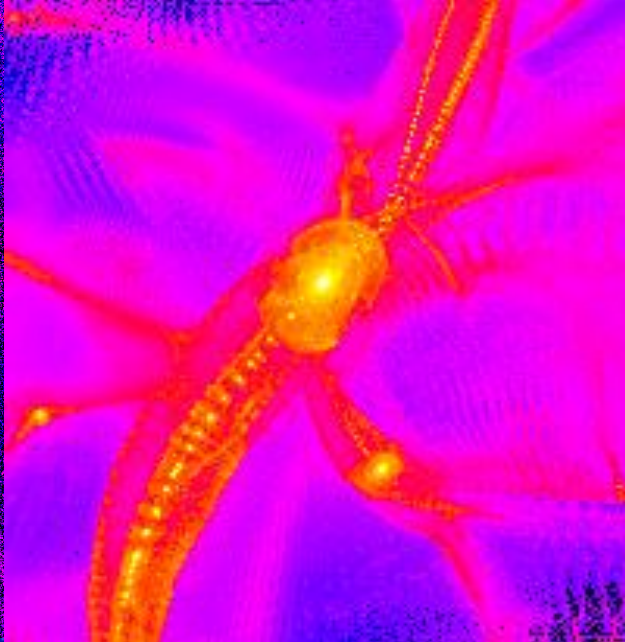
- Simulations of CDM + Gas
AGN Feedback
Supernova Feedback
Star Formation
Etc..
- Fine Tuned

<<There is no evidence for any dark matter core formation in our simulations, even so they include repeated baryonic outflows by supernova-driven winds and black hole quasar feedback.>> Marinacci, Pöker & Springel 2014

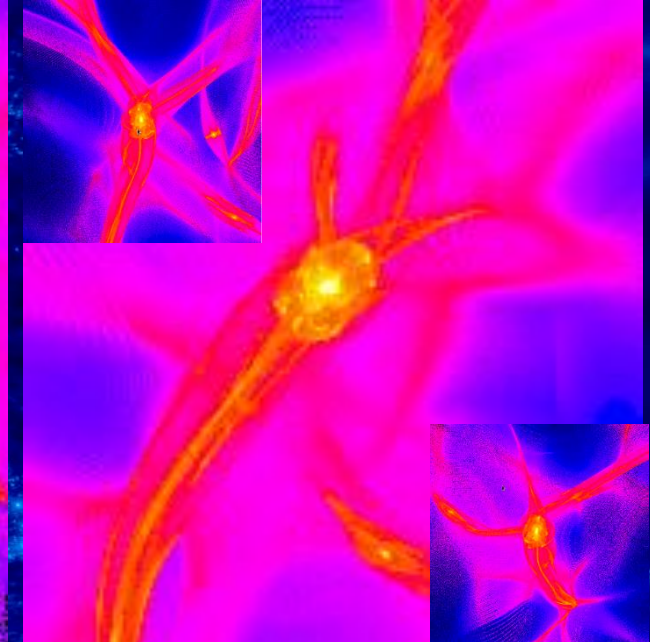
- Solve missing satellites problem, lose the core and vice versa
- Proper KeV simulations + baryonic physics are yet to be performed
- Quantum effects – Fermionic pressure



"Low" resolution

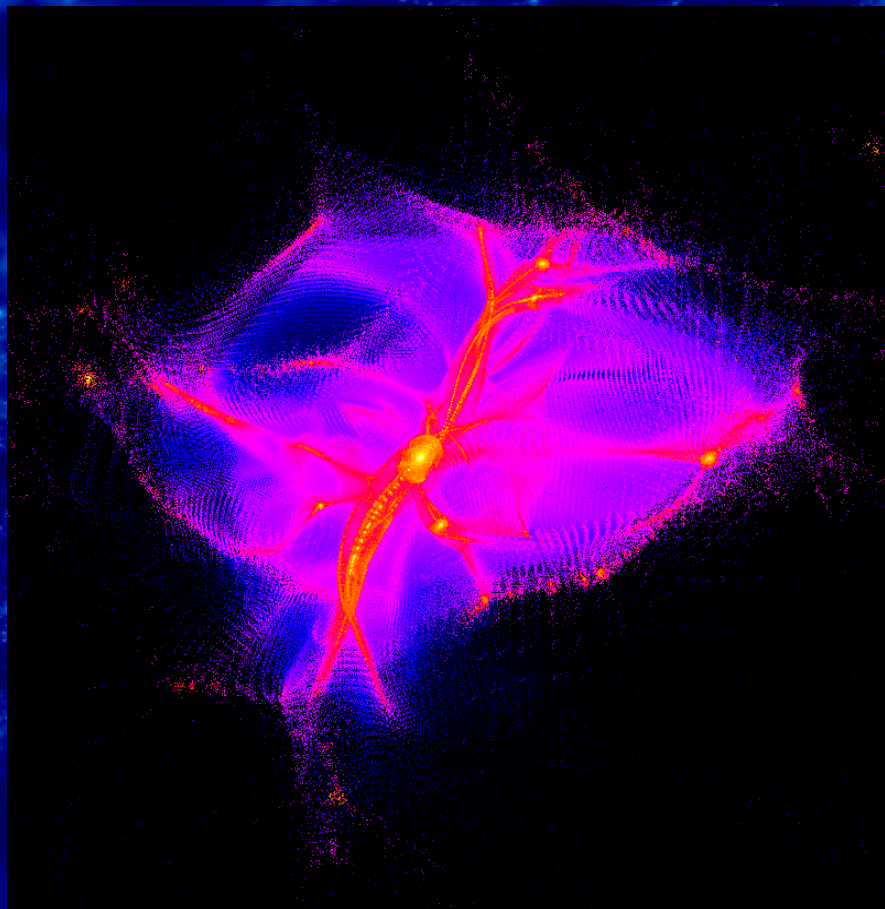


"High" resolution

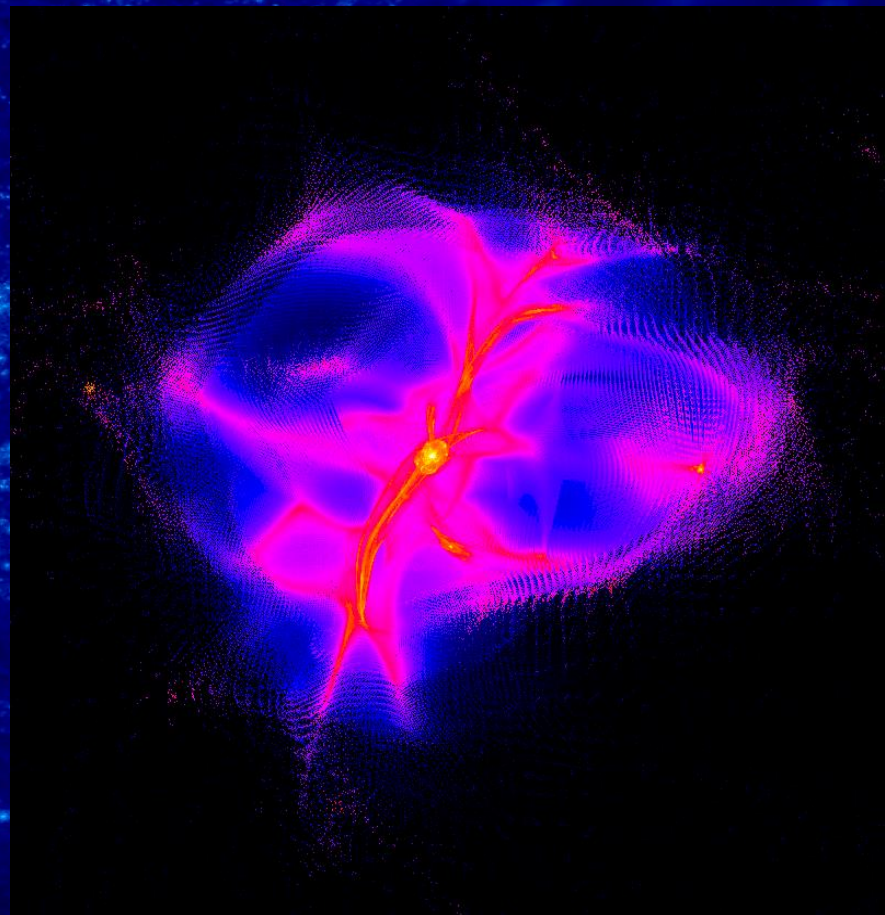


"High" resolution
Large Softening

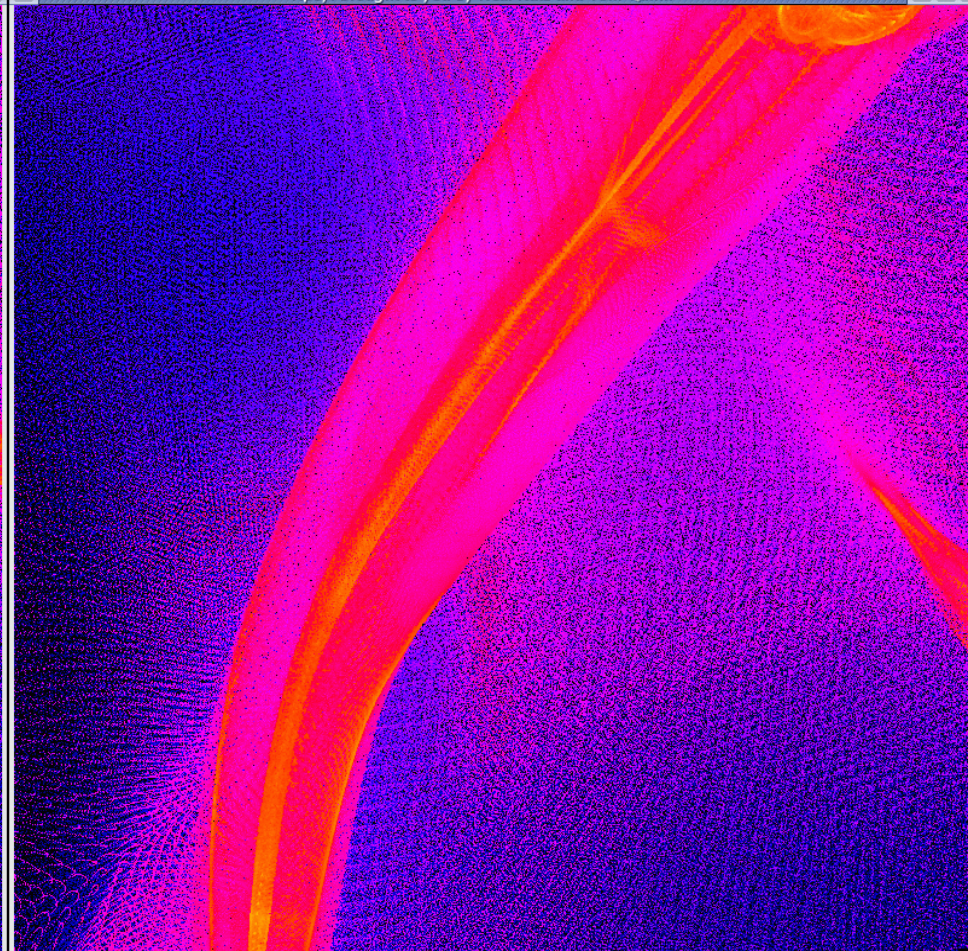
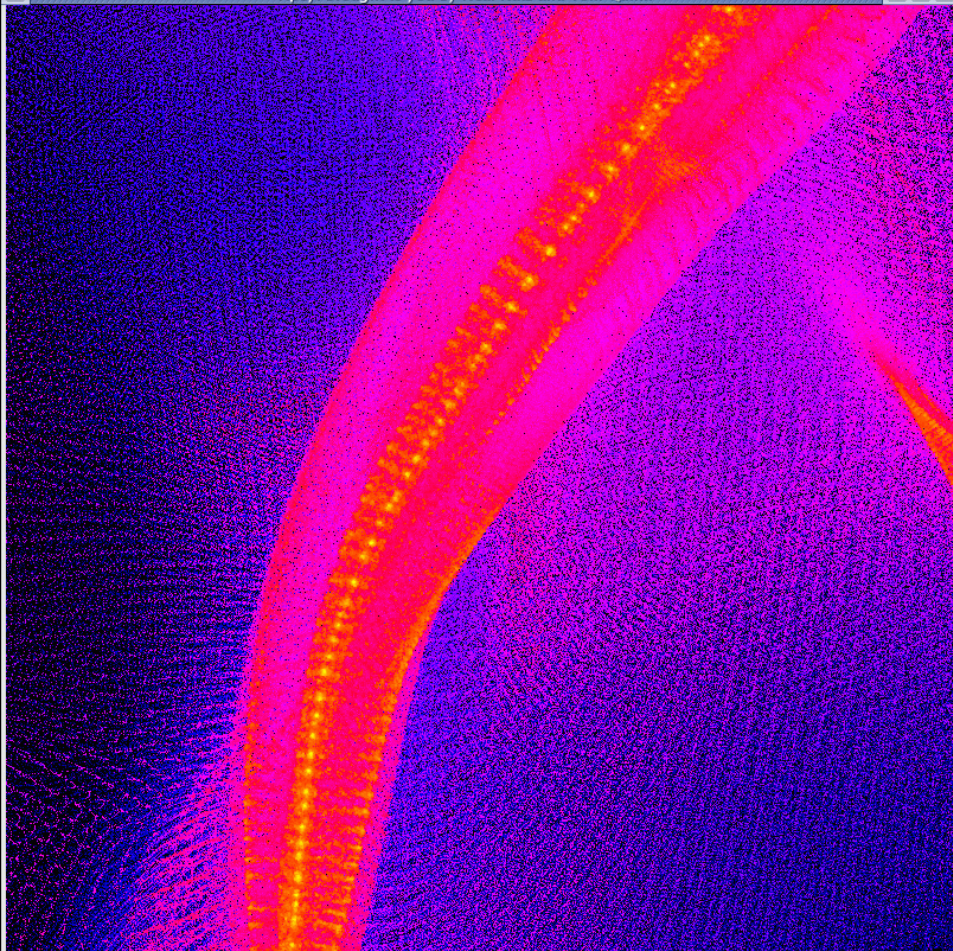




High resolution



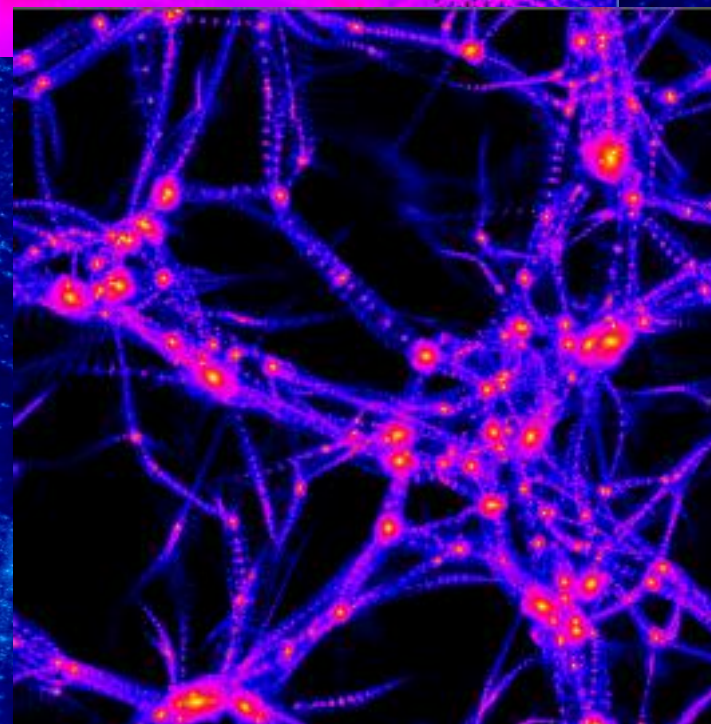
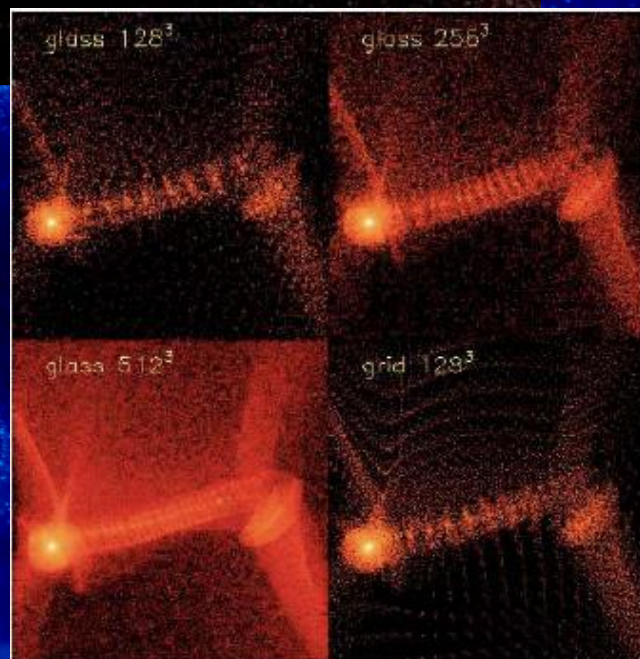
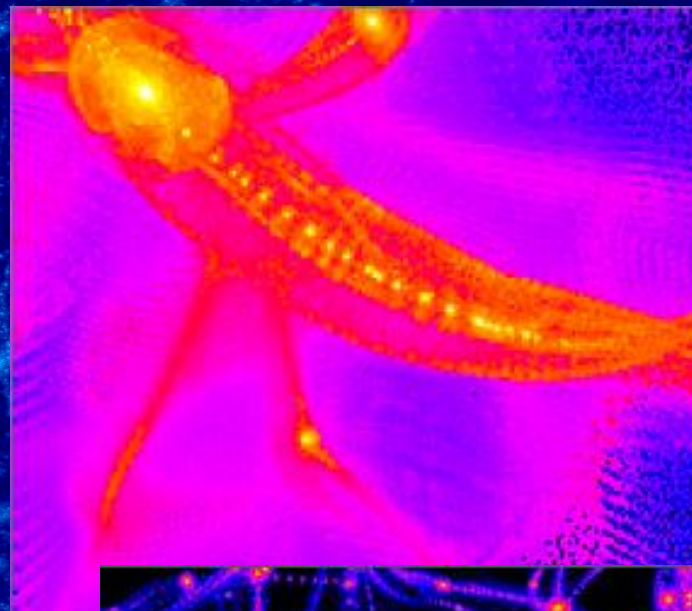
High resolution + larger softening



High resolution filament

High resolution + larger softening

grid 128^3
Wang & White 2007



CONCLUSIONS and COMPLICATIONS

- Formation of haloes in WDM models differs from CDM.
Hybrid mechanism - Top-Down & Hierarchical; long distance & nearest neighbours
Looking at high redshift galaxies for T-D memory.
- The exact recipe for structure formation seems to depend only on the morphology and architecture of the environment
- ☼ Quantum Pressure; Baryons and their physics
- Warm dark matter haloes contain visible caustics and shells.
- Galaxies that do not suffer mergers
- The turn over in PSD results in constant density core with characteristic size.
- Spurious fragmentation below the free streaming scale hard to overcome
\$\$ Adaptive softening? NovA – Hobbs et al. 2015
- The velocity dispersion is crucial in describing warm dark matter particles!
There is no universal one to one correspondence between mass and thermal velocity
Constraints on the particle's mass are not accurate.

Outlook and Perspectives

- **Phase space density studies for dwarf galaxies**
- **High resolution halos without mergers**
- **High resolution simulations of WDM + baryons**
- **Shells, caustics and 3.5 keV line**
- **Simulations for different WDM particles (power spectra)**
- **Softening and resolution scaling – convergence studies**
- **Arcs in the sky and early supermassive black holes**

