Using Dark Globular Cluster and Dwarf Galaxy Data to Constrain the Properties of the Dark Matter Particle

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OUTLINE

Observational Trends for MW dSphs:

- Mass Modeling via velocity dispersion data
 - Determine best-fit Burkert halo parameters (r_0, ρ_0)
 - Find strong correlations between the half-light radii (r_{hf}) and r_0 and ρ_0
- Phase Space Density measurements
 - r_{hf} correlations also found for stellar PSD
 - A model for σ_{DM} / σ_*
 - PSD of DM

<u>A Physical Mechanism for the r_{hf}-r₀ & r_{hf}-p₀ correlations:</u>

Baryonic infall & adibatic compression of DM

Implications of the r_{hf} correlations for the first galaxies:

- Observations of Dark $\overline{\overline{G}}$ lobular Clusters (DGCs) vs. Classical GCs
 - Evidence of DM (DGCs) vs. No evidence of DM (GCs)
 - Simulations suggest DGCs originally ~107 M_{Θ}
 - Is $10^7 M_{\Theta}$ a special scale? Yes!
 - Suggests $10^7 M_{\odot}$ = fundamental building block of galaxies (FSS)

Resulting FSS, LSS, & PSD limits all point to m_{DM,thermal} ~ 2 keV.

PART 1: Observational Trends

MW Cl. dSphs Velocity Dispersions (Gerringer-Sameth et al. 2015)



MW UF dSphs Velocity Dispersions (Gerringer-Sameth et al. 2015)



Eliminated UF dSphs: Data



Eliminated UF dSphs: Data & Tidal Disruption



MW Classical + UF dSphs Data Set



Mass Modeling: Jeans Analysis

$$\frac{1}{\nu}\frac{d}{dr}(\nu\bar{v_r^2}) + 2\frac{\beta\bar{v_r^2}}{r} = -\frac{GM(r)}{r^2}$$

Stellar Density - Plummer Profile:

$$\nu(r) = 3L(4\pi r_{half}^3)^{-1}[1 + r^2/r_{half}^2]^{-5/2}$$

For $\beta \sim 0$ and \sim flat velocity dispersion profiles:

$$M(r) = -\frac{r^2 \bar{v_r^2}}{G\nu} \frac{d\nu}{dr} = \frac{5r_{half} \sigma^2 \left(\frac{r}{r_{half}}\right)^3}{G\left[1 + r^2/r_{half}^2\right]}$$

Determine Best-fit Burkert Profile:

$$M_B(r) = 4\pi \int_0^r s^2 \rho_B(s) ds$$

$$\rho_{\rm B}(r) = \frac{\rho_0}{(1+x)(1+x^2)}, \text{ where } \mathbf{x} = \mathbf{r}/\mathbf{r}_0$$

The Half-Light Radius: r_{hf} or r_e



Best-Fit Burkert Mass Profiles



Best-Fit Burkert Profiles

Strigari et al. (2009)

Gerringer-Sameth et al. (2014)











Phase Space Density Overview I

$$Q \propto rac{
ho}{\sigma^3}$$

• For a fermionic thermal relic, Hogan & Dalcanton (2001) find:

$$Q_{\rm HD} = \frac{\rho}{(3\sigma^2)^{3/2}} = AQ_* \left(\frac{m}{\rm keV}\right)^4$$

• where $A = 5 \ge 10^{-4}$ and Q_*

$$_{\rm k} = \frac{M_{\odot}/pc^3}{({\rm km \ s^{-1}})^3}$$

- adiabatic invariant
- strongly mass-dependent

Phase Space Density Overview II

- Hogan & Dalcanton's assume a 1-D velocity disperson.
- As in Horiuchi et al. (2014), we assume MB:

$$Q = \frac{\rho}{(2\pi\sigma^2)^{3/2}} \simeq 0.33Q_{\rm HD}$$

$$Q_P = AQ_* \left(\frac{m}{\text{keV}}\right)^4$$

• where A = 1.65 x 10⁻⁴ and $Q_* = \frac{M_{\odot}/pc^3}{(\text{km s}^{-1})^3}$

Connecting the Past to the Present

• Galaxy formation processes alter Q by an unknown factor Z:

$$Z = \frac{Q_P}{Q_0}$$

- De Vega & Sanchez (2010) explored a number of analytical methods to find Z, concluding that
 - $-1 \le Z \le 10^4$, in agreement with simulations
 - the mass of a thermal relic DM particle is ~ keV:

$$\frac{m_{\rm th}}{\rm keV} = \left(\frac{Q_p}{A}\right)^{1/4} = \left(\frac{ZQ_0}{A}\right)^{1/4} \simeq 1 - 10$$

PSD Goals

- 1. Determine Z directly from the dwarf galaxy data to produce a model-independent mapping between Q_p and Q_0 .
- 2. Use this empirical Z factor to determine the DM particle mass both for thermal and non-thermal relics.
- 3. Identify primordial dwarf galaxies i.e., systems for which $Q_0 \approx Q_{P^*}$
- 4. Draw insights from these primordial objects about the formation and evolution of galaxies.

Dwarf Galaxy Data (Sample) Data for 23 dSphs from Walker et. al. (2009)

	σ (km/s)			ρ (Μ _Θ pc ⁻³)				r _{hf}			M(r _{hf})		
Dwarf							(pc)			(10 ⁷ ${\sf M}_{_{\Theta}}$)			
Carina	6.6	<u>+</u>	1.2	0.1	±	0.04	241	<u>+</u>	23	0.61	<u>+</u>	0.23	
Draco	9.1	±	1.2	0.3	±	0.08	196	<u>+</u>	12	0.94	<u>+</u>	0.25	
Fornax	11.7	±	0.9	0.042	±	0.007	668	<u>+</u>	34	5.3	<u>+</u>	0.9	
Leo I	9.2	<u>+</u>	1.4	0.19	<u>+</u>	0.06	246	<u>+</u>	19	1.2	<u>+</u>	0.4	
Leo II	6.6	<u>+</u>	0.7	0.26	±	0.06	151	<u>+</u>	17	0.38	<u>+</u>	0.09	
Sculptor	9.2	±	1.1	0.17	±	0.05	260	<u>+</u>	39	1.3	<u>+</u>	0.4	
Sextans	7.9	±	1.3	0.019	±	0.007	682	<u>+</u>	117	2.5	<u>+</u>	0.9	
U Minor	9.5	<u>+</u>	1.2	0.16	±	0.04	280	<u>+</u>	15	1.5	<u>+</u>	0.4	
C Ven I	7.6	<u>+</u>	0.4	0.025	±	0.003	564	<u>+</u>	36	1.9	<u>+</u>	0.2	
U Ma II	6.7	<u>+</u>	1.4	0.32	±	0.14	140	<u>+</u>	25	0.36	<u>+</u>	0.16	

Q – r_{hf} Power-Law Relation

• The power-law relations from Walker et al. (2009):



Phase Space Density of the DM

- Q_0 shown in the previous plot is based on *stellar* velocity dispersions, σ_* .
- What about the DM velocity dispersion, σ ?
- Simulations show, e.g., Horiuchi et al. (2014)

$$\eta_{-} = \sigma/\sigma_* = 1.5 \pm 0.2$$

• What other constraints are possible?

<u>A model for σ</u>:

Consider an equivalent form of the Jeans Equation for the stars:

$$\frac{GM(r)}{r} = \langle \sigma_*^2 \rangle (\gamma_* - 2\beta_* - \alpha)$$

$$\gamma_* = -d\log\nu/d\log r \qquad \alpha = d\log\langle \sigma_*^2 \rangle$$

The LHS is the same for DM, so α must also be the same:

$$\frac{GM(r)}{r} = \langle \boldsymbol{\sigma}^2 \rangle (\gamma - 2\beta - \alpha)$$

$$\gamma = -d\log\rho/d\log r$$

$$\alpha = d \log \langle \sigma^2 \rangle / d \log \eta$$

It follows that:

$$\frac{\langle \sigma^2 \rangle}{\langle \sigma_*^2 \rangle} = \frac{(\gamma_* - 2\beta_* - \alpha)}{(\gamma - 2\beta - \alpha)} \longrightarrow \frac{5}{3}$$

$$\eta = \sigma / \sigma_* \sim 1.3$$

 $d\log r$

<u>Phase Space Density of DM – a model for σ :</u>

Simulations and observations suggest $\eta \sim constant$.

What are the implications from the Jeans equation?

$$\sigma_*^2 = \left(\frac{\rho_*}{\rho^{\eta}}\right)^{1/\eta - 1}; \quad \sigma^2 = \eta \sigma_*^2$$

$$\sigma_* = \sigma_0 \left(\frac{\left[(1+x)(1+x^2)\right]^{\eta}}{(1+y^2)^{5/2}}\right)^{1/2(\eta - 1)}; \quad x = \frac{r}{r_0}, \quad y = \frac{r}{r_{hf}}$$

with $\eta = \frac{5 - 2\beta_* - \alpha}{3 - 2\beta - \alpha}.$

Test 1: agrees with numerical integration of Jeans equation with best-fit Burkert profile.

Test 2 for the constant η **model**:

$$\sigma_* = \sigma_0 \left(\frac{\left[(1+x)(1+x^2) \right]^{\eta}}{(1+y^2)^{\frac{5}{2}}} \right)^{1/2(\eta-1)}; \quad x = \frac{r}{r_0}, \quad y = \frac{r}{r_{hf}}$$

with $\eta = \frac{5-2\beta_* - \alpha}{3-2\beta - \alpha}.$

Find the best-fit results for σ_0 and r_0 for the 12 MW dSphs:

$$\sigma_{0} = (5.2 \pm 1.1) \left(\frac{r_{hf}}{100 \text{pc}} \right)^{p}; \quad p = (8.7 \pm 0.8) \times 10^{-2}$$

with $\beta = \beta_{*} = 0, \ \alpha = 0.7 \pm 0.4, \ \eta = 1.37 \pm 0.1,$
and *the same* $r_{0} - r_{hf}$ relationship found for mass modeling!

The combination of α and $r_0(r_{hf})$ reproduces all features



Phase Space Density of the DM

- Based on the constant-η model, we can find η(r_{hf}) for the dSphs in the Walker et al. (2009) data set.
- Applying this correction factor to Q_{*}, yields

$$Q_{0,\text{DM}} = (1.61 \pm 0.42)Q_* \left(\frac{r_{hf}}{\text{pc}}\right)^{-n}$$

where
$$n = 2.27 \pm 0.15$$
 and $Q_* = \frac{M_{\odot}/pc^3}{(\text{km s}^{-1})^3}$

Using $Q(r_{hf})$ to find Z

- We can rewrite the $Q(r_{hf})$ power-law in terms of:
 - the unknown, primordial $\boldsymbol{Q}_{\boldsymbol{p}}$ and

– an unknown radial scale, r_p:

$$Q_0 = Q_P \left(\frac{r_p}{r_{hf}}\right)^n = Q_P / Z_{\rm em}$$

$$Z_{\rm em} = (r_{hf}/r_p)^n$$

Thus, determining r_p is the key to the empirical Z factor.

Empirical Upper Limits on r_P

Q can only decrease (Liouville's Theorem), so

$$Z = (r_{hf}/r_p)^n \ge 1$$

$$r_p \leq r_{hf,min}$$

- Minimum r_{hf} values:
- Willman 1: $r_{hf} = 25 \pm 6 pc$
- -Segue 1: $r_{hf} = 29 \pm 7 pc$
- Segue 2: $r_{hf} = 34 \pm 5 pc$

$$r_p \le 19 - 39 \,\mathrm{pc}$$

<u>A physical foundation for the r_{hf} correlations</u>

- Consider baryonic infall, e.g., Blumenthal et al. 1986; Ryden & Gunn 1987, etc.
- Begin with pseudo-isothermal profiles for baryons and DM:

$$\rho_{b,i} = \frac{\rho_{b0,i}}{(1+q^2)}; \rho_{d,i} = \frac{\rho_{d0,i}}{(1+q^2)} \quad \Rightarrow \frac{\rho_{b0,i}}{\rho_{d0,i}} = \frac{\Omega_{b,0}}{\Omega_{d,0}}; q = \frac{r}{R_{vir}}.$$

Allow baryons to evolve to a Plummer profile.

• Given
$$M_i(R_{vir}) = M_f(R_{vir})$$
,

$$\begin{split} \mathbf{M}_{tot,\,i} &- \mathbf{M}_{b,f\,(Plummer)} = \mathbf{M}_{d,f} \text{ matches } \mathbf{M}_{d,f\,(Burkert)} \\ \text{when we assume } \mathbf{r}_0 \text{-} \mathbf{r}_{hf} \text{ and } \rho_0 \text{-} \mathbf{r}_{hf} \text{ correlations.} \end{split}$$

- Correlations also simultaneously satisfy
 - $r_i M_i(r_i) = r_f M_f(r_f)$ (adiabatic invariant for spherically symmetric systems – e.g., Blumenthal et al. 1986)

$$- L_{i,d}(\mathbf{r}_i) = L_{f,d}(\mathbf{r}_f)$$
 (DM angular momentum)

PART 2: Implications of DGC Observations and r_{hf} correlations for the First Galaxies

DGC Observations and the First Galaxies

Dark Globular Clusters (DGCs - Taylor et al., 2015)

- Recently found in Cen A
- Observations suggest they contain a significant amount of DM.
- Sizes and masses are intermediate between <u>DM-free</u> Classical GCs and <u>DM-dominated</u> dSphs
- Suggests DGCs and similar compact stellar structures (CSSs e.g., Janz et al. 2015) may occupy the smallest dark matter halos that can form,
- i.e., they could be associated with the free-streaming scale (FSS) and be the fundamental building block of galaxies.

Are CSSs just stripped down relics of larger halos?

Evidence of tidal stripping in CSSs:

- Directly observered tidal streams of stars (Huxor et al. 2013; Foster et al. 2014; Jennings et al. 2015)
- SMBHs expected for higher mass galaxies (Kormendy et al. 1997; Seth et al. 2014)
- Stellar populations like those of more massive galaxies (e.g. Chilingarian et al. 2009; Francis et al. 2012; Sandoval et al. 2015).

Evidence that CSSs are *not* **tidally stripped remnants:**

- Follow high mass extrapolations of GC luminosity function (LF) in galaxies with sufficiently rich GC populations (e.g. Fellhauer & Kroupa 2005; Hilker 2006; Norris & Kannappan 2011; Mieske et al. 2012).
- Some cEs follow low-mass extrapolations of the Ell. Galaxy LF (Kormendy et al. 2009)
- UCDs and cEs are found in all environments from the field to dense clusters ((Norris & Kannappan 2011; Huxor et al. 2013; Paudel et al. 2014; Norris et al. 2014; Chilingarian & Zolotukhin 2015)

We will consider the origin of the latter objects – associating them with the $M_{\rm fs}$ and the first, lowest mass galaxies.

DGCs and the Free-streaming mass scale: M_{fs}

Observed DGC Masses in Cen A:

- Range: $5 \times 10^5 < M/M_{\odot} < 5 \times 10^6$; median $\sim 10^6 M_{\odot}$
- Simulations suggest tidal stripping removes 80-90% of original mass of small halos e.g., Wang et al. (2015)

Inferred Original/Peak Masses:

• Range: $5 \times 10^6 < M/M_{\odot} < 2.5 \times 10^7$; median $\sim 10^7 M_{\odot}$

If $M_{fs} \sim 10^7 M_{\odot}$, $10^7 M_{\odot}$ should be a special scale.

Strigari et al. (2009) found M(300 pc) \geq (0.4-2.0) x 10⁷ M_{\odot} for 18 MW dSphs, despite variations of up to 10⁶ in luminosity. **Strigari et al. 2009 Results**



Analysis of the Free-Streaming Scale (FSS):

$$\lambda_{fs} = \frac{0.107 \text{Mpc}}{(1+z)} \left(\frac{\Omega_{m,0}h^2}{0.1371}\right)^{1/3} \left(\frac{m_{\text{th}}}{\text{keV}}\right)^{-4/3}$$

$$M_{fs} = 4/3\pi\rho_{0,m}(1+z)^3 \left(\frac{\alpha\lambda_{fs}}{2}\right)^3$$
$$= 2.0 \times 10^8 M_{\odot} \left(\frac{\alpha}{2}\right)^3 \left(\frac{m_{\rm th}}{\rm keV}\right)^{-4},$$

where $\alpha \lambda_{fs}/2$ is the scale at which significant suppression of galaxy formation occurs.

For
$$\mathbf{M}_{\mathrm{fs}} = (0.4 - 2.0) \ x \ 10^7 \ M_{\Theta}$$
 , we find

$$m_{\rm th} = 2.12^{+0.55}_{-0.34} \left(\frac{\alpha}{2}\right)^{3/4} \rm keV$$

For $\alpha \sim 2$, $m_{th} \sim 2$ keV - as in many other studies (see conclusions).

Collapse Redshift of the First Galaxies

Set $M_{fs} = M_{vir}$

$$M_{\rm vir} = 4/3\pi (200\rho_{0,m}(1+z_{\rm coll})^3)R_{\rm vir}^3$$

Choosing $R_{vir} = 300$ pc, we find

$$1 + z_{\rm coll} = 22 \pm 6$$

This result is independent of α .

Internal Properties of the First Halos – r_{hf,fs} Assume baryonic infall will lead to the same r_{hf} correlations in the first galaxies:

$$r_0 = (123 \pm 20) \operatorname{pc} \left(\frac{\mathrm{r_{hf}}}{100 \mathrm{pc}}\right)^{0.94 \pm 0.1}$$
$$\rho_0 = (1.13 \pm 0.15) M_{\odot} \mathrm{pc}^{-3} \left(\frac{\mathrm{r_{hf}}}{100 \mathrm{pc}}\right)^{-1.63 \pm 0.2}$$

Integrate $\rho_{Burkert}(\rho_0, r_0)$ to $R_{vir} = 300 \text{ pc}$ and set $M_{vir} = (1.0 \pm 0.3) \times 10^7 \text{ M}_{\Theta}$:

$$M_{\rm vir,B}(r) = 4\pi \int_0^{R_{\rm vir}} r^2 \rho_B(r) dr$$
$$= \pi \rho_0 r_0^3 \left(\ln[(1 + c_{\rm vir}^2)(1 + c_{\rm vir})^2] - 2 \arctan(c_{\rm vir}) \right),$$
where $\mathbf{c}_{\rm vir} = \mathbf{R}_{\rm vir}/\mathbf{r}_0.$

Internal Properties of the First Halos – r_{hf,fs}

Solving $M_{vir} = M_{vir}(r_{hf})$ for $r_{hf,fs}$, we find:

$$r_{hf,fs} = 40^{+37}_{-21}$$
 pc

Applying the $\rho_0(r_{hf})$ and $r_0(r_{hf})$ correlations yields:

$$r_{0,fs} = 51^{+45}_{-22} \text{pc}, \ \rho_{0,fs} = 5.1^{+8.1}_{-3.4} M_{\odot} \text{pc}^{-3}$$

$$c_{\rm vir} = R_{\rm vir}/r_0 = 5.8^{+4.3}_{-2.7}$$

Associate r_{hf,fs} with r_P to find PSD mass limits

Recall
$$r_p \leq r_{hf,min}$$

<u>Maximum r_p values:</u> - Willman 1: $r_{hf} = 25 \pm 6 \text{ pc}$

Sets ceiling on r_{hf, fs}:

$$r_{hf,fs} = 40^{+37}_{-21}$$
 pc

$$r_p = 25 \pm 6 \text{ pc}$$

Q_p + DM Particle Mass with $r_p = 25 \pm 6 pc$

- Max/Min Q_0 ratio is ~ 10^4
- Max/Min Q_p differ by ~ 4.5

 $\mathbf{Q}_{\mathbf{P}} = \mathbf{Z}_{\mathbf{em}} \mathbf{Q}_{\mathbf{0}}$

• Max/Min m_{th} values differ by ~ 1.5

$$\frac{m_{\rm th}}{\rm keV} = \left(\frac{Z_{\rm em}Q_0}{A}\right)^{1/4} = \left(\frac{\left(\frac{r_{hf}}{r_p}\right)^n Q_0}{A}\right)^{1/4}$$

Including all galaxy data uncertainties

• $1 < Z < 10^4$

•
$$m_{\rm th} = 2.02 \pm 0.35 \; {\rm keV}$$

PSD of DM



Internal Properties of the First Halos Refined

With the refined limits on r_{hf,fs} from the PSD analysis:

$$r_{hf,fs} = 25 \pm 6 \mathrm{pc}$$

Limits on $r_0(r_{hf})$, $\rho_0(r_{hf})$, and c_{vir} now become:

$$r_{0,fs} = 33.3 \pm 7.6 \text{pc},$$

 $\rho_{0,fs} = 10.7^{+6.0}_{-3.2} M_{\odot} \text{pc}^{-3},$
 $c_{\text{vir,fs}} = R_{\text{vir,fs}}/r_{0,fs} = 9.0 \pm 2.7$

Final Refinement – constraints from simulations

For the 12 MW dSphs in our sample, we can use velocity dispersion data and the r_{hf} correlations to explore:

$$z_{\text{coll}}, M_{\text{vir}}, \text{ and } c_{\text{vir}}(M_{\text{vir}}, z_{\text{coll}})$$

Compare to $c_{vir}(M_{vir}, z_{coll})$ found in the best simulations, e.g., Prada et al. (2011).

We find statistically self-consistent results for

$$c_{\rm vir}(M_{\rm vir}, z_{\rm coll}) = 10.2 \pm 1.1$$

Tightest Constraints on the First Halos

With
$$c_{\text{vir,fs}} = 10.2 \pm 1.1$$
 and $r_{0,fs} = R_{\text{vir,fs}}/c_{\text{vir,fs}}$,
we find
 $r_{0,fs} = 29.3 \pm 3.5 \text{pc.}$
 $r_{hf,fs} = 22 \pm 2.6 \text{pc}$
 $\rho_{0,fs} = 13.4 \pm 2.6 M_{\odot} \text{pc}^{-3}$

And applying the refined r_{hf,fs} to the PSD data yields:

$$m_{\rm th} = 2.19 \pm 0.35 \; {\rm keV}$$

Galaxy Constraints Satisfied by 2 keV Thermal Dark Matter Particle (Abazajian 2014)

- Local Group Phase Space Density and Subhalo Counts: $m_{th} > 1.7 \text{ keV}$ (Horiuchi et al. 2014), $m_{th} \sim 2 \text{ keV}$ (de Vega & Sanchez 2010.12)
- High Redshift Galaxy Counts: m_{th} > 1.3 keV (Schultz et al. 2014)
- Abundance, Radial Distribution, and Inner Density Profile Crises of Milky Way Satellites solved if: $m_{th} \approx 2 \text{ keV}$ (e.g., Lovell et al. 2012 and Abazajian 2014 for additional references)
- A *non*-thermal particle can produce similar LSS:
- For instance, a 7.14 keV Shi-Fuller v_s with L = 7 x 10⁻⁴ behaves like $m_{th} \approx 2$ keV. (Abazajian 2014)

v_s Transfer Functions II: Lyα Constraints Scalar Decay, Shi-Fuller, DW (Merle & Schneider 2014)



v_s Halo Mass Function: Scalar Decay, Shi-Fuller, DW (Merle & Schneider 2014)



Summary and Conclusions

Observational Trends for MW dSphs:

- Mass Modeling via velocity dispersion data
 - Determined best-fit Burkert halo parameters (r_0, ρ_0) for 12 MW dSphs
 - Found strong correlations between the half-light radii (r_{hf}) and r_0 and ρ_0
- Phase Space Density measurements
 - r_{hf} correlations also found for stellar PSDs.
 - Constant $\eta = \sigma_{DM} / \sigma_*$ model for σ_{DM} obeys same $r_0 r_{hf}$ correlations
 - Determined PSD of DM

Discussed foundation for the r_{hf} **-** r_0 **&** r_{hf} **-** ρ_0 **correlations:**

Baryonic infall & adibatic compression of DM

Applied r_{hf} **correlations to study the first galaxies:**

- Interpretted *some* DGCs/CSSs as the lowest mass halos (FSS)
 - Suggests ~ $10^7 M_{\odot}$ is the fundamental building block of galaxies (FSS)

Resulting FSS, LSS, & PSD limits all point to m_{DM,thermal} ~ 2 keV.

Non-thermal DM

• If the DM particle is a sterile neutrino, we can use the following transformation equations (e.g., Viel et al. 2005; Abazajian 2014) to find the corresponding non-thermal limits:

$$m_{s,\rm DW} = 4.27 \text{keV} \left(\frac{m_{\rm th}}{\text{keV}}\right)^{4/3} \left(\frac{\Omega_{\rm m,0} h^2}{0.1371}\right)^{-1/3} \simeq 1.5 m_{s,\rm SF}$$

- Applying these transformations, we find:
 - 2.9 < m/keV < 22.1 (Dodelson-Widrow) X (Watson et al. 2012)
 - **1.9 < m/keV < 14.7 (Shi-Fuller)** Bulbul et al. (2014) OK
- Alternative transformations (deVega & Sanchez 2013):

$$m_{\nu}^{\rm DW} = 2.85 \text{keV} \left(\frac{m_{\text{th}}}{\text{keV}}\right)^{4/3}; m_{\nu}^{\rm SF} \cong 2.55 m_{th}$$

1.9 < m/keV <</th>14.7 (Dodelson-Widrow)X (Horiuchi et al. 2014)1.9 < m/keV <</td>8.6 Shi-FullerBulbul et al. (2014) OK