PROGRESS REPORT ON QUANTUM ASPECTS OF STANDARD - SLOW ROLL SINGLE FIELD INFLATION

D. Boyanovsky[1-3], H.J. de Vega[2,3], N. Sanchez

1) Dept. of Physics - Univ. of Pittsburgh
2) observatoire de Paris - LERMA
3) LPTHE, Universite Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII)

INFLATON DECAY DURING INFLATION: IMPLICATIONS FOR POWER SPECTRUM.

D. BOYANOVSKY, H.J. de VEGA, N.G. SANCHEZ

INFLATON FLUCTUATIONS ⇒ (scalar) density perturbations.

- Adiabatic (primarily)
- Gaussian
- ALMOST scale invariant.
- A-causal (superhorizon)

Basic "paradigm" validated by WMAP.

Brief review of BASIC implementati

Single field inflation (generalized later

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$$

$$\phi(\vec{x},t) = \phi_0(t) + \delta\phi(\vec{x},t)$$

"Drives" FRW EXPANSION (Classical)

Fluctuations (Quantum)

$$H^2 = \frac{8\pi}{3M_p}V(\phi_0) = \left(\frac{\dot{a}}{a}\right)^2$$

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \frac{dV(\phi_0)}{d\phi_0} = 0$$

Gaussian approx: $\chi(\vec{x},t) = a\,\delta\phi(\vec{x},t)$

(+ Space F.T.) $\Rightarrow$ CONFORMAL TIME

$$d\eta = \frac{dt}{a(t)}$$

$$\chi_k''(\eta) + \left[k^2 + M^2 a^2(\eta) - \frac{a''}{a}\right]\chi_k(\eta) = 0$$

$$M^2 = \left.\frac{d^2V}{d\phi^2}\right|_{\phi_0(t)}$$

- De Sitter: $\phi_0 =$ CONSTANT

$$\chi_k = \alpha_k S_\nu(k;\eta) + \alpha_{-k}^{+} S_\nu^{*}(k;\eta)$$

$$S_\nu(k,\eta) = \frac{1}{2} i^{-\nu-\frac{1}{2}} \sqrt{\pi\eta}\, H_\nu^{(2)}(k\eta)$$

$$\alpha_k |0\rangle_{B.D.} = 0$$

$$\nu = \left[\frac{9}{4} - \frac{M^2}{H^2}\right]^{1/2}$$

SUPERHORIZON: $k\eta \to 0$

$$S_\nu(k,\eta) \xrightarrow{\sim} k^{-\nu}\, \eta^{\frac{1}{2}-\nu}$$

Sub Horizon: $k\eta \gg 1$

$$S_v(k,\eta) \sim \frac{e^{-ik\eta}}{\sqrt{2k}}$$

Determines normalization!

Power spectrum:

$$P(k) = \frac{k^3}{2\pi^2} \langle 0 | \delta\phi_k^{(+)} \delta\phi_{-k}^{(+)} | 0 \rangle$$

$$= \frac{k^3}{2\pi^2} \frac{\langle 0 | \chi_k(\eta) \chi_{-k}(\eta) | 0 \rangle}{a^2(\eta)}$$

$$= \frac{k^3}{2\pi^2} \frac{|S_v(k,\eta)|^2}{a^2(\eta)}$$

$$P(k) \xrightarrow[k\eta \to 0]{} \left(\frac{H}{2\pi}\right)^2 \left[\frac{k}{aH}\right]^{3-2\nu}$$

Scale inv: $\Rightarrow \nu \approx \frac{3}{2} \Rightarrow M/H \ll 1$

$\Rightarrow V''(\phi_0)/H^2 \ll 1 \rightarrow$ SLOW-ROLL.

SLOW-ROLL ("QUASI" de SITTER)

$$3H\dot{\phi}_0 \simeq -V'(\phi_0)$$

$$\epsilon_V = \frac{M_P^2}{2}\left(\frac{V'(\phi_0)}{V(\phi_0)}\right)^2 \ll 1 \Rightarrow \text{SLOW VARIATION OF } \phi_0$$

$$\eta_V = M_P^2\left(\frac{V''(\phi_0)}{V(\phi_0)}\right) \ll 1 \Rightarrow \text{FLAT POT. (massless)}$$

$$\xi_V = M_P^4\left(\frac{V'}{V}\right)\left(\frac{V'''}{V}\right) \ll 1 \Rightarrow \text{SMALL NON-LINEARITIES (couplings)}$$

$$\Delta_R^2 = \frac{H^2}{8\pi^2\epsilon_V M_P^2} \quad \text{(curvature Pert)} \ll 1$$

(TEMP. FLUCTS)

$$a(\eta) \simeq -\frac{1}{H\eta}\left(1+\epsilon_V+\cdots\right)$$

$\Rightarrow$ SLOW-ROLL: same mode eqns

with

$$\nu = \frac{3}{2} + \underbrace{\epsilon_V - \eta_V}_{\text{Depart.}} \text{ from Scale Inv.}$$

IN GAUSS. APPX.

Summary:

- Gaussian approx ⟺ Neglect interactions
- Slow roll ⟺ Near scale invariance

Amplitud determined when modes are SUB-HORIZON (or at horizon X-ing)

⇒ SMALL (slow-roll) parameter

$\Delta = \frac{3}{2} - \nu$ $\quad (= \eta_V - \epsilon_V$ for inflaton)

⇒ ALMOST MASSLESS FIELDS.

INTERACTIONS:

- With other fields: necessary for REHEATING (inflaton decay AFTER INFLATION → Reheating.

- SELF-INTERACTION:

$$V(\phi) = V(\phi_0) + V'(\phi_0)\delta\phi + V''\frac{(\delta\phi)^2}{2} + V'''\frac{(\delta\phi)^3}{6} + \dots$$

$V(\phi_0)$ ↓ FRW; $V'(\phi_0)$ ↓ EQN. OF MOT.; V'' ↓ MASS; V''' ↓ INTER.!!

S.I. Constrained by WMAP!!

Dimensionless Coupling $= \frac{g}{H} \equiv \frac{V'''(\phi_0)}{2H}$

- SLOW ROLL:

$$\frac{g}{H} \simeq 3\pi \xi_v \left(\Delta_R^2\right)^{1/2} + \ldots$$

↳ Constrained by amp. of curv. pert.

Constrained by $\frac{dn_k}{d\ln k}$ (running index)

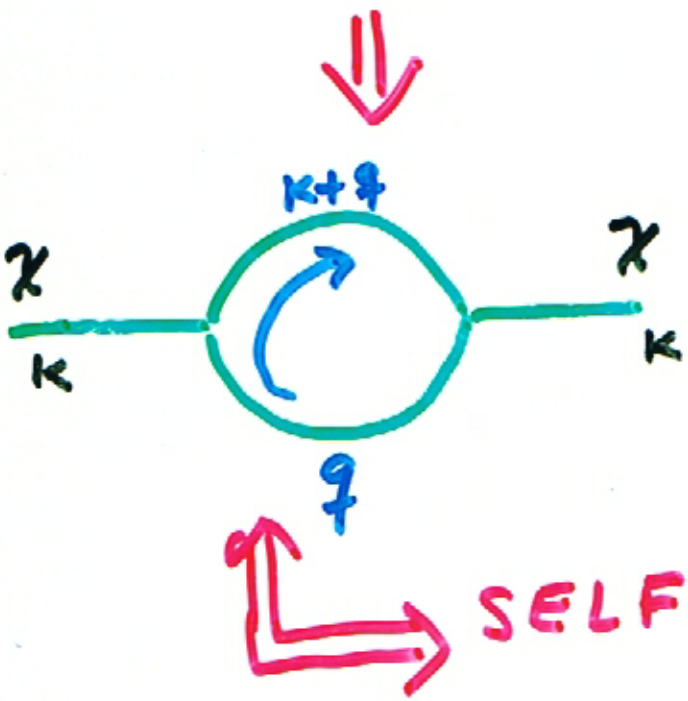
SLOW ROLL + WMAP → $\frac{g}{H} \sim 10^{-5}$

Scalar flucts WITH SELF-INTERACTI

Method: EQN of motion of expe value of flucts ⟹ TIME EVOL. OF FLUCTS. Need NON-EQ. Q.F.T

$$\chi_k''(\eta) + \left[k^2 + M^2 a^2 - \frac{a''}{a}\right]\chi_k(\eta) + \frac{2g^2}{\eta H^2}\int_{\eta_0}^{\eta}\frac{d\eta'}{\eta'}\,K(k,\eta,\eta')\,\chi_k(\eta') = 0$$

INTERACTION



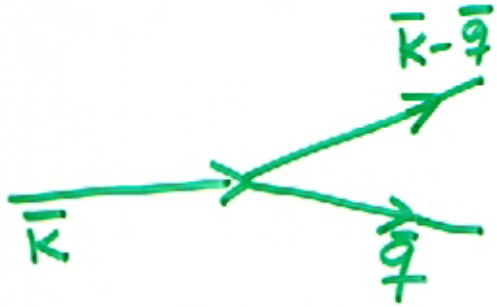
SELF-ENERGY = $K(k,\eta,\eta')$

NOVEL PROCESSES:

- Expansion → NO ENERGY CONSERVATION

Forbidden in Minkowski → Allowed during INFLATION

INFLATON FLUCTS SELF DECAY



- SLOW-ROLL $\Rightarrow$ $M/H \ll 1$ (ALMOST SCALE INV $\Rightarrow$ ALMOST MASSLESS STRONG I.R. BEHAVIOR: SUPER-HORIZON MODES $\approx \eta^{\frac{3}{2}-\nu}$

  $\Delta = \frac{3}{2} - \nu \ll 1$ I.R. REGULATOR

SELF-ENERGY:

- STRONG I.R.
- LONG TIME ($\eta \to 0$) DIVERGENCE
- Q.F.T: "RE" SUM $\Rightarrow$ DECAY

I.R. ENHANCEMENT (consequence of scale INVARIANCE!)

$$k[\kappa,\eta,\eta'] = \int d^3q\, I_m\Big[\underbrace{S_\nu(q,\eta)\, S^*_\nu(q,\eta')}_{\approx q^{-2\nu} \text{ as } q\to 0}\; S_\nu(|\vec{k}-\vec{q}|,\eta)\; S^*_\nu(|k-$$

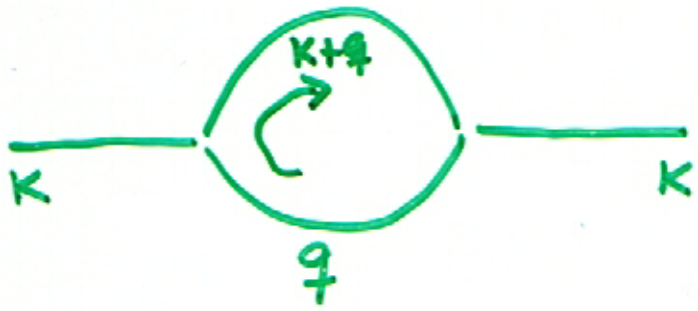
$$\Rightarrow \sim \frac{1}{\left(\frac{3}{2}-\nu\right)}$$: I.R. sing. in the scale INV. limit o Power spect.

Solve in $\mathcal{F}.T$: SUPERHORIZON

$$\chi_{k\sim 0}(\eta) \cong \underbrace{\chi^{(0)}_{k\sim 0}(\eta)}_{\text{NO INT.}} \Big[1 + \Gamma \underbrace{\ln\left(\frac{\eta}{\eta_0}\right)}_{\text{long time } (\eta\to 0) \text{ divergence!}} + \cdots\Big]$$

USE D.R.G. to SUM...

$$\chi_{k\sim 0}(\eta) \simeq \chi^{(0)}_{k\sim 0}(\eta) \underbrace{\left(\frac{\eta}{\eta_0}\right)^{\Gamma}}_{\text{DECAY of amplitude.}}$$



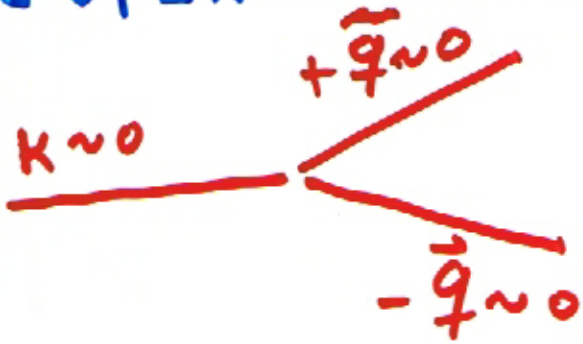
$$\sim \int d^3q\, (q\eta)^{-2\nu} \approx |\chi_q|^2$$

$$\Rightarrow \chi_k \sim \frac{1}{\Delta^2} \sim \frac{1}{(\frac{3}{2}-\nu)^2}$$

SMALL COUPLING $\sim \frac{g}{H} \sim 10^{-5}$ bu

I.R. ENHANCED!!

SUPERHORIZON MODES $k\eta \approx 0$



DECAY INTO "BACK-TO-BACK" PAIRS OF SUPERHORIZON MODES

Growing mode:

$$\chi_k(\eta) \sim \chi_k^{(0)}\, \eta^{\Gamma}$$

↑ UNPERTURBED

Γ = DECAY RATE =

$$\frac{32\, z_v^2\, \Delta_R^2}{\left(n_s - 1 + \frac{\Gamma}{4}\right)^2}\left[1 + \mathcal{O}(\text{slow roll})\right]$$

$$\approx 3.6 \times 10^{-9} \quad (\text{WMAP})$$

Sub-Horizon FLUCTS: $k\eta \gg 1$

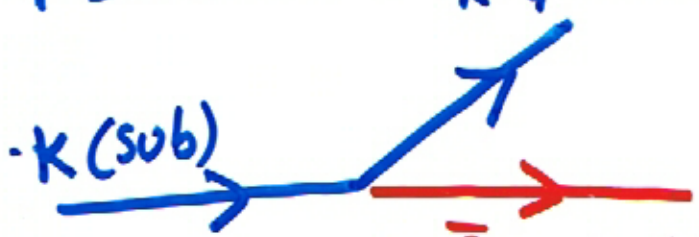
$$\chi_k(\eta) = \underbrace{\chi_k^{(0)}(\eta)}_{\text{unpert}}\, e^{-\Gamma[k,\eta]}$$

$$\Gamma[k,\eta] \sim \frac{g^2}{32\pi^2 H^2}\,\frac{H}{k_{ph}(\eta)}\left[1 + \frac{2H}{\pi k_{ph}(\eta)}\left(\frac{1}{\Delta} + \ln\left(\frac{k_{ph}(\eta)}{H}\right)\right)\right]$$

$k_{ph} = k/a(\eta)$

Bremm-radiation of SUPERHORIZ

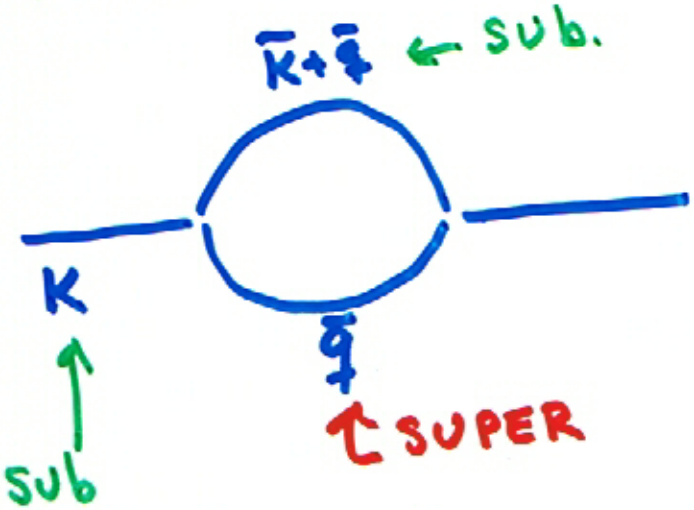
FLUCTS: $\bar{k} - \bar{q} \sim \bar{k}$ (sub)



$\bar{q} \approx 0$ (super) $\rightarrow \frac{1}{\Delta}$

NOTE: QUANTUM LOOP EFFECTS ⇒ CAUSAL CONNECTION BETWEEN SUB and SUPER-HORIZ

FLUCTUATIONS:



QUANTUM EMISSION OF SUPERHORIZON QUANTA !!

Suppres. by PHASE SPACE, but NOT by causality!

Sub-Horizon FLUCTS DECAY into SUPER-HORIZON fluctS

Phase space suppressed by $1/k_{ph}$

CONSEQUENCES: DECAY OF AMPLITU

OF sub-Horizon fluctS – Hasten

as $k_{ph} \to H$

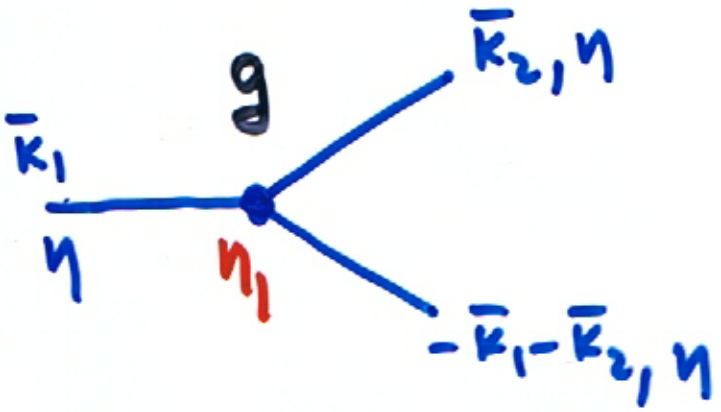
$\Rightarrow$ MATCHES ONTO SUPERHORIZON

WITH SMALLER AMPLITUDE!

CONNECTION WITH NON-GAUSS:

"Bi-SPECTRUM"

$$\langle 0 | \chi_{\bar{k}_1}(\eta)\, \chi_{\bar{k}_2}(\eta)\, \chi_{-\bar{k}_1-\bar{k}_2}(\eta) | 0 \rangle =$$



$\sim \int d\eta_1 \to$ long time divs

(No loop $\int$ ..)

$$\langle 0|\chi\,\chi\,\chi|0\rangle = \frac{g}{H}\,\underbrace{F[k,g,\eta]}$$

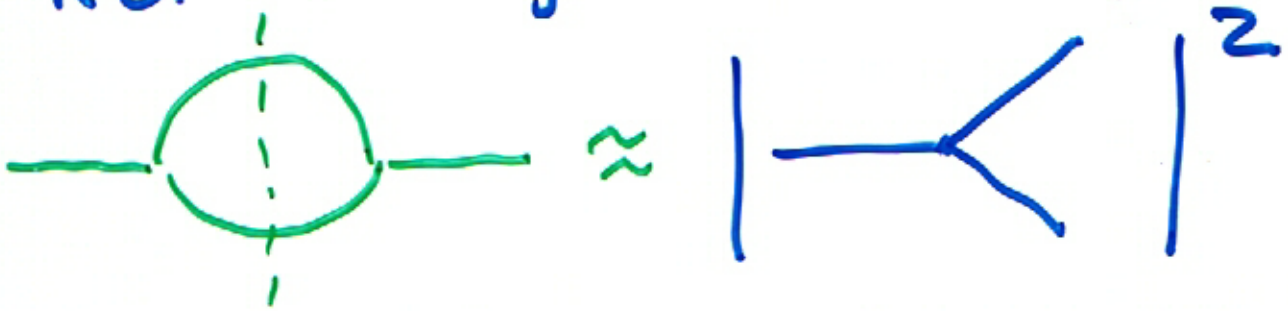
$\approx \ln[k\eta]$ for

$g \sim 0$; $k\eta \to 0$

SAME LONG-TIME DIV OF 

SELF-ENERGY!

Related by "OPTICAL THM":



$\Rightarrow$ SELF-INTERACTIONS $\Rightarrow$

- Suppression of Amplitudes by decay
- "Running" of index? (sub-Hu zon decay)
- NON-Gaussian "bi-spectrum"

Summary:

- Rapid Cosmological Expansion allows NOVEL QUANTUM PROCESSES: ENERGY NOT CONSERVED
- INFLATON FLUCS SELF-DECAY:
  - WMAP CONSTRAINS COUPLING
  - Slow-Roll ⇒ STRONG I.R. ENHANCEMENT CONSEQUENCE OF SCALE INV. POW. SPEC.
  - Superhorizon FLUCTS: DECAY IN "back-to-back" pairs.

$$\Gamma = \frac{32\,\xi_v^2\,\Delta_R^2}{(n_S - 1 + \Gamma/4)^2}$$

  - Growing MODE FEATURES ANOMALOUS SCALING
- Sub-Horizon FLUCTS decay by bremmstr. RAD. of SUPERHORIZ QUANTA (causal through Loops!

Consequences:

- Non-Gaussianity: Bi-Spectrum related to DECAY by "OPTICAL THM"

- Suppression of Amplitude of Super-Horizon fluctuations by decay

- Quantum processes allow decay of SUB into SUPER with phase space suppression

  Running Index?

D.B + H.J. de Vega + N.G. Sanchez
astro-ph 0409406

D.B. + H.J. deVega
P.R. D70, 063508 (04)