

QUANTUM TRANS-PLANCKIAN ORIGIN OF THE UNIVERSE AND MULTIVERSES

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(Dated: February 26, 2025)

Abstract: We describe the Universe quantum asymptotic gravitational Vacuum: the initial (in) Vacuum "*Nothing*", the late Vacuum State, ("*Everything*") today, and connect them through a unitary S -Matrix as a Classical-Quantum Dual Transform across the Planck scale. The **minimal** or initial time and **entropy** are in the range $0 < t_{in} \leq 10^{-61} t_P$, and $0 < S_{in} \leq 10^{-122}$ (t_P , and s_P being their Planck values). The late time and entropy being : $t_{today} = 10^{-61} t_P \leq t_{late} \leq t_{max}$, $S_{today} = 10^{122} \leq S_{late} \leq S_{max}$. The gravitational phase transitions in this context are discussed, early and last stages of expansion and contraction (bangs and crunches). The Initial Vacuum is a quantum gravitational foam of birthing and dying (crushing) universes, quantum and decaying black holes, particles and or strings and radiation. The truly quantum gravity (trans-Planckian) gravitational entropy $S_Q = 10^{-122}$ *dominates* the tunneling probability from "Nothing", which is almost equal to 1, strongly supporting the Universe "birth" as a quantum tunneling from the Vacuum. (In the non-complete picture the gravitational euclidean action is classical/semiclassical with huge value $S_\Lambda = 10^{+122}$ and extremely small tunneling, while in the *complete quantum picture* the effect is the *opposite*). The global quantum de Sitter space-time is found: The Penrose-de Sitter diagram is extended to the quantum level and a new diagram found, the physical spectrum covering all gravity domains. There are **no singularities** of any kind here. The imaginary time *quantum* de Sitter instanton is complete, its quantum gravity partition function, temperature and entropy are found. These results are all non-perturbative and strongly support the feature that a complete quantum theory of gravity must be a *finite theory* and *of pure numbers*, which is much more than a renormalizable theory.

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I. INTRODUCTION AND RESULTS

Most systems in the Universe have a "lifetime", they originate, decay, end (birth, grow, evolution, *transformation...*). We know *approximately* how systems in the universe begin or structure: planets, stars, galaxies, clusters, superclusters,...

A black hole originates as the final state of a (massive enough) star $M_{\star} > M_{crit \star}$, in fact the star *transforms* into a black hole. A black hole emit quantum mechanically and at the final evaporation stages when reaching the Planck mass scale (or a mass of the order of it), it *transforms*, e.g a phase transition occurs, into a quantum composite state which does decay into particles and radiation, as quantum heavy objects do, Refs [1], [2], [3], [4], [5], [6]. In turn, these quantum particles can collide themselves, desintegrate, or decay into another particle states ...*In Nature, nothing is lost, all is transformed...* [7]. A Scattering matrix or

S operator could be then associated to the process, which is unitary and CPT invariant [1], [8]

Let us physically formulate the question of how the Universe had started. (Or, more generally, how Multiverses started). By definition, the entire world is "Everything", (including, if any, the Multiverses). That is to say, the beginning was "Nothing", and had passed by a series of phase transitions, transforming eras, and evolutions, *from Nothing or "Nothing(s)" to Something(s) and to Everything*. The concept of entropy, gravitational entropy in particular, is well appropriated to describe "the arrow" of time and the cosmic gravitational history.

"Nothing" means Vacuum which, depending of its properties or state can have high activity and energy (quantum vacuum) or, on the contrary, be of very low energy, dilute and empty (classical vacuum), with the corresponding semi-classical or semi-quantum vacuum states. In fact, there are many and different vacuum states, e.g. *vacua*.

Our Universe today is mostly *empty*, filled by a very low energy (dark energy), e.g., a very dilute cosmological vacuum: This is a *classical* or semiclassical gravity vacuum, that is to say, our Universe *today* is *not* in a quantum gravity vacuum state, even if of course, a complete quantum theory of gravity must account for it too in its low energy limit.

The *full* quantum gravity *domain* is not reduced to be fixed to the Planck scale or to the neighborhoods of it, but it extends deeply in the highly quantum trans-Planckian domain:

$$E_{Planck} \leq E < \infty, \quad 0 < L \leq l_{Planck}, \quad 0 < t \leq t_{Planck}$$

The era or state of "Nothing" from which the Universe could have started is a *genuine* quantum trans-Planckian era, which is the vacuum period before the Planck time, (*the crossing scale*), followed by the known inflation period which is a semiclassical (or semi-quantum) gravity vacuum state, and followed by the different classical gravitational eras, radiation and matter dominated eras, until the today low energy vacuum phase (today dark energy).

The state of our Universe today is the most classical gravitational state: the most classical cosmological vacuum, the most diluted and empty, termed *voids* by the astronomical observations, with very low energy, (10^{-122} in Planck units) and very high gravitational entropy

(10^{122} , the highest physical magnitude in the Universe until now). This is the classical or semiclassical dark energy de Sitter state of the Universe today.

The fact that the latest classical/semiclassical state of the Universe and the early quantum/semiclassical inflation state of the Universe are both de Sitter (or quasi-de Sitter) vacua states, is a manifestation of the classical-quantum (wave-particle, de Broglie) duality of Nature *extended* across and beyond the Planck scale to include gravity in my 2019 trilogy, see Ref [8].

In this paper we provide new quantum physical understanding of the origins of the Universe *and Multiverses* with Planckian and trans-Planckian physics, and new results with classical-quantum gravity duality and quantum space-time in this context. The Standard Model of the Universe based on General Relativity and Quantum Field Theory is a semiclassical gravity Model, the best Model until now to describe the Universe in agreement with observations, but is not complete (quantum gravity is not included in the most early Planck time and before (Planckian and trans-Planckian phases). We complete this Model with quantum gravity back in time *in the far past before cosmic inflation*, but also in the late classical dual Universe: today and the far future, in agreement with observations.

Two main phases compose the quantum gravity completed Universe, the Planck scale being the *transition scale* between the two, (t_P being the Planck time): **(i)** The quantum early trans-Planckian phase $0 < 10^{-61} t_P \leq t \leq t_P$, (truly quantum gravity phase), and **(ii)** the semiclassical and most classical gravity post-Planckian universe $t_P \leq t \leq t_{today} = 10^{61} t_P$. The pre-Planckian era could be tested indirectly through its post-Planckian observables: as primordial gravitons, inflation, Cosmic Microwave Background observations and the today vacuum energy (dark energy).

In this paper we complete the above picture by including its *quantum gravity origins*, and the very last phases (the last classical dual stage once the Universe in the far future starts to *contracts* and then (after a time of about $10^{61} t_P$ in the contracting phase) decays, ending into regular (non singular) quantum gravity big bangs, all of them *finite* without any divergency nor singularity here, which then again, with different probabilities will lead to other Universe(s) like ours (Multiverse), as we describe here in Sections III and VII (

String theory entails *multivacua* or "*multiverses*", although it have serious problems to

describe or reproduce our own universe, (except to introduce fine tunings or additional hypothesis or principles), and similarly in order to reproduce the observed/detected particle physics phenomenology described by the robustly tested Standard Model of Particle Physics.

The aim of a theory describing the origin of the Universe from "Nothing" could be :

"From the theory of Nothing to the theory of Everything".

In general, the Quantum trans-Planckian Vacuum is necessarily of de Sitter type or Anti-de Sitter type nature, depending of the physical context involved, that is to say: **(i)** de Sitter vacuum (positive constant curvature) in the Universe and black hole interiors. **(ii)** Anti-de Sitter vacuum (negative constant curvature) for the particle physics vacuum. Vacuum dominance at the trans-Planckian era (eg the Universe arising from vacuum) implies that the Universe is a de Sitter or quasi de Sitter phase in its most earliest stage.

In Sections IV and V of this paper we describe the quantum de Sitter universe covering the different de Sitter regimes. The de Sitter vacuum is a simple and smooth constant curvature background without any physical singularity, it is maximally symmetric and can be described as a hyperboloid embedded in Minkowski space- time with one more spatial dimension.

We describe **in Section V of this paper** the de Sitter manifold with imaginary time, namely the *complete (quantum) instanton*: The classical gravity Gibbons-Hawking instanton, is cutted at the horizon: The classical de Sitter instanton is regular but *not* complete. The quantum trans-Planckian region which naturally does appear in quantum space-time allows to *complete* the classical de Sitter instanton. The *quantum* de Sitter instanton is thus both: regular *and complete*.

With these results we describe (**in Section VI here**) the *quantum* de Sitter Partition function covering all (classical and quantum) gravity regimes, and the total de Sitter entropy, that is, including the quantum gravity (trans-Planckian) entropy. We then clarify the comparison between the Quantum Field Theory (point particle) entropy (without gravity), the gravitational entropy, and the quantum extended object (string) entropy, particularly enlightened in terms of ordered and non ordered partitions.

Our studies here and in recent work see Ref [8] show clearly that the vacuum energy everywhere in the universe is the remnant of the initial trans-Planckian vacuum energy: the known (post-Planckian) inflation era is the direct (and more diluted) continuation of this quantum gravity vacuum. And in turn, a remnant of this vacuum does yield the dark energy today: A Bose-Einstein condensate, with the axion (and a lightest neutrino) both in the $\text{meV} = 10^{-3} \text{ eV}$ mass scale, do appear as the best candidate for dark energy, Ref [9].

The Hubble constant is in fact a Hubble $H(t)$ rate with today value $H_0 = H(t_0)$. In the early universe, $H(t)$ is truly associated to an early $\Lambda(t)$ rate, which most classical remnant in the Universe is the cosmological vacuum energy today $\Lambda(t_0) = \Lambda_0$. These results allow for a clarification of dark energy as the vacuum cosmological energy or cosmological constant, Ref [8]. And the very recent results of DESI Ref [10] on *varying dark energy evidence* support our results and predictions. Moreover, it should be interesting to apply our very recent (2025) quantum work Ref [11] on the description under which a quantum system *does appear classical* to the quantum gravity work presented here.

In this paper we analyze too the new quantum vacuum region inside the Planck scale hyperbolae which delimitate the quantum light cone in the de Sitter space-time. In Section IV we describe the global quantum de Sitter space-time structure: The *quantum* Penrose-de Sitter diagramme.

The quantum space-time is **totally regular**. This is also true in the quantum Schwarzschild-Kruskal space-time. The singularity is *removed* out from the quantum space-time, Refs [1], [3], [8]. More results of this paper and remarks are summarized in the Conclusions Section VIII here.

This paper is organized as follows:

In **Section II** we describe the cosmic history extended back in time before inflation, thus covering its different gravity phases: classical, semiclassical and quantum-Planckian and transplanckian- phases and their properties, including the gravitational entropy and temperature. In **Section III** we discuss the different notions of *vaccua* and the origin of the universe and *multiverses* from "*Nothing*" which is truly trans-Planckian. In **Section IV** we

describe the quantum de Sitter space-time structure with *quantum space-time coordinates*. In **Section V** we describe the *complete*, (eg with the quantum gravity phase included), imaginary time de Sitter instanton. **Section VI** we discuss the complete Partition Function and the complete quantum gravity de Sitter entropy. **Section VII** presents the expansion and contraction phases of the Universe and multiverses (bangs and crunches) as a quantum gravity foam of borning and dying universes, quantum and decaying back holes, particles and / or strings and radiation. **Section VIII** presents our conclusions.

II. COSMIC GRAVITATIONAL HISTORY. THE NEW EARLIEST TRANS-PLANCKIAN ERA

The gravitational history of the universe before the Inflation era and the current picture can be extended by including the quantum precursor phase within the standard model of the universe in agreement with observations. Quantum physics adds a new quantum Planckian and trans-Planckian phase of the Universe from the Planck time t_P until the extreme past $10^{-61}t_P$, which is an upper bound for the origin of the Universe, with energy $H_Q = 10^{61}h_P$, in a similar manner the present age of the Universe is a lower bound to the (unknown) future age.

The classical large dilute Universe today and the highly dense very early quantum trans-Planckian Universe are classical-quantum duals of each other in the precise meaning of the classical-quantum duality, here the classical-quantum gravity duality, at work in all cosmological eras and its associated gravitational quantities, temperatures, and entropies.

Cosmological evolution goes from a quantum trans-Planckian vacuum energy phase to a semi-classical accelerated era (de Sitter inflation), then to the classical known eras until the present classical de Sitter phase.

The classical-quantum or wave-particle-gravity duality specifically manifests in this evolution, between the different gravity regimes, and is the basis of a mapping between the asymptotic (in and out) early and late (today) states of the Universe characterized by the sets U_Q and U_Λ and thus as a scattering-matrix (S-matrix in time) description: The most early quantum trans-Planckian state in the remote past being the "in-state", and the very late classical dilute state being the far future or today "out-state", Refs[8], [9].

The quantum space-time structure consists of discrete levels of odd numbers Refs [?], [8]

$$X_n^2 = (2n + 1) \quad , \quad T_n^2 = (2n + 1) \quad (\text{in Planck units}) \quad , \quad n = 0, 1, 2, \dots \quad (2.1)$$

$$(X_n, T_n) \text{ and the } \textit{mass levels} \text{ being } \sqrt{(2n + 1)} \quad (2.2)$$

The Planck scale : $(X, T) = 1$ (in Planck units) is the fundamental level ($n = 0$) from which the levels (X_n, T_n) go to the quantum post Planckian and semiclassical levels (low and intermediate n) levels, to the classical (large n) levels. Asymptotically, for very large n the space-time becomes continuum.

While the Quantum trans-Planckian levels go as :

$$(X_n, T_n)_Q = 1 / \sqrt{(2n + 1)}, \quad (2.3)$$

For very high n , the quantum $(X_n, T_n)_Q$ space-time levels are highly excited (deep in the trans-Planckian domain), and desexcite gradually till $n = 0$. The total quantum levels cover *all* the range of scales from the smallest ones in the $10^{-61}l_P$ scales in the extreme past $10^{-61}t_P$ of the initial trans-Planckian phase to the largest cosmological scales and time $10^{61}t_P$ today, passing through the Planck scale (l_P, t_P) , thus covering the two principal phases: post and pre Planckian phases respectively.

III. VACUUM AND VACUA. UNIVERSE AND MULTIVERSES

If the Universe started from Nothing, which is the possibility we consider here as the most *natural* to address the question of the *origin* of the Universe, the Vacuum state from which the Universe have started must be the most remote past vacuum state, the state with the *lowest entropy*, entropy being the appropriate concept for describe this quantum trans-Planckian (or quantum gravity) era. We consider this Quantum Gravity (QG) Vacuum of the origin as the more general initial "In" Vacuum, the "Absolute Nothing" or Total Vacuum

$$| \Psi_{Total Vacuum} \rangle,$$

as a superposition of the various possible vacua, multivacua or "relative vacua":

$$| \Psi_{(vac 1)} \rangle, | \Psi_{(vac 2)} \rangle, \dots, | \Psi_{(vac n)} \rangle,$$

with different amplitude probabilities: A_1, \dots, A_n , from which the different Universes, Multiverses, are created, in particular our Universe, namely:

$$| \Psi_{QG Vacuum} \rangle = A_1 | \Psi_{(vac 1)} \rangle + A_2 | \Psi_{(vac 2)} \rangle + \dots + A_n | \Psi_{(vac n)} \rangle, \quad n = 1, 2, \dots \quad (3.1)$$

The origin initial state:

$$| In Vacuum \rangle_{t=t_{in}} \equiv | Nothing \rangle_{t=t_{min}} \equiv | \Psi_{QG-Vacuum} \rangle,$$

is a state of quantum trans-Planckian nature, and therefore with high Planckian and trans-Planckian energy fluctuations, namely of ”**foam**” nature, that is to say:

We consider this **foam** vacuum as an ensemble or ”soup” of borning and ”crushing” (or ”dying”) universes. The big crushing of the dying universes are Planckian and trans-Planckian events which are in turn borning events or big fluctuations (**regular** ”big bangs”) from which borning universes arise: These are **all finite** energy realisations, that is, **regular** realisations of the ”big bang(s)”.

Notice that all these explosions, bangs, crushings do **not** reach any infinity energy nor infinity curvature, nor any other infinity magnitude. They are totally *finite*, without singularity. The classical - quantum gravity duality (namely, the wave - particle duality including gravity) *precludes* realisations of physical magnitudes having infinity values, Refs [1], [8]. This is not a ”principle” nor an ”assumption” but the consequence of the ”classical - quantum duality of Nature, which is universal, irrespective of the type of the fundamental interaction, number or type of dimensions, existence or not of compactifications, etc.

Moreover, and consistently, this framework strongly supports the idea that a Quantum Theory of Gravity must be a *Finite Theory* (which is more than a renormalizable theory), as we discussed too in Ref [1]. The idea of a renormalizable theory does not apply to quantum gravity, already at the *conceptual level*, and besides that, as is known, it does not work at the technical level neither.

The **finite** quantum gravity scale explosions (regular big crushings and regular big bangs) in the most remote state, which is the ”*In state*” $| In Vacuum \rangle_{t=t_{in}}$ of the Universe could include too the **final states of the black hole evaporations**: At these last Planckian and trans-Planckian stages, black holes are quantum composite particles which necessarily

decay as *pure* quantum particles states: This *quantum decay* is like the decay of a heavy particle or a soliton and is *non-thermal*.

The *In Vacuum State* $| In Vacuum \rangle_{t=t_{in}}$, or Total Absolute Vacuum $|\Psi_{Total Vacuum}\rangle$, is the Trans-Planckian Vacuum State at the initial minimal time of the Universe t_{in} :

$$0 < t_{in} \ll t_{Planck} \equiv t_P$$

and with initial minimal Entropy S_{in} :

$$0 < S_{in} \ll S_{Planck} \equiv s_P$$

In the Universe evolution, $0 < t \leq t_{today} = 10^{61} t_P$ the classical-quantum gravity applied to our Universe fix the most remote past time or *the minimal time*, and the *minimal Entropy* of the Universe to be in the range:

$$0 < t_{in} \leq 10^{-61} t_P : \quad \mathbf{minimal\ time\ range} \quad (3.2)$$

$$0 < S_{in} \leq 10^{-122} s_P : \quad \mathbf{minimal\ entropy\ range} \quad (3.3)$$

The S-matrix as a Classical-Quantum Gravity Duality Transform:

The trans-Planckian (Quantum Gravity) phase is linked to the non trans-Planckian phases $t > t_P$, (Semiclassical Gravity and Classical Gravity phases), by a Quantum-Classical Duality transform across the Planck scale, *the crossing scale*, and conversely.

The Semiclassical Gravity phase is the usual (post-Planckian) Inflation era, followed by the known (radiation and matter dominated) stages until the *today era*, which is the *most classical gravitational state*: This state have the biggest known size, largest time, maximal known Entropy (the biggest physical number 10^{122} in Planck units), and the minimal Λ vacuum energy (10^{-122}).

There is a classical- quantum duality transform among the different stages through the Universe evolution, which acts as a mapping or **S- matrix transform** between the asymptotic initial Vacuum state, namely $| In Vacuum \rangle$, ("*Nothing*") at an initial time t_{in} , and the late Vacuum State $| Late Vacuum (t) \rangle$, ("*Everything*") at the today era:

$$| Late Vacuum (t) \rangle = S(t) | In Vacuum \rangle \quad (3.4)$$

$$| \Lambda(t) \rangle = S(t) | \Lambda(t = t_{in}) \rangle \quad (3.5)$$

The Evolution operator is like a Scattering matrix $S(t)$ in time, and must be unitary $S^+ S = S S^+ = 1$. Or, in terms of the *today time* t_0 :

$$| \Lambda(t) \rangle = S(t) S^{-1}(t_0) | \Lambda(t = t_0) \rangle \quad (3.6)$$

This S -*matrix* evolution is a classical-quantum gravity duality mapping relating small quantum (trans-Planckian) scales to large classical scales, passing across the Planck scale (the crossing scale): After the Planck scale, the semi-classical (or semi-quantum) gravity scales and regimes do occur: as Inflation and those stages of classical gravity plus QFT particle physics, until the present era (today dark energy).

A component of the Vacuum energy of the initial $| In Vacuum \rangle$ state goes to the Vacuum (GUT energy) state of Inflation $| Inflation \rangle$, which dilutes afterwards into the remaining Vacuum energy of the today state (Dark Energy), namely $| \Lambda_{(t=t_0)} \rangle$, and we have the relation:

$$| \Lambda(t_{inflation}) \rangle = S(t_{inflation}) | \Lambda_{(t=t_{in})} \rangle \quad (3.7)$$

Thus,

$$| \Lambda(t) \rangle = S(t) S(t_{inflation})^{-1} | \Lambda(t_{inflation}) \rangle, \quad (3.8)$$

which implies in particular the today state: $| \Lambda(t_0) \rangle$ for $t = t_0$.

The dis-excitement of the states along the cosmic evolution goes from the most highly excited *quantum discrete* Vacuum trans-Planckian levels ($n = 10^{122}$), to the Planck level ($n = 0$) at the Planck time t_P , and as time increases (and n increases $n = 1, 2, \dots, 10^{122}$) it goes into a *continuum* classical gravitational Vacuum today of very low energy: $\Lambda_{today} \equiv \Lambda(t_0) = 10^{-122}$ (in Planck units), today dark energy. Such quantum trans-Planckian phase as well as the inflation phase and the today phase are de Sitter phases.

The cosmological Vacuum energy of the Universe today $| \Lambda(t_0) \rangle$ is well described self-consistently (with back reaction included) by Quantum Field Theory in a semiclassical expanding FRW universe, as a Bose-Einstein condensate, with the best candidate being an axion in the $meV = 10^{-3}eV$ mass scale and a lightest neutrino in the same $10^{-3}eV$ mass scale, Ref [9].

Cosmological Vacuum Energy: We prefer the concept name **cosmological vacuum energy** (and not cosmological "constant") because:

The classical-quantum gravity duality and its quantum levels imply a varying vacuum energy, e.g a varying Λ_n at each discrete level n in time, and yield support to it: H_n varies and Λ_n varies too, even if of course this variation is very mild. Inflation needs too such a vacuum Λ variation in time. Moreover, what it is called Hubble constant is in fact a **Hubble rate** $H(t)$ which present value today $H(t_0)$ is the Hubble constant H_0 . Λ is precisely associated to H . Thus, what is called Λ cosmological *constant* would be in fact a Λ **rate**: $\Lambda(t)$ with $\Lambda(t_0) = \Lambda_0$ its present value (the cosmological constant today), as $H(t_0) = H_0$ is the present value of H (the Hubble constant).

We thus consider the S-matrix for the general universe evolution, from its origin trans-Planckian era $0 < t \leq t_P$ to its post-Planckian eras: (Inflation and late known eras), as a mapping or quantum - classical gravity duality transform from a state into another, from the initial to the final state. Quantum gravity implies a quantum discrete spectrum of levels for the gravitational physical magnitudes: mass, energy, size, time, entropy. Refs [1], [3], [8]. And such evolution imply inherently cosmic gravitational phase transitions:

Cosmic Gravitational Phase Transition: The passing from the quantum gravity discrete levels to the semiclassical gravity and the classical gravity continuum states can be considered as the effect of a gravitational phase transition into a semiclassical gravity spectrum and classical gravity continuum spectrum. And this phase transition could arise a graviton condensate state:

The remote past quantum trans-Planckian "soup" or "foam" is a gaz of borning (expanding) and dying (crushing) universes, each of them being of the de Sitter nature, which is the typical vacuum state, because among other properties, de Sitter state has *Maximal Symmetry*, it is a constant density vacuum, of constant curvature, totally regular, without any singularity, fully appropriate for a vacuum state, and having negative quantum pressure. We do correlate here with the Coleman statement (although in another, pure particle physics context, not in de Sitter context) Ref [12]:

The Symmetries of the Vacuum are the Symmetries of Nature.

- **The crushing dying universes** would be the corresponding **reverse time** events of the borning and evolving universes. This ensemble of states can also include the latest states of black hole evaporation because they are of similar trans-Planckian de Sitter nature and then decaying (non thermally) in pure (non mixed) states as gravitons, particles and radiation. In such a self-gravitational gaz (whatever being their components: particles, black holes, and or strings), a **typical phase transition** towards a condensate, larger "single" state, does appear.
- **Such gravitational phase transition** is a common feature to several contexts: from Hagedorn and the Carlitz phase transition to the "long string" (Refs [13], [14]), the de Vega - Sanchez self-gravitating bosonic phase transition (Refs [2], [15], [16]), the different types of black hole - string-particle phase transitions, (Refs [4], [5], [6]). Such a borning or emergent condensate state is a regular (*without any singularity*) constant curvature gravitational vacuum : the de Sitter vacuum.
- **Such a gravitational phase transition** with its vacuum condensate does occur at the two most separated asymptotic universe states or cosmic stages: the initial or most past remote quantum state ($0 < t \leq t_P$) and the latest classical state, the today dark energy era, (besides of occurring under different physical conditions in other intermediate eras of the universe and its known astrophysical systems which are not the subject here). Both *extreme* cosmic states (initial and today stages) are precisely classical- quantum *cosmological duals* of each other in the precise meaning of the classical- quantum gravity duality, here *classical- quantum cosmology duality*.

The dynamical explicit description of such *gravitational phase transitions* and the expressions for their corresponding complete physical magnitudes, is beyond the scope of this paper, but on the basis of all our previous results (Refs above) these gravitational phase transitions should all have a similar type, and approximate *a square root branch point* at the transition, eg in the temperature, entropy or equivalent magnitude).

Let us recall that the Planck mass $m_{Planck} = 10^{-5} \text{ gr}$ is the heaviest mass for a quantum elementary particle compatible with locality, (point particles with masses larger than m_{Planck} have quantum sizes L_Q - Compton lengths - smaller than their classical gravitational radius

L_G). Quantum gravitational states are necessarily *trans-Planckian*, with masses larger than m_{Planck} , and being *composite*, eg having internal structure and discrete levels, Refs [1], [8].

In the Euclidean (imaginary time) path integral of gravity Ref [19], the Euclidean action provides the zero order term to the gravitational entropy S and the tunneling probability is

$$P = A \exp[-S] \quad (3.9)$$

with A the factor amplitude. Is not our aim here to discuss the well known several proposals or prescriptions for the wave function amplitude, but our aim is to include here the new component with which the trans-Planckian part of the gravitational action does *complete* this picture.

We derived the *total* $Q\Lambda$ (classical and quantum) gravitational entropy $S_{Q\Lambda}$ in Ref [8] with the result:

$$S_{Q\Lambda} = 2 s_P + S_\Lambda + S_Q = 2 s_P \left[1 + \frac{1}{2} (10^{+122} + 10^{-122}) \right] \quad (3.10)$$

$$s_P = (\kappa_B a_P) / (4l_P^2) = (\pi\kappa_B)$$

s_P being the Planck entropy.

The *total* $Q\Lambda$ gravitational entropy is the sum of its three gravitational components : classical gravity entropy (subscript Λ), quantum trans-planckian entropy (subscript Q) and Planck value entropy (subscript P) corresponding to the tree gravity regimes. The term $2s_P$ arises from the duality between the quantum and classical lengths L_Q and L_Λ across the Planck scale. The factor 2 reflects the complete (global) covering: The Planck scale being the bordering or crossing scale common to the two (classical and quantum) Q and Λ domains.

Therefore, the *total* (complete) probability is:

$$P = \exp[-(2 s_P + S_\Lambda + S_Q)] = \exp[-s_P (2 + 10^{+122} + 10^{-122})] \quad (3.11)$$

The **new results** and implications of this expression in our context here are the following:

(i) The truly quantum gravity (trans-planckian) component of the gravitational entropy $S_Q = 10^{-122}$ *dominates* the tunneling probability P of the birth of the Universe from "Nothing", which is almost equal to 1. Therefore, this clearly shows that *the transplanckian*

component is entirely consistent with (and does strongly support) the "creation" of the Universe as a quantum process (tunneling) Ref [17] from the Vacuum.

(ii) In the non- complete picture (without the trans-Planckian domain), the gravitational euclidean action is entirely classical or semiclassical, ie $S_\Lambda = 10^{+122}$, its value is huge, and the tunneling probability is extremely small, while in the complete picture here the value is the opposite.

(iii) The gravitational entropy S_Λ of the present time large *classical* universe is a very *large number*, consistent with the fact that the universe today contains a very huge amount of information. Moreover, to reach such a huge size and entropy today 10^{+122} , the universe in its very beginning state should have been in a hugely energetic quantum initial vacuum, consistently with its $\Lambda_Q = 10^{+122}$.

$S_\Lambda = 10^{+122}$ and Λ_Q are the same numbers (in Planck units). As well as $S_Q = 10^{-122}$ and Λ are the same numbers: S_Λ and S_Q are quantum gravity duals one of each other, as well as Λ and Λ_Q .

(iv) The gravitational cosmic history can be described through *its vacuum energy* and their relevant physical properties: *size, age, gravitational entropy and temperature*, from the extreme past trans- Planckian phases, to the Planck scale and the post-Planckian phases:

trans-Planckian cosmic stages \longrightarrow Planck Scale \longrightarrow post-Planckian cosmic eras

(v) Post-Planckian phases go from Semiclassical gravity inflation, and the classical gravity phases until today most classical gravity phase (dark energy). The whole duration (of the trans-Planckian plus post-Planckians eras) is precisely: $t_{in} \leq t \leq 10^{+61} t_P$, (being $t_P = 10^{-44}$ sec the Planck time), and t_{in} is within the range $0 < t_{in} \leq 10^{-61} t_P$, Eq. (3.2).

(vi) That is to say, *each* time component *naturally* dominates in each phase: classical time component 10^{+61} in the classical era, quantum Planck time t_P in the quantum trans-Planckian era. The present time of the universe $10^{+61} t_P$, is a *lower bound* for the future (if any) age of the Universe, while its quantum dual $10^{-61} t_P$, is an *upper bound* for the most remote past t_{in} at the origins of the Universe.

IV. QUANTUM DE SITTER SPACE-TIME

De Sitter space-time is a hyperboloid embedded in Minkowski space-time with one more spatial dimension. The classical and quantum lengths: L_H, L_Q can be extended to the more complete length L_{QH} which contains both: the Q and H lengths).

It contains two non-zero lengths (L_H, L_Q) or two relevant scales (H, l_P) enlarging the possibilities for the space-time phases: classical, semiclassical, quantum (trans-Planckian), Planckian, and Minkowski (flat) regimes. The total (classical and quantum) QH Hubble constant H_{QH} , curvature R_{QH} and Λ_{QH} from the QH de Sitter length L_{QH} are described in Ref [8], [and their constant Planck scale values (l_P, h_P, m_P) only depending on (c, \hbar, G)], precisely characterize the classical, semiclassical, Planckian and quantum (super-Planckian) de Sitter regimes.

In Refs [8] we showed that de Sitter space-time can be described as an (inverted) harmonic oscillator. The g_{00} component determined by the Einstein Equations for the de Sitter metric entails the harmonic oscillator potential, e.g.

$$d/dr (r g_{00}) = 1 - (8\pi\rho_0 + \Lambda)r^2$$

and

$$X^2 - T^2 = H^{-2}(1 - H^2 r^2),$$

containing the (inverted) harmonic oscillator potential.

Inverted harmonic oscillator means that it has the minus sign in the potential or in the oscillator equations, that is to say, a imaginary frequency, the oscillator constant being precisely $\kappa_{oscill} = H^2$,

$$H \equiv \sqrt{(8\pi G \Lambda)/3} \tag{4.1}$$

The length of the oscillator is the Hubble radius and the Hubble constant is precisely the *surface gravity* of the universe (similar to the black hole surface gravity, the inverse of the its radius).

Quantum space-time can be described as a quantum oscillator with its quantum algebra. From the quantum oscillator (X, P) variables in global phase space, the space-time coordinates are promoted to quantum non-commuting operators. The relevant new and quantum light-cone structure refers to (X, T), and not to the transverse spatial X_j coordinates.

The quantum de Sitter space-time coordinates (X, T) can be considered quantum oscillator coordinates $(X, T = iP)$, including quantum space-time fluctuations and quantized levels.

The relevant quantum structure of the space-time arises from the relevant non-zero *space-time commutator* $[X, T]$, or non-zero quantum uncertainty $\Delta X \Delta T$ by considering *quantum* coordinates (X, T) . All other commutators are zero. The remaining transverse spatial coordinates X_{\perp} have all their commutators zero. This is enough to capture the essential quantum light cone structure (and this is the case in the quantum black hole space-time too).

In terms of the de Sitter phase-space representation (x^*, p^*) , the global de Sitter (X, P) coordinates are given by:

$$X = \exp(\kappa x^*) \cos(\kappa p^*), \quad P = \exp(\kappa x^*) \sin(\kappa p^*) \quad (4.2)$$

$$(X^2 + P^2) = \exp(2\kappa x^*) = 2 H_{osc}, \quad (4.3)$$

$$(X^2 - P^2) = \exp(2\kappa x^*) \cos(2\kappa p^*) = 2 H_{inv-osc} \quad (4.4)$$

with the star coordinate x^* :

$$\exp(Hx^*) = H^{-1} \sqrt{1 - H^2 r^2} \exp(Hr), \quad Hr < 1 \quad (4.5)$$

being the Hubble constant $H = \kappa$ the dimensionless (in Planck units) gravity acceleration or surface gravity.

In terms of the quantum (non-commutative) star coordinates (x^*, t^*) Eq (4.5) we have for the global quantum de Sitter representation:

$$X = \exp(\kappa x^*) \cosh(\kappa t^*), \quad T = \exp(\kappa x^*) \sinh(\kappa t^*) \quad (4.6)$$

$$(X^2 - T^2) = \exp(2\kappa x^*) \cosh(\kappa [x^*, t^*]) \quad (4.7)$$

$$(X^2 + T^2) = \exp(2\kappa x^*) \cosh(2\kappa t^*) \quad (4.8)$$

$$[X, T] = \exp(2\kappa x^*) \sinh(\kappa [x^*, t^*]) \quad (4.9)$$

where we used the usual exponential operator product:

$$\exp(A) \exp(B) = \exp(B) \exp(A) \exp([A, B])$$

New terms appear due to the quantum commutators. At the *classical level*: $[X, T] = 0$, $[x^*, t^*] = 0$, the known classical de Sitter equations are recovered.

The equation for the quantum hyperbolic de Sitter "trajectories" is

$$(X^2 - T^2) = \pm \sqrt{\exp(4\kappa x^*) + [X, T]^2} \quad (4.10)$$

$$= \pm \sqrt{(1 - H^2 r^2)^2 \exp(2Hr) + [X, T]^2} \quad (4.11)$$

The characteristic lines and what classically were the light-cone generating horizons

$X = \pm T$, (at $Hr = 1$, or $x^* = -\infty$), are now *quantically*:

$$X = \pm \sqrt{T^2 + [X, T]^2} \quad \text{at } Hr = 1: \quad X \neq \pm T, \text{ no horizons} \quad (4.12)$$

We see that $X \neq \pm T$ and the null horizons are quantum mechanically *erased*. In this respect, this is similar to the *quantum* Schwarzschild - Kruskal space-time structure Ref [1], or to the *quantum* Rindler - Minkowski space-time, although the de Sitter manifold is totally *regular*, without curvature singularity *already* at the classical level.

In Figure 1 we display the known classical (non quantum) Penrose diagram of the **classical** (non quantum) de Sitter space-time.

In Figure 2 we display the **new (quantum) Penrose diagram** of the **quantum** de Sitter space-time.

"External and internal" regions of de Sitter space are now with respect to the hyperbolae $X^2 - T^2 = \pm 1$, which correspond to the Planck scale (in Planck units). The internal region is entirely a finite (non-singular) quantum gravity (trans-Planckian) domain. The external region is the semiclassical and classical gravity domain. This applies too in Black holes of all types and whatever their masses, Ref [1].

The quantum space-time description with non commutative coordinates (X, T) , naturally provides discrete space - time levels (X_n, T_n) , $n = 0, 1, 2, \dots$ and discrete magnitudes, the Planck scale hyperbola $X^2 - T^2 = \pm 1$ corresponds to $n = 0$, (the fundamental level).

A new quantum trans-Planckian vacuum region is present inside the Planck scale hyperbolae, which is absent classically, and in which interior: there are no future and past concepts, because the four space-time parts merge, Fig 1. *The merging* of the four space-time regions in the trans-Planckian light cone structure region is a quantum gravity consequence of quantum space-time.

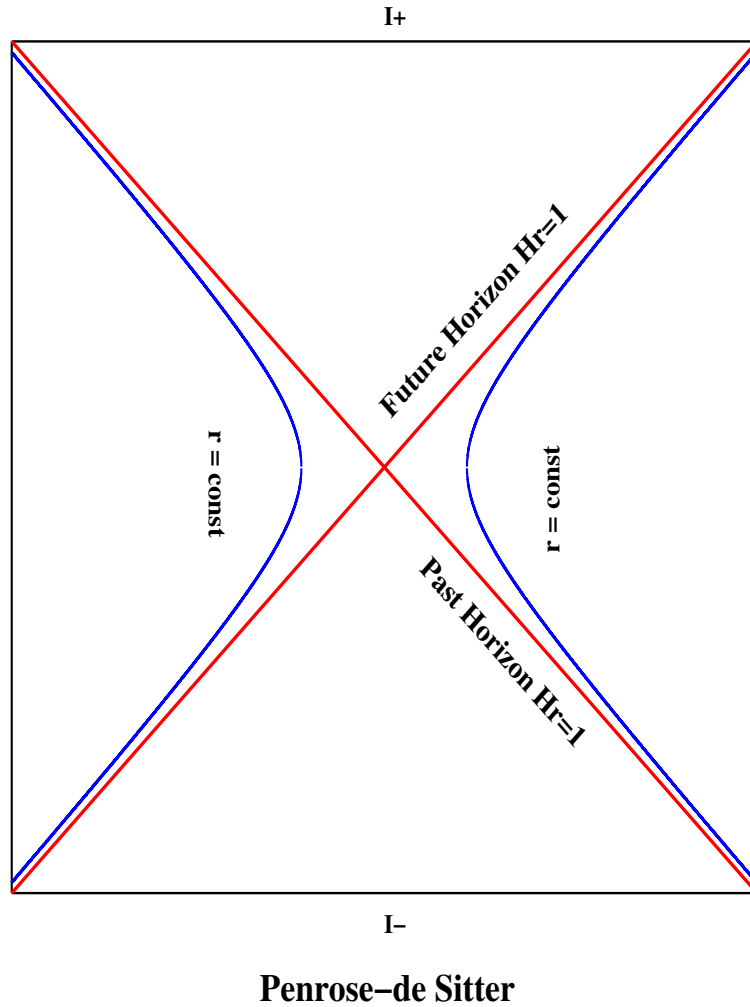
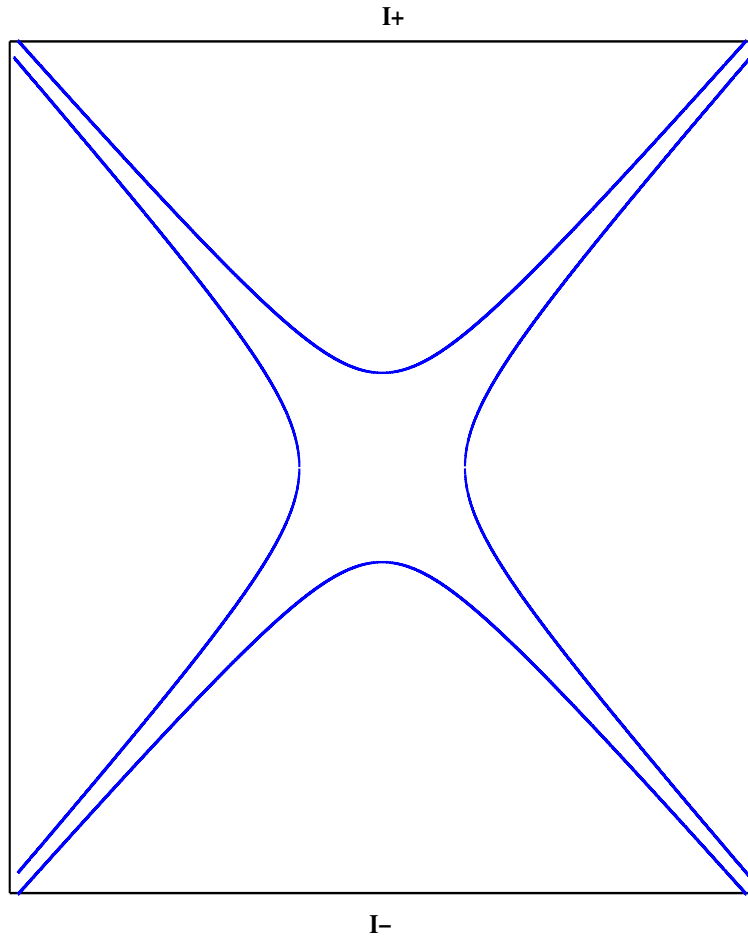


FIG. 1. **The known classical (non quantum) Penrose diagram of the classical (non quantum) de Sitter space-time.** The classical null horizons $X = \pm T$. and the classical light cone structure: The future and past null horizons. The classical hyperbolic trayectories $r = \text{const}$ and the four disconnected regions are displayed.

V. IMAGINARY TIME AND THE COMPLETE QUANTUM GRAVITY DE SITTER INSTANTON

As is known, in classical (non-quantum) space-time, taking imaginary time $T = i \mathcal{T}$, $t = i \tau$, transforms the hyperbolic space-time structure into circular structure: The characteristic lines $(X^2 - T^2) = 0$ collapse to $X = \pm \mathcal{T} = 0$, and similarly in the cart $x = \pm \tau = 0$.



Quantum Penrose–de Sitter

FIG. 2. **The new (quantum) Penrose diagram of the quantum de Sitter space-time found here.** The quantum hyperbolae $X^2 - T^2 = \pm l_P^2$ due to the quantum uncertainty or commutators $[X, T] = l_P^2$ generalize (and replace) the classical null horizons $X = \pm T$. In the classical limit both coincide. The new central quantum gravity region is a truly tran-Planckian energy region of constant curvature. The other four regions are the semiclassical/classical ones. From the Planck scale hyperbolae the space-time classicalizes. Asymptotically, the space-time structure is the same as the classical one. This is accomplished by the discrete space-time quantum levels in the central region which become quasi classical and classical (continuum) asymptotically, as it must be.

Therefore, the classical de Sitter horizon $X = \pm T$ ($Hr = 1$) collapses to the origin, and in the **classical de Sitter instanton**, (as well as in the black hole one), no horizon $Hr = 1$, (and no singularity), does appear in the instanton.

The euclidean (imaginary time) **quantum de Sitter instanton** clearly shows the *new* trans-Planckian region because for $Hr = 1$, $(X^2 + \mathcal{T}^2)$ is *not* zero but have Planck length radius: The equation for the quantum instanton "trajectories" are

$$(X^2 + \mathcal{T}^2) = \pm \sqrt{\exp(4\kappa x^*) - [X, \mathcal{T}]^2} \quad (5.1)$$

$$= \pm \sqrt{(1 - H^2 r^2)^2 \exp(2Hr) - [X, \mathcal{T}]^2} \quad (5.2)$$

What classically was the zero radius: $X = \pm \mathcal{T} = 0$ at $Hr = 1$ or $x^* = -\infty$, are now *quantically*:

$$X = \pm \sqrt{\mathcal{T}^2 - [X, \mathcal{T}]^2} \quad \text{at } Hr = 1: \quad X \neq \pm \mathcal{T} = 0, \text{ no horizon} \quad (5.3)$$

We see that for the quantum de Sitter instanton, it is:

$$X \neq \pm \mathcal{T} \neq 0 \quad \text{at} \quad Hr = 1.$$

The classical horizons corresponding to the origin $X = \pm \mathcal{T} = 0$ in the euclidean signature space (instanton) are *quantum* mechanically *replaced* by the circle of Planck length radius:

$$(X^2 + \mathcal{T}^2) = [X, \mathcal{T}]^2 = 1.$$

That is to say, quantum theory *consistently extends* the instanton manifold: classically the instanton is "*cutted*" at the "horizon" $r = 1/(2\kappa)$, while at the quantum level it *extends beyond it*: It contains the quantum region of Planck length radius l_{Planck} , which is necessarily trans-Planckian and is *absent* at the *classical* level.

This means the following: The *complete*, quantum and regular imaginary time de Sitter space, (*quantum gravitational instanton*), is the usual classical/semiclassical instanton for radius larger than the Planck length *plus a highly dense quantum domaine* of Planck length radius, and of high finite curvature, which is the trans- Planckian initial stage of the universe, and which is *absent* classically.

The imaginary time τ in the classical instanton is *periodic* with period $\beta = 1/\kappa$:

$$0 \leq \tau \leq \beta = 1/\kappa \quad (5.4)$$

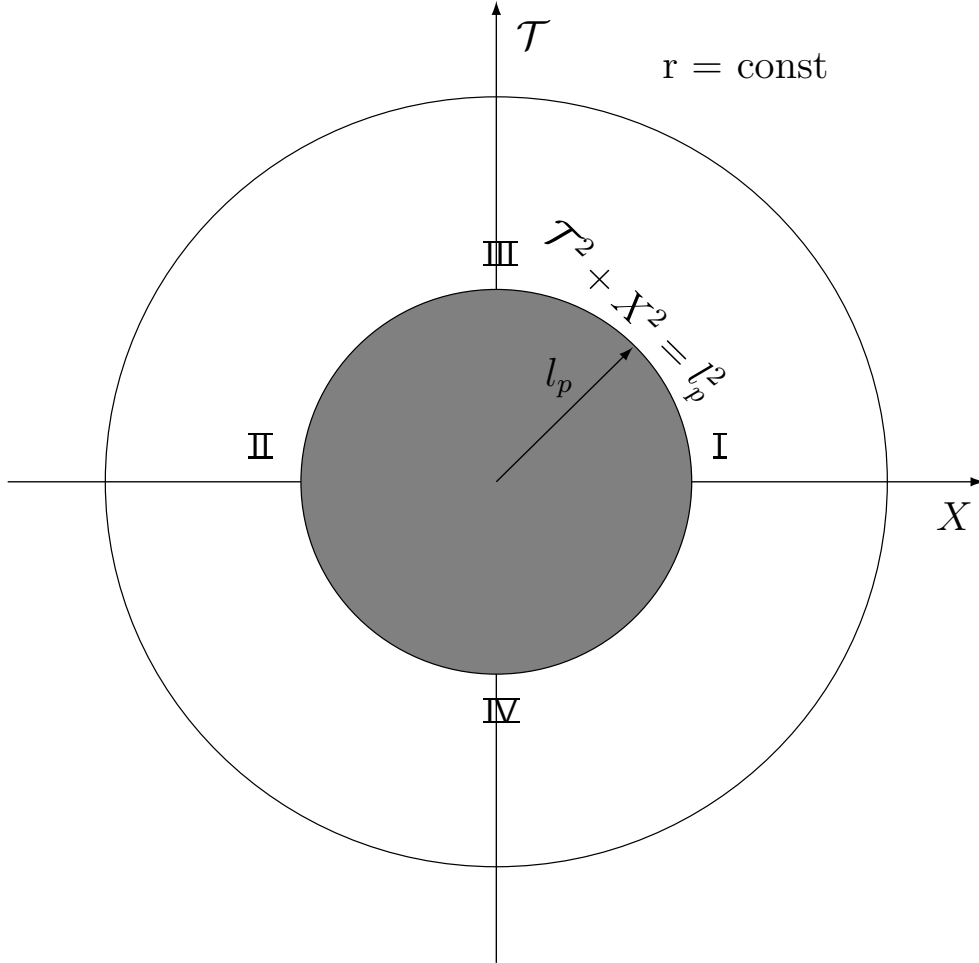


FIG. 3. **The new quantum gravity de Sitter instanton (imaginary time: $T = i\mathcal{T}$, $t = i\tau$).** The classical null horizons corresponding to the origin $X = \pm\mathcal{T} = 0$ in the *classical* de Sitter (Gibbons-Hawking) instanton are *extended* at the quantum level to the Planck length circle of $(X^2 + \mathcal{T}^2) = [X, \mathcal{T}] = 1$, (in Planck units). The de Sitter instanton manifold is **consistently extended** by the Quantum theory : The classical (non quantum) instanton is regular but is *not complete* because it is "cutted" at the null horizon $Hr = 1$, while at the quantum level it is *both: regular and complete*: It is the known classical instanton for distances larger than the Planck length plus the **new** highly dense *quantum region* of Planck length radius, and of high finite curvature, which is *absent* classically. The difference between the four regions disappears in the euclidean manifold, (indicated here for "classical" memory). The imaginary time τ in the **classical** instanton is *periodic* with period $\beta = 2L_G = 1/\kappa$: $1/\beta$ being the Hawking de Sitter temperature. In the complete **quantum** gravity instanton, the imaginary time is periodic with *complete* $L_{QG} = (L_G + L_Q)$ including both quantum and classical magnitudes. The complete de Sitter Temperature T_{QG} , Entropy S_{QG} and density of states all include the truly quantum gravity (trans-Planckian) domain, described in Sections V and VI.

In the complete (quantum) instanton, the imaginary time is periodic as in Eq. (5.4) but with the *total* κ completed with the quantum trans-Planckian magnitudes:

The imaginary time τ in the **classical de Sitter instanton** is *periodic* with period $\beta = 2L_H = 1/\kappa_H$

$$0 \leq \tau \leq \beta = 2L_H = 1/\kappa_H, \quad (\text{classically}) \quad (5.5)$$

$1/\beta$ being the intrinsic manifold semiclassical temperature: the Gibbons-Hawking de Sitter Temperature

$$T_Q = t_P \left(\frac{l_P}{2L_H} \right) = t_P \left(\frac{H}{2h_P} \right) = t_P \left(\frac{M_Q}{m_P} \right) \quad (5.6)$$

$t_P = (m_P c^2 / 2\pi\kappa_B)$ being the Planck temperature.

In the complete or *total* **quantum de Sitter instanton**, the imaginary time is periodic as in Eq.(5.5) but with the *complete* L_{QH} which includes the quantum Planckian and trans-Planckian magnitudes:

$$0 \leq \tau \leq \beta = 2L_{QH} = 2(L_H + L_Q) = 1/\kappa_{QH}, \quad (5.7)$$

$$\kappa_{QH} = \kappa_P (l_P/L_{QH}), \quad \kappa_Q = \kappa_P^2/\kappa_H, \quad \kappa_P = c^2/2l_P \quad (5.8)$$

$$\kappa_{QH} = \frac{H}{[1 + (H/h_P)^2]} = H \sum_{n=0}^{\infty} (-1)^n \left(\frac{H}{h_P} \right)^{2n} = \frac{H_Q}{[1 + (H_Q/h_P)^2]} \quad (5.9)$$

In the classical/semiclassical gravity domaine : $\kappa_H \ll \kappa_P$, ($H \ll h_P$), it yields the usual Hubble constant or classical surface gravity κ_H of the large universe stages, (eg massive M_H) with masses $M_H \gg m_P$.

In the quantum domaine of sizes and masses $L_H < l_P$, $H > h_P$, $M_H < m_P$: For $\kappa_Q \ll \kappa_P$, it yields the quantum $\kappa_Q = (h_P/H)\kappa_P = (M_H/m_P)\kappa_P$. The corresponding **complete** temperature being :

$$T_{QH} = t_P \kappa_{QH} / (2\pi\kappa_P), \quad T_Q = t_P^2 / T_H = t_P (H/h_P) \quad (5.10)$$

$$T_{QH} = \frac{T_H}{[1 + (T_H/t_P)^2]} = \frac{T_Q}{[1 + (T_Q/t_P)^2]} \quad (5.11)$$

In the late (dark energy) de Sitter stage of large universe radius, (large mass M_H), it yields the Gibbons-Hawking de Sitter temperature $T_Q = (M_Q/m_P) t_P = (H/h_P) t_P$, which is a very low temperature in this classical/semiclassical stage $T_Q \ll t_P$, as it must be.

For small radius and masses ($0 < L_H \leq l_P$), ($0 < M_H < m_P$), it yields the usual temperature $T_G \ll t_P$, proportional to the mass M_H , and $T_Q \gg t_P$, as it must be in the quantum gravity early de Sitter phase (the remote past trans-planckian phase). This is also manifest in the de Sitter Partition function (described in Section VI below) and in the corresponding *complete* de Sitter entropy (eg including the trans-Planckian entropy).

The Temperature is a measure of the length (in units of κ_B), $T_H = t_P (L_H/l_P)$, $T_Q = t_P (L_Q/l_P)$, while the gravitational entropy is a measure of the area. In this respect, is interesting to notice that:

$$S_{QFT} = s_P (L/l_P)^3 \Rightarrow n \quad (5.12)$$

$$S_H = s_P (L/l_P)^2 = s_P (M/m_P)^2 \Rightarrow (\sqrt{n})^2 \quad (5.13)$$

$$S_{string} = s_s (L/l_s) = s_s (L/l_s)^2 = s_s (M/m_s) \Rightarrow \sqrt{n} \quad (5.14)$$

$$S_Q = s_P (l_P/L)^2 = s_P (m_P/M)^2 \Rightarrow 1/(\sqrt{n})^2 \quad (5.15)$$

In quantum particle physics without gravity, that is Quantum Field Theory (QFT) the entropy is proportional to the volume (with a cut-off at the Planck scale l_P because of the well known ultraviolet divergences of QFT).

In string theory, the string entropy S_{string} is proportional to the length, with the string constant α' , (subscript s) playing the role of c^2/G^2 which is a tension.

In gravity theory, the gravitational entropy is proportional to the area, whatever be the classical/semiclassical gravity entropy S_H or the quantum gravity entropy S_Q . Thus, we do understand clearly that the area (gravitational entropy) "*interpolates*" between the volume (non-gravitational entropy S_{QFT}) and the length (string entropy) S_{string} .

Thus, the many questions formulated along the years about the difference between the gravity (or black hole, or de Sitter) entropy proportional to the area, and the usual QFT proportional to the volume, find in our Section here, a precise clarification. And the Section below on the Partitions functions provide strong support to these results, without invoking principles or hypothesis.

S_H is, in particular the known Bekenstein-Hawking entropy which is the a classical/ semiclassical gravitational entropy, valid for $L \gg l_P$ (or equivalently, $M_H \gg m_P$, $\kappa_H \ll \kappa_P$, $T_Q \ll t_P$):

$$S_H = s_P (T_H / T_Q) = (M_H c^2 / T_Q)$$

VI. QUANTUM GRAVITY DE SITTER PARTITION FUNCTION

As is known, $(D + 1)$ dimensional quantum field theory with imaginary periodic time ($0 \leq \tau \leq \beta$) corresponds to a classical statistical mechanics or field theory with temperature $(1/\beta)$, which is used in the euclidean path integral of gravity Ref [19],

$$\mathcal{Z} = Tr \exp(-\beta\mathcal{H}), \quad (6.1)$$

\mathcal{H} being the euclidean Hamiltonian \mathcal{H} , the "evolution" generator in imaginary time, with the trace implying periodic evolution ($0 \leq \tau \leq \beta$).

D-dimensional de Sitter space-time is the hyperboloid embedded in $(D + 1)$ Minkowski space-time. The $(D + 1)$ Minkowski imaginary time instanton with periodic time is the statistical mechanics theory in D-space dimensions. The $(D - 1)$ dimensional de Sitter instanton is the $(D - 1)$ sphere embedded in a D-dimensional euclidean space.

The complete (including *both* classical and quantum) de Sitter radius and temperature are L_{QH} and T_{QH} and T_{QH} as discussed in Section V above. The complete (whole range) discrete quantum levels are discussed in Section II, Eqs (2.1), (2.2), (2.3).

The total gravitational entropy $S_{Q\Lambda}$ of the total or complete (classical and quantum) de Sitter euclidean manifold, is the sum of the classical, quantum and Planck scale entropies:

$$d_{Q\Lambda} = \exp [S_{Q\Lambda}] \quad (6.2)$$

$$S_{Q\Lambda} = 2 \left[s_P + \frac{1}{2} (S_\Lambda + S_Q) \right], \quad (6.3)$$

$$S_\Lambda = \frac{\kappa_B}{4} \frac{A_\Lambda}{l_P^2}, \quad S_Q = \frac{\kappa_B}{4} \frac{A_Q}{l_P^2}, \quad s_P = \frac{\kappa_B a_P}{4 l_P^2} = \pi \kappa_B, \quad (6.4)$$

s_P is the Planck entropy, $A_\Lambda = 4\pi L_\Lambda^2$, $A_Q = 4\pi L_Q^2$ are the classical and quantum areas respectively (recall $L_Q = l_P^2/L_\Lambda$):

$$A_\Lambda = a_P \left(\frac{L_\Lambda}{\lambda_P} \right)^2, \quad A_Q = a_P \left(\frac{\lambda_P}{L_\Lambda} \right)^2 = \frac{a_P^2}{A_\Lambda}, \quad a_P = 4\pi l_P^2 \quad (6.5)$$

$$S_\Lambda = s_P \frac{\rho_Q}{\rho_P} = s_P \frac{\lambda_P}{\Lambda} = s_P 10^{+122} \quad (6.6)$$

$$S_Q = s_P \frac{\rho_\Lambda}{\rho_P} = s_P \frac{\Lambda}{\lambda_P} = s_P 10^{-122} \quad (6.7)$$

a_P being the Planck area, and λ_P the Λ Planck scale value : $\lambda_P = (3 h_P^2/c^4) = 3/(l_P^2 c^2)$. The *complete* (eg quantum gravity) de Sitter entropy is then:

$$S_{Q\Lambda} = 2 s_P \left[1 + \frac{1}{2} \left[\frac{\lambda_P}{\Lambda} + \frac{\Lambda}{\lambda_P} \right] \right] = 2 s_P \left[1 + \frac{1}{2} \left[\frac{h_P^2}{H^2} + \frac{H^2}{h_P^2} \right] \right], \quad (6.8)$$

and consistently, the **complete de Sitter partition function** is:

$$\mathcal{Z}_{Q\Lambda} = \exp[S_{Q\Lambda}] = z_P \mathcal{Z}_Q \mathcal{Z}_\Lambda \quad (6.9)$$

In the quantum trans-Planckian phase, which is the most early or *precursor* phase ($t_{in} \leq t \leq t_P$), the **total de Sitter entropy** S_{QG} is dominated by the Planck entropy s_P , the quantum entropy S_Q being extremely low: $S_Q = 10^{-122}$. The most remote past state $|In(t = t_{in})\rangle$ corresponds to the minimal time in the range ($0 \leq t_{in} \leq 10^{-61} t_P$) and minimal entropy S_{min} in the range ($0 \leq S_{in} \leq 10^{-122} s_P$).

The **total de Sitter entropy** S_{QH} is high in the post-Planckian (eg semiclassical and classical) eras: ($t_P < t \leq t_{today} = 10^{61} t_P$) and dominated by the Gibbons- Hawking de Sitter entropy S_H which is a semiclassical gravity entropy, being maximal in the today de Sitter phase: $S_H = 10^{122}$ (dark energy).

The discrete levels $n = 0, 1, 2, \dots$, cover *all* gravity regimes: from the quantum gravity (trans-Planckian and Planckian) phases to the post- Planckian universe (semiclassical and classical) gravity phases.

For ($0 \leq t \leq t_P$), ie in the trans-Planckian phase, the **quantum de Sitter levels**(Q denoting quantum) are:

$$H_{Qn} = \sqrt{(2n+1)}, \quad \Lambda_{Qn} = (2n+1), \quad S_{Qn} = 1/(2n+1), \quad n = 0, 1, 2, \dots \quad (6.10)$$

For ($t_P < t \leq t_{today} = 10^{61}t_P$), ie in the post-Planckian universe, the levels for the Hubble constant H_n , vacuum energy Λ_n , and gravitational (Gibbons-Hawking) entropy S_n (in Planck units) are:

$$H_n = 1/\sqrt{(2n+1)}, \quad \Lambda_n = 1/(2n+1), \quad S_n = (2n+1), \quad n = 0, 1, 2, \dots \quad (6.11)$$

The corresponding associated mass M -levels and scalar curvature R -levels in the respective classical/semiclassical gravity and quantum gravity (Q) stages are:

$$M_{Hn} = H_{Qn}, \quad M_{Qn} = H_n; \quad R_n = S_{Qn}, \quad R_{Qn} = S_n \quad (6.12)$$

Moreover, the **de Sitter density of states** in the respective gravity stages are:

$$d_{\Lambda n} = \exp(2n+1) = \exp(M_{Hn}^2) = \exp(\Lambda_{Qn}) \quad (6.13)$$

$$d_{Qn} = \exp[1/(2n+1)] = \exp(M_{Qn}^2) = \exp(\Lambda_n) \quad (6.14)$$

$$d_{Q\Lambda n} = \exp[(2n+1) + 1/(2n+1)] = \exp(\Lambda_{Qn} + \Lambda_n) \quad (6.15)$$

- The **complete de Sitter (Q Λ) density of states** $d_{Q\Lambda n}$ contains both: the classical/semiclassical gravity density $d_{\Lambda n}$ with the known (Bekenstein-Hawking) entropy $S_{\Lambda n}$, and the quantum gravity de Sitter density d_{Qn} with the new trans-Planckian entropy S_{Qn} .
- In the initial quantum region, n diminishes from the most highly excited trans-Planckian levels, until $n = 0$ and then increasing in the semiclassical and classical universe. The n -levels completely cover from the highest excited levels in the remote past of the universe until the lowest excited ones in the present era today, fully covering the twofold dual branches, classical and quantum, passing through the Planck ($n = 0$) level, the *fundamental* level, the crossing scale.
- As n increases, radius and mass of the Universe, H_n and Λ_n both decrease, S_{Hn} increases and *consistently* the Universe *classicalizes*.

- The most remote past trans-Planckian level t_{in} corresponds to the maximum trans-Planckian level n_{max} with the **most quantum** values: maximum curvature R_{Qmax} , maximum vacuum Λ_{Qmax} and minimum entropy S_{Qmin} :

$$10^{122} \leq n_{max} < \infty; \quad 10^{122} \leq (R_{Qmax}, \Lambda_{Qmax}) < \infty, \quad (6.16)$$

$$0 < S_{Qmin} \leq 10^{-122} \quad (6.17)$$

n decreases crossing the Planck ($n = 0$) level: $H_{Planck} = 1 = \Lambda_{Planck} = S_{Planck}$, and then enters the post-Planckian universe : $n = 1, 2, \dots, n_{infl} = 10^{12}, \dots, n_{cmb} = 10^{114}, \dots, n_{reion} = 10^{118}, \dots, n_{today} = 10^{122}, \dots, n_{max}$, with the **most classical** values:

$$H_{min} \leq H \leq H_{today} = 10^{-61}, \quad \Lambda_{min} \leq \Lambda \leq \Lambda_{today} = 10^{-122},$$

$$10^{122} = S_{today} \leq S \leq S_{max} .$$

For n in the range:

$$n \in [n_{max}, 0] : \quad \mathbf{Quantum\ Trans-Planckian\ Gravity},$$

the gravity spectrum is fully quantum and trans-Planckian.

For n in the range:

$$n \in [0, n_{max}] : \quad \mathbf{Semiclassical\ and\ Classical\ Gravity},$$

the gravity spectrum is quantum for the low n which are quantum space-time levels with energies smaller than the Planck energy. The spectrum becomes semiclassical for intermediate n , and classical for the high and very high n until reaching n_{max} , where

$$10^{+122} = n_{max\ today} \leq n_{max} < \infty : \quad \mathbf{Classical\ Gravity},$$

n is **bounded** by n_{max} and **finite** because of the classical-quantum gravity duality:

- n_{max} corresponds to both most quantum state and its most classical dual state, that is to say: n_{max} is the highest excited trans-Planckian level, for example the most quantum state in the earliest phase of the Universe, which removes out the initial singularity. Similarly, in the black hole case, it corresponds to the most central internal level in the constant curvature interior trans-Planckian core of the black hole which quantically removes the classical central singularity.

- n_{max} corresponds too to the lowest excited level of the post-Planckian Universe , eg. the most classical gravity and empty state of the Universe with maximal classical/semiclassical entropy, $S_\Lambda \text{ today} = 10^{+122}$ which in this regime is the Gibbons - Hawking entropy, and vacuum energies in the $meV = 10^{-3}eV$ (today dark energy).
- Consistently, in the Universe post-Planckian phase, the lowest excited level n_{max} corresponds to the lowest energy and most classical gravity state eg, the most large size region, with maximal Bekenstein-Hawking entropy and the minimal vacuum energy Λ (dark energy).

VII. EARLY AND LAST STAGES OF THE UNIVERSE. EXPANSION AND CONTRACTION (BANGS AND CRUNCHS)

Our results here and mainly the time, Hubble H and Λ spectrum in Section VI not only have implications for the initial and late phases of the Universe but also for its *future* evolution and *final* phases.

(X_n, T_n) are given in Planck (length and time) units. Eqs. (2.1) and (2.2) translate into the discrete levels for the Universe radius, Hubble constant and vacuum energy Λ

$$L_n = \sqrt{(2n+1)} = 1/ H_n, \quad n = 0, 1, 2, \dots \quad (7.1)$$

The Universe size , vacuum energy Λ , Hubble rate and their associated gravitational magnitudes : curvature and entropy have discrete levels from the most fundamental one ($n = 0$), going to the semiclassical stage (intermediate n), to the classical ones (large n) which yield a continuum classical space-time, radius and mass, as it must be. This is clearly seen from the global mass levels $M_{n\pm}$, (and similarly for the radius levels):

$$M_{n\pm} = [M_n \pm \sqrt{M_n^2 - m_P^2}], \quad (7.2)$$

The whole spectrum is covered. The condition $M_n^2 \geq m_P^2$ simply corresponds to the whole spectrum $n \geq 0$:

$$M_{n\pm} = m_P [\sqrt{2n+1} \pm \sqrt{2n}] \quad (7.3)$$

$$M_{(n=0)+} = M_{(n=0)-} = M_{Q(n=0)} = m_P, \quad n = 0 : \text{ Planck mass} \quad (7.4)$$

$$M_{n+} = m_P \left[2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2}) \right], \quad \text{branch (+) : masses } > m_P \quad (7.5)$$

$$M_{n-} = \frac{m_P}{2\sqrt{2n}} + O(1/n^{3/2}), \quad \text{branch (-) : masses } < m_P \quad (7.6)$$

- Large n levels are semiclassical tending towards a classical continuum classical /semi-classical phase. Low n are quantum, the lowest mode ($n = 0$) being the Planck scale. The dual (\pm) branches are manifest in the local variables ($\sqrt{2n+1} \pm \sqrt{2n}$) corresponding to the large and small n behaviours and covering the *whole* spectrum: from the largest entropy and scales (branch (+)) to the quantum smallest entropy and scales (branch (-)) passing by the Planck entropy and length.
- The last stage of the universe of decreasing radius ("big crunch") and its quantum decay are in the quantum mass branch (-) with Planck scale radius, Planck mass and smaller.
- Branch (+) covers all *macroscopic*, classical universe phases as well as the semiclassical radius quantization $\sqrt{2n+1}$ until nearby the Planck length radius ($n = 0$).
- The *microscopic quantum* de Sitter universes (with masses near the Planck mass and smaller), ie originated from Planckian and trans-Planckian primordial fluctuations, or as a consequence of big crunchs, belong to the branch (-).
- Branches (+) and (-) cover respectively the early and late phases of the Universe. The Universe in its evolution from its remote past to today and the far future goes from the branches (-) to (+), and would return to branch (-) (big crunch), and then again in a big expansion. Black holes in the process of their formation and evaporation do the opposite, going from classical to quantum: the branches (+) to (-), decaying and then passing again from branch (+) to (-).
- There are no quantum objects at such heavy mass as the Planck mass which would remain stable. They naturally decay quantum mechanically in all particles, mainly gravitons and radiation. Therefore, the end of universes in the last crunch, quantum trans-Planckian states, is the decay in gravitons, radiation, and other elementary particles.

Consequently, the whole process of the universe formation, expansion and end by collapse and crunch can be considered in terms of a Scattering-matrix between the asymptotic states of such phases.

The Universe (U) formation through the quantum tunneling from the Vacuum can be described as a S-matrix evolution (\mathcal{S}_U):

$$| \Psi_U (t) \rangle = \mathcal{S}_U (t) | \Psi_{Vacuum} (t = t_{in}) \rangle \quad (7.7)$$

And in general:

$$| \Psi_U (t) \rangle = \mathcal{S}_U (t) | \Psi_{QU} (t_{in}) \rangle \quad (7.8)$$

It can be expressed in terms of the final Vacuum state at $t = t_{final}$, in the final expansion phase that is to say, the future time at which the Universe reverses from expansion to contraction.

In addition, microscopic de Sitter states formed from the crunching universe phase can in turn decay, and end into a gaz of particles and radiation which eventually, under gravity and pressure conditions, forms again de Sitter states, stars and black holes. That is to say, the quantum initial gravitational Vacuum can be arised from the decay of the crunching Universe states (or at least a part of it):

$$| \Psi_{QU} (t_{in}) \rangle = \mathcal{S}_{QU} (t_{in}) | \Psi_{QU} (t_{final}) \rangle \quad (7.9)$$

Therefore,

$$| \Psi_U (t) \rangle = \mathcal{S}_U (t) \mathcal{S}_{QU} (t)^{-1} | \Psi_{QU} (t) \rangle \quad (7.10)$$

This description applies too to black holes, Ref [1]. These are relevant examples in which *gravitational unitarity* does hold in a whole complete quantum process: the *S*-matrix in the whole process is unitary $SS^+ = 1 = S^+S$ as it must be [7]: *"In Nature nothing is lost, all is transformed"*.

VIII. CONCLUSIONS

The Introduction and Sections of this paper include themselves some of the novel results of this paper, and therefore we will not include all of them here. We summarize Conclusions and outlook below.

- We provide a description of gravitational Vacuum: the initial $| \textit{In Vacuum} \rangle$, ("*Nothing*"), and the late Vacuum State $| \textit{Late Vacuum} (t) \rangle$, ("*Everything*") today, characterized these two asymptotic states as "in" and "out" states, and connected them through an S - Matrix as a Classical-Quantum Dual Transform through the Planck scale, and which is unitary as it must be. The **minimal or initial time** is in the range $0 < t_{in} \leq 10^{-61} t_P$, with the minimal entropy range $0 < S_{in} \leq 10^{-122}$ (t_P , and s_P being the Planck time and entropy). The late time is in the range $10^{-61} = t_{today} t_P \leq t_{late} \leq t_{max}$ with the maximal entropy $S_{today} = 10^{122} \leq S_{late} \leq S_{max}$. This is accompanied with the discussion of the **gravitational phase transitions in this context** and the description of the early and last stages of the Universe as expansion and contraction phases (**quantum gravitational bangs and crunches**). The Initial Vacuum is thus a quantum gravitational foam of birthing and dying (crushing) universes, quantum and decaying black holes, particles and or strings and radiation.
- For the **quantum origin** of the Universe from the Vacuum ("*Nothing*") we have very **new** results : The truly quantum gravity (trans-planckian) component of the gravitational entropy $S_Q = 10^{-122}$ **dominates the tunneling probability** P of the birth of the Universe from "*Nothing*", which is almost equal to 1. Therefore, this clearly shows that *the transplanckian component* is entirely consistent with (and does strongly support) the "creation" of the Universe as a quantum process (**tunneling from the Vacuum**).
- Notice that in the non- complete picture (without including the quantum gravity trans-Planckian domain) discussed until now in the literature, the gravitational euclidean action is entirely classical or semiclassical, ie $S_\Lambda = 10^{+122}$, its value is huge, and the quantum tunneling probability is extremely small, while in the **complete quantum picture here** the Probability value is almost one, and **the tunneling effect is relevant**.
- The **global quantum structure of de Sitter space-time** is found from quantum coordinates, and a consistent quantum picture of de Sitter space-time does appear from the internal quantum trans-Planckian region, to the semiclassical and classical late regions until (today Universe), accompanied with their physical magnitudes and spectrum. The **quantum de Sitter Penrose diagramme** is found.

- There are **no singularities** of any kind here. Besides of being a useful system of units, the Planck scale has full physical meaning as the quantum gravity onset scale, the Planckian and trans-Planckian domaine, which precludes to extrapolate until zero time or length. de Sitter vacuum is a smooth constant curvature vacuum **without any curvature singularity**. The small quantum de Sitter vacuum have a high but **bounded** trans-Planckian constant curvature and therefore **without singularity**. No divergencies, no infinities arise here. This is a theory of **finite** magnitudes and numbers. This is also consistent with the fact that the **number of Multiverses** is **finite** and in agreement with Hawking (and Hartle) last work [21].
- In the imaginary time manifold (**instanton**), we have **quantum completed here the classical de Sitter Gibbons-Hawking instanton**, which is cutted at the horizon: The classical de Sitter instanton is *not* complete. The de Sitter quantum instanton is complete. The **quantum gravity de Sitter partition function**, temperature and entropy clearly include the new highly excited and dense trans-Planckian de Sitter core of radius l_P .
- **The earliest quantum universe states** from the trans-Planckian phase (**regular "big bangs"**) reach the Planck mass smoothly into more and more massive decoherentized states (known expansion evolution phases including inflation). In the inverse process (**regular "big crunchs"**), the Universe states reach the Planck mass m_P from the semiclassical or classical gravity "side" (as black holes do), and undergo a phase transition into pure (non mixed) quantum gravity states. They then decay into gravitons, other elementary particles and radiation in the similar way heavy particles, quantum strings, or Planck mass black holes do decay.
- **The results of this paper** should open new research directions and new understanding : for quantum theory in its own right, for gravity, eg quantum gravitational signals, for e-LISA [27] for instance, after the successul results of LIGO [28],[29], as well as for other observable cosmological effects [34],[30], [31],[32], [39],[40] [41],[42] although the discussion of them is beyond the scope of this paper.
- A Scattering-matrix between the asymptotic (in and out) very different de Sitter vacua is described as a Dual Classical-Quantum mapping across the Planck scale. The

most early quantum trans-Planckian Vacuum is the in-state, and the most late: the far future or today classical gravity dilute Vacuum, is the out-state: dark energy today.

- **The results of this paper** provide further strong support to the conceptual general feature described by this author in Ref [1] that a complete quantum theory of gravity is a *finite theory*, which and a *theory of pure numbers*.
- **The results of this paper** on the cosmological initial and late Vaccua should also help for a better understanding or resolution of the present Hubble constant H_0 problem or other tensions, which are beyond the scope of this paper and we not discuss them here. In addition, a post-inflation early low vacuum energy (remnant of the inflationary vacuum) could not be excluded in principle, if appropriately implemented. Such vacuum energy rate encompasses a Hubble $H_0(t)$ rate, related to a early $\Lambda(t)$ rate, which most classical remaining in the present Universe is the cosmological vacuum energy $\Lambda(t_0)$, e.g Λ_0 today, e.g. dark energy, and the very recent results from DESI reported in Ref [10] on **varying dark energy support our results and predictions**. Finally, it should be interesting to apply our very recent quantum work (2025, Ref [11]) on the description under which a quantum system does appear classical to the quantum gravity work presented here.

ACKNOWLEDGEMENTS

I acknowledge discussions and communications with Gerard 't Hooft, Adam Riess and Roger Penrose on various occasions.

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