Axion-Sterile-Neutrino Dark Matter

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Mainly based on

Salvio, Scollo, Universe 7 (2021) 354 arXiv:2104.01334

keV Warm Dark Matter in Agreement with Observations in Tribute to Héctor J. De Vega

The Standard Model (SM) and its needed extensions

The SM is very successful but, certainly, it has to be extended:
e.g. it does not include gravity and does not (completely) account for

- **Dark Matter** (DM)
- Neutrino Oscillations.
  (Obvious candidates to solve this problem are right-handed neutrinos N_i)
- Baryon Asymmetry of the Universe (BAU)

Other SM problems

(besides DM, neutrino oscillations and BAU)

Electroweak vacuum metastability

In order to ensure the absolute stability of the electroweak (EW) vacuum one needs

$$M_t < (171.09 \pm 0.15_{\rm th} \pm 0.25_{\alpha_3} \pm 0.12_{M_h})\,\text{GeV} = (171.09 \pm 0.31)\,\text{GeV}$$

[Salvio (2017)], *[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2013)]*

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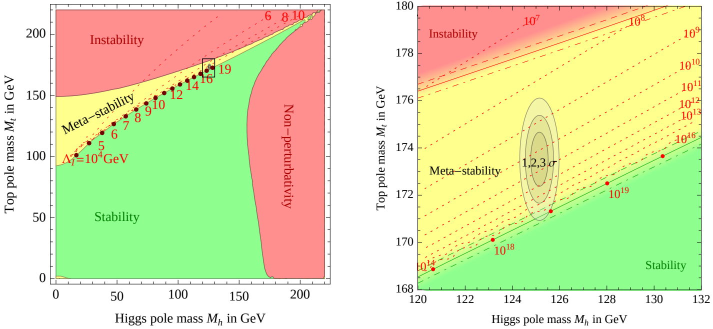
- Phase diagram of the SM:

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2013)]



The metastability is a SM problem during inflation

- During inflation the energy were so high that transitions to the true minimum were possible $\rightarrow$ interesting upper bounds on the Hubble rate during inflation *[Joti, Katsis, Loupas, Salvio, Strumia, Tetradis, Urbano (2017)]*

- The condition to have SM Higgs inflation is very similar to the stability bound

$$M_t < (171.43 \pm 0.12_{\rm th} \pm 0.28_{\alpha_3} \pm 0.12_{M_h})\,{\rm GeV} = (171.43 \pm 0.32)\,{\rm GeV}$$

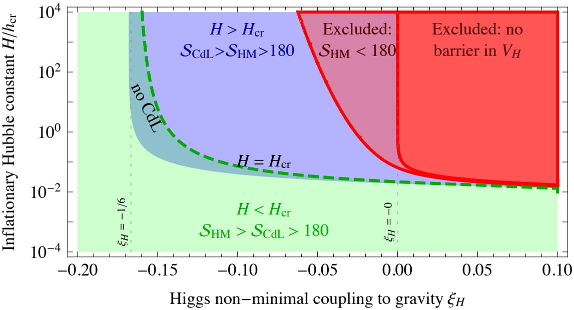
then we need new physics to account for inflation

Upper bounds on the Hubble rate during inflation

The model:

$$\mathscr{L} = \mathscr{L}_{\rm EH} + \mathscr{L}_{\rm SM} + \xi_H |H|^2 R$$

The results:



The strong CP problem

One can add a P and CP violating term to the QCD Lagrangian:

$$-\frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu},$$

where

$$G^a_{\mu\nu} \equiv \text{gluon field strength}, \qquad \tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} G^a_{\alpha\beta}$$

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example: $d_{\text{neutron}} \sim |\theta| e \frac{m_\pi^2}{m_{QCD}^3} \sim 10^{-16} |\theta| e \times \text{cm}$

[Baluni (1978); Crewther, Di Vecchia, Veneziano, Witten (1979)]

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This suppression is not enough: experimentally $d_{\rm neutron} \lesssim 10^{-26} e \times$cm

$$\rightarrow |\theta| \lesssim 10^{-10}$$

Peccei-Quinn symmetry

Idea by *Peccei and Quinn (1977)*: promote θ to a dynamical variable such that changes in θ are equivalent to redefinitions of the various fields and so have no physical effect.

This is implement through a global chiral $U(1)$ (the Peccei-Quinn symmetry, $U(1)_{\mathrm{PQ}}$): some colored fermions are charged under $U(1)_{\mathrm{PQ}}$

$\Longrightarrow$ because of chiral anomaly a field redefinition leads to

$$\theta \to \theta + \Delta\theta$$

Field redefinitions cannot affect physics so any value of θ is equivalent to

$$\theta = 0$$

(the P and CP conserving value)

Peccei-Quinn symmetry and axions

In the presence of fermion masses

$\rightarrow$ the condensing field, which gives mass to fermions, is charged under $U(1)_{\mathrm{PQ}}$

Since any colored fermion is (or seems to be) massive
$\rightarrow U(1)_{\mathrm{PQ}}$ is spontaneously broken

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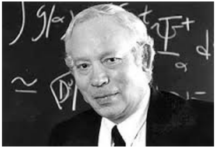
This leads to a pseudo Goldstone boson called *the axion*
[Weinberg (1978); Wilczek (1978)]



A simple solutions for the problems above: add axions, right-handed (sterile) neutrinos

The $a\nu$MSM *[Salvio (2015, 2018)]*

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_N + \mathscr{L}_{\rm axion} + \text{gravity part}$$

Right-handed neutrino sector:

$$\mathscr{L}_N = i\overline{N}_i \partial\!\!\!/ N_i + \left(\frac{1}{2} N_i M_{ij} N_j + Y_{ij} L_i H N_j + \text{h.c.}\right)$$

Axion sector (KSVZ):

$$\mathscr{L}_{\rm axion} = i \sum_{j=1}^{2} \overline{q}_j D\!\!\!\!/\, q_j + |\partial_\mu A|^2 - (y\, q_2 A q_1 + h.c.)$$
$$-\lambda_A (|A|^2 - f_a^2/2)^2 - \lambda_{HA}(|H|^2 - v^2)(|A|^2 - f_a^2/2)$$

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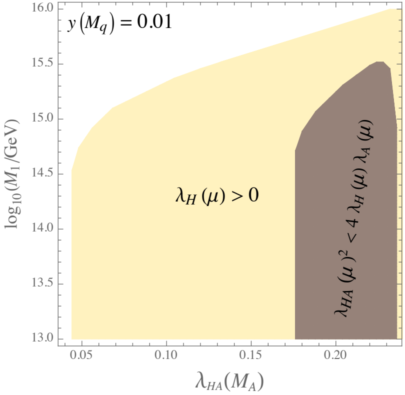
In some region of the parameter space the model accounts for

- **Neutrino oscillations** (through right-handed neutrinos which symmetrize the SM field content)
- **DM** **(through the axion and the lightest sterile neutrino)**
- **Baryogenesis** (through leptogenesis triggered by the right-handed neutrinos)

The $a\nu$MSM and absolute stability



- In the plot we set
  - the SM and low-energy neutrino parameters around the central values
  - the lightest neutrino mass $m_1 = 0$, $M_2 = 10^{14}$GeV
  - $f_a = 10^{11}$GeV and $\lambda_A(M_A) = 0.05$

The $a\nu$MSM and inflation

[Salvio (2015, 2018)]

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_N + \mathscr{L}_{\rm axion} + \text{gravity part}$$

This model was further studied by several scientists, e.g.
[Ballesteros, Redondo, Ringwald, Tamarit (2016)]
who proposed a variant where the Majorana masses M_i are generated by $\langle A \rangle$

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Inflation can be triggered by the Higgs and/or by $|A|$
[Salvio (2015, 2018)], *[Ballesteros, Redondo, Ringwald, Tamarit (2016)]*

By sitting at the frontier between stability and metastability (criticality) one can avoid further new physics or strong coupling at subplanckian energies in the case of Higgs inflation *[Salvio (2017, 2018)]*

The $a\nu$MSM and criticality

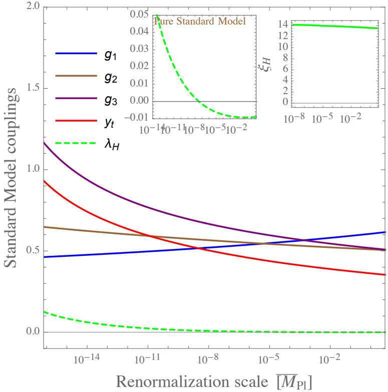

Figure: *Representative RG evolution of the relevant SM parameters close to criticality (λ_H is nearly zero at the Planck scale).*

The $a\nu$MSM and inflation: results

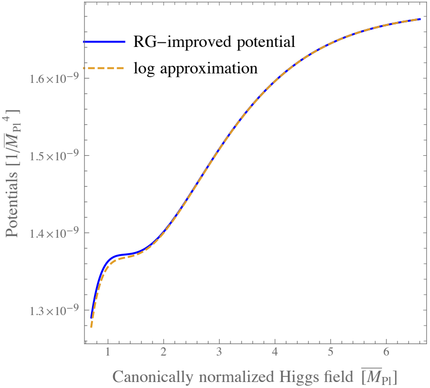

Figure: *RG-improved potential and its log-approximation close to criticality.*

Inflationary observables: $n_s \approx 0.96$, $r \sim 0.01$, $A_s \approx 2.1 \times 10^{-9}$ in agreement with the most recent Planck results (2018)

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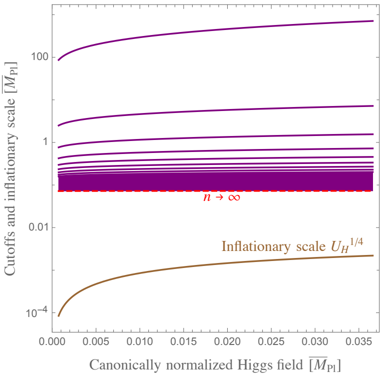


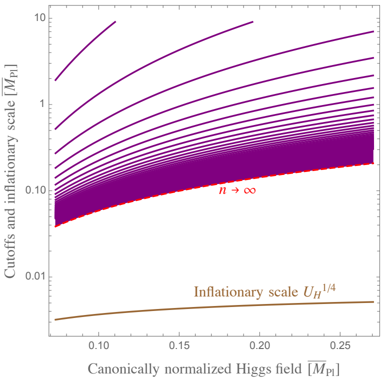


Figure: *The cutoff of the theory obtained by reading the coefficients of the dimension-n operators $\delta h'^n$ (for $n > 4$ and varying n) is compared to the inflationary scale.*

The $a\nu$MSM and dark matter (DM)

Work with Simone Scollo

There are three possible sources of DM in the aνMSM

- axion
- lightest sterile neutrino
- Primordial black holes?

Axion dark matter

The axion is a good dark matter candidate

Axions are produced non-thermally through

Misalignement mechanism: *[Preskill, Wise, Wilczek (1983); Abbott, Sikivie (1983); Dine, Fischler (1983); Turner (1986)]*

A recent calculation gives *[Ballesteros, Redondo, Ringwald, Tamarit (2016)]*

$$\Omega_a h^2 = (0.12 \pm 0.02) \left(\frac{f_a}{1.92 \times 10^{11} \mathrm{GeV}} \right)^{1.165}$$

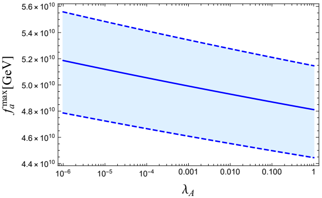
If $\Omega_a = \Omega_{\rm DM}$ this fixes f_a

Higgs inflation features a high reheating temperature, $T_{\rm RH} \gtrsim 10^{13}$ GeV, thanks to the sizable couplings between the Higgs and other SM particles *[Bezrukov, Gorbunov, Shaposhnikov (2008)]*, *[Bellido, Figueroa, Rubio (2009)]*.

Thus the PQ phase transition occurs after inflation in this case

String decay: *[Davis (1986); Harari, Sikivie (1987); Davis, Shellard (1989); Battye, Shellard (1997); etc]*

It was estimated in the KSVZ model by *[Ballesteros, Redondo, Ringwald, Tamarit (2016)]*



Sterile-neutrino DM

The lightest sterile neutrino N_1 with mass m_s can contribute a fraction Ω_s of $\Omega_{\rm DM}$

It can be produced through a mixing θ with the active neutrinos. θ can receive a contribution from the mixing $\theta_{\alpha 1}$ of N_1 with the active neutrino of any flavour $\alpha \in \{e, \mu, \tau\}$:

$$\theta^2 = \sum_{\alpha=e,\mu,\tau} |\theta_{\alpha 1}|^2$$

- Non resonantly *[Dodelson, Widrow (1994)]*

  For a standard quark-hadron crossover transition, $T_{\rm QCD} \approx 170\ \rm MeV$, one obtains *[Abazajian (2005)]*

$$m_s \approx 3.4\ \mathrm{keV} \left(\frac{\sin^2(2\theta)}{10^{-8}} \right)^{-0.615} \left(\frac{\Omega_{\rm s}}{0.26} \right)^{0.5}$$

- Resonantly *[Shi, Fuller (1998)]*: similar to the Dodelson-Widrow mechanism but there is a resonant enhancement due to a primordial lepton asymmetry

Sterile-neutrino DM

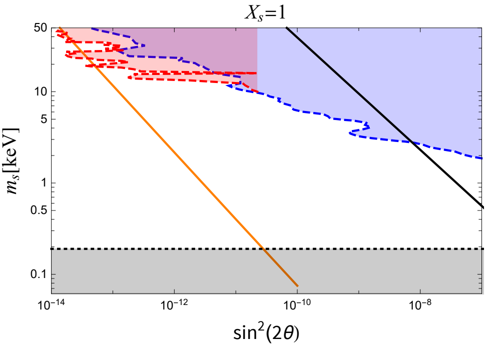
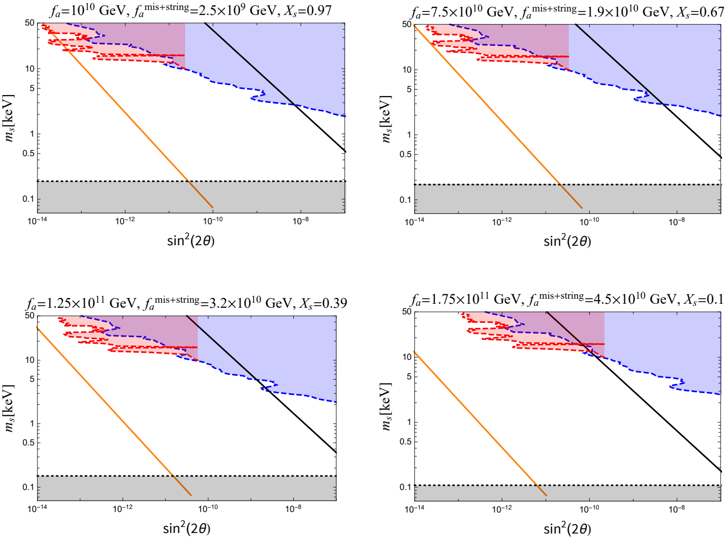


Figure:
The black line is the non-resonant sterile-neutrino production.
The region between the black and orange line is the resonant sterile-neutrino production.
The upper constraints are given by X-rays searches and the bound in dashed black is a phase-space bound related to Pauli's exclusion principle

The allowed regions have $m_s \sim$ keV and a very small θ

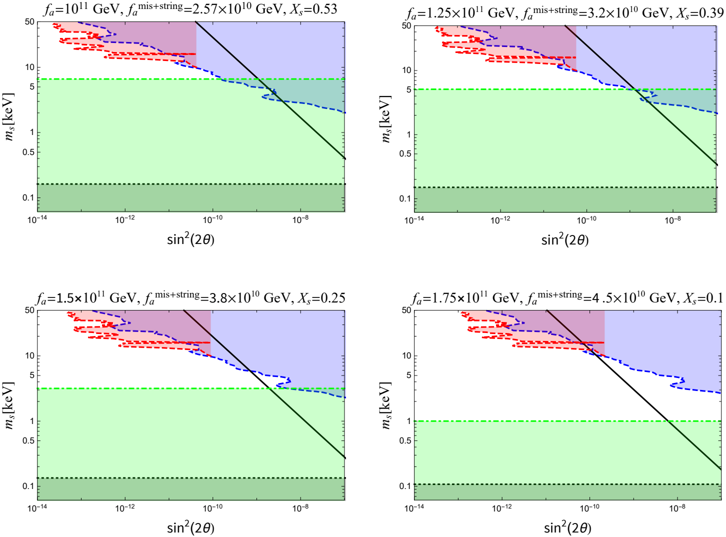
Axion-sterile-neutrino DM



If you include some estimate (subject however to large uncertainties) of structure formation bounds one finds a small region of parameter space allowed for resonantly produced sterile neutrino DM.

Axion-sterile-neutrino DM

Adding the structure formation bounds in the non-resonant case (green dot-dashed lines) *[Palazzo, Cumberbatch, Slosar, Silk (2007)]*



An allowed region appears only for $X_s \lesssim 0.3$

Primordial black holes?

- Primordial black holes may be generated if the curvature power spectrum has a peak of order $\sim 10^{-2}$*[Hertzberg, Yamada (2017)]*.
- This is about 7 orders of magnitude larger than at ~ 60 e-folds before the end of inflation (the $A_s \sim 10^{-9}$ measured by Planck).

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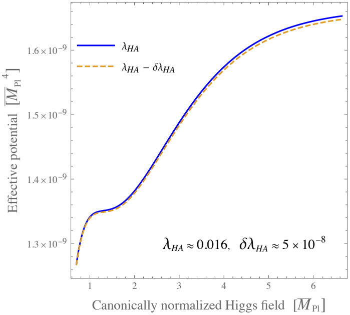
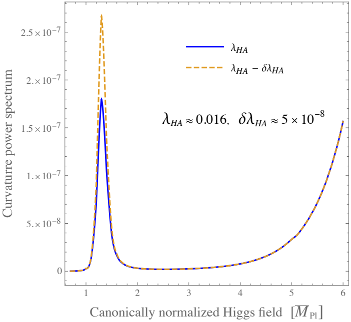




We approach criticality by lowering λ_{HA} but the number of e-folds become too large before reaching the required height

Summary

It was proposed a model (aνMSM) that combine the idea of axions and right-handed neutrinos and accounts for *all* the observational evidence for new physics as well as inflation and solve the strong-CP problem as well as the metastability issue of the SM. In particular we have discussed:

1. Critical Higgs inflation can be implemented in a viable way, but primordial black holes cannot contribute to DM
2. Multicomponet axion-sterile-neutrino DM (work with Simone Scollo)

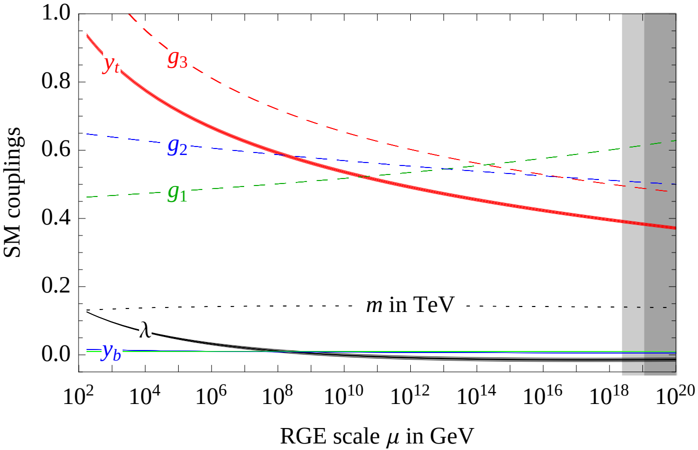
This can be achieved accounting for neutrino oscillation, baryogenesis, absolute stability and inflation at the same time

Thank you very much for your attention!

Extra slides

The consistency seems ok (up to the Planck mass $M_{\rm Pl}$)

Some couplings diverge as a function of the energy μ (Landau poles), but above $M_{\rm Pl}$



Solutions of the renormalization group equations (RGEs) of the most relevant SM parameters

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Qualitative origin of the stability bound

$$V_{\rm eff} = V + V_1 + V_2 + ...$$

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2, \quad V_1(h) = \frac{1}{(4\pi)^2}\sum_i c_i m_i(h)^4 \left(\ln\frac{m_i(h)^2}{\mu^2} + d_i\right), \quad ...$$

where $h^2 \equiv 2|H|^2$ and c_i and d_i are ~ 1 constants

By substituting bare parameters $\to$ renormalized ones

$$\Longrightarrow \frac{\partial V_{\rm eff}}{\partial \mu} = 0$$ and one is free to choose μ to improve perturbation theory

- Since at large fields, $h \gg v$, we have $m_i(h)^2 \propto h^2$, we choose $\mu^2 = h^2$, then

$$V_{\rm eff}(h) = \frac{\lambda(h)}{4}\left(h^2 - v(h)^2\right)^2 + ... = -\frac{m(h)^2}{2}h^2 + \frac{\lambda(h)}{4}h^4 + ...$$

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So for $h \gg v$

$$V_{\text{eff}}(h) \approx \frac{\lambda(h)}{4}h^4$$

- M_h contributes positively to $\lambda \rightarrow$ lower bound on M_h
- y_t contributes negatively to the running of $\lambda \rightarrow$ upper bound on M_t

Procedure to extract the stability bound

Steps of the procedure

- $V_{\rm eff}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called *threshold corrections* or *matching conditions*) at the EW scale (e.g. at M_t) ...

Finally impose that $V_{\rm eff}$ at the EW vacuum is the absolute minimum!

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State-of-the-art loop calculation

- Two loop $V_{\rm eff}$ including the leading couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop RGEs for $\{\lambda, y_t, g_3, g_2, g_1\}$ and one loop RGE for $\{y_b, y_\tau\}$ ...
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Previous calculations

[Cabibbo, Maiani, Parisi, Petronzio (1979); Casas, Espinosa, Quiros (1994, 1996); Bezrukov, Kalmykov, Kniehl, Shaposhnikov (2012); Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia (2012)]

Input values of the SM observables

(used to fix relevant parameters: λ, y_t, g_1, g_2)

$$M_h = (125.09 \pm 0.21_{\text{stat.}} \pm 0.11_{\text{syst.}})\,\text{GeV}.$$

[ATLAS and CMS Collaborations (2015)]

M_W	=	80.384 ± 0.014 GeV	Mass of the W boson [1]
M_Z	=	91.1876 ± 0.0021 GeV	Mass of the Z boson [2]
M_h	=	125.15 ± 0.24 GeV	(source quoted above)
M_t	=	$173.34 \pm 0.76 \pm 0.3$ GeV	Mass of the top quark [3]
$V \equiv (\sqrt{2}G_\mu)^{-1/2}$	=	246.21971 ± 0.00006 GeV	Fermi constant [4]
$\alpha_3(M_Z)$	=	0.1184 ± 0.0007	$\text{SU}(3)_c$ coupling (5 flavors) [5]

[1] TeVatron average: FERMILAB-TM-2532-E. LEP average: CERN-PH-EP/2006-042

[2] 2012 Particle Data Group average, pdg.lbl.gov

[3] ATLAS, CDF, CMS, D0 Collaborations, arXiv:1403.4427. Plus an uncertainty $\mathcal{O}(\Lambda_{\text{QCD}})$ because of non-perturbative effects [Alekhin, Djouadi, Moch (2013)]

[4] MuLan Collaboration, arXiv:1211.0960

[5] S. Bethke, arXiv:1210.0325

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Step 1: effective potential

RG-improved tree level potential (V)

Classical potential with couplings replaced by the running ones

One loop (V_1)

V_{eff} depends mainly on the top, W, Z, h and Goldstone squared masses in the classical background h: in the Landau gauge ... they are

$$t \equiv \frac{y_t^2 h^2}{2}, \quad w \equiv \frac{g_2^2 h^2}{4}, \quad z \equiv \frac{(g_2^2 + 3g_1^2/5)h^2}{4}, \quad m_h^2 \equiv 3\lambda h^2 - m^2, \quad g \equiv \lambda h^2 - m^2$$

$\rightarrow (4\pi)^2 V_1$ is (in a suitable renormalization scheme, called $\overline{\text{MS}}$)

$$\frac{3w^2}{2}\left(\ln\frac{w}{\mu^2} - \frac{5}{6}\right) + \frac{3z^2}{4}\left(\ln\frac{z}{\mu^2} - \frac{5}{6}\right) - 3t^2\left(\ln\frac{t}{\mu^2} - \frac{3}{2}\right) + \frac{m_h^4}{4}\left(\ln\frac{m_h^2}{\mu^2} - \frac{3}{2}\right) + \frac{3g^2}{4}\left(\ln\frac{g}{\mu^2} - \frac{3}{2}\right)$$

In order to keep the logarithms in the effective potential small we choose

$$\mu = h$$

Indeed, t, w, z, m_h^2 and g are $\propto h^2$ for $h \gg m$

Two loop (V_2)

It is very complicated, but always depend on t, w, z, m_h^2, g plus g_i

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Step 2: running couplings

For a generic parameter p we write the RGE as

$$\frac{dp}{d\ln\mu^2} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4} + \ldots$$

They were computed before in the literature up to three loops

(very long and not very illuminating expressions at three loops)

One loop RGEs for λ, y_t^2, g_i^2 and m^2

$$\begin{aligned}
\beta_\lambda^{(1)} &= \lambda\left(12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}\right) - 3y_t^4 + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2 g_1^2}{40}, \\
\beta_{y_t^2}^{(1)} &= y_t^2\left(\frac{9y_t^2}{2} - 8g_3^2 - \frac{9g_2^2}{4} - \frac{17g_1^2}{20}\right), \\
\beta_{g_1^2}^{(1)} &= \frac{41}{10}g_1^4, \quad \beta_{g_2^2}^{(1)} = -\frac{19}{6}g_2^4, \quad \beta_{g_3^2}^{(1)} = -7g_3^4, \\
\beta_{m^2}^{(1)} &= m^2\left(6\lambda + 3y_t^2 - \frac{9g_2^2}{4} - \frac{9g_1^2}{20}\right)
\end{aligned}$$

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Step 3: threshold corrections

$$
\begin{aligned}
\lambda(M_t) &= 0.12604 + 0.00206\left(\frac{M_h}{\text{GeV}} - 125.15\right) - 0.00004\left(\frac{M_t}{\text{GeV}} - 173.34\right) \pm 0.00030_{\text{th}} \\
\frac{m(M_t)}{\text{GeV}} &= 131.55 + 0.94\left(\frac{M_h}{\text{GeV}} - 125.15\right) + 0.17\left(\frac{M_t}{\text{GeV}} - 173.34\right) \pm 0.15_{\text{th}} \\
y_t(M_t) &= 0.93690 + 0.00556\left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.00042\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.00050_{\text{th}} \\
g_2(M_t) &= 0.64779 + 0.00004\left(\frac{M_t}{\text{GeV}} - 173.34\right) + 0.00011\,\frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}} \\
g_Y(M_t) &= 0.35830 + 0.00011\left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.00020\,\frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}} \\
g_3(M_t) &= 1.1666 + 0.00314\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} - 0.00046\left(\frac{M_t}{\text{GeV}} - 173.34\right)
\end{aligned}
$$

The theoretical uncertainties on these quantities are much lower than those used in previous determinations of the stability bound

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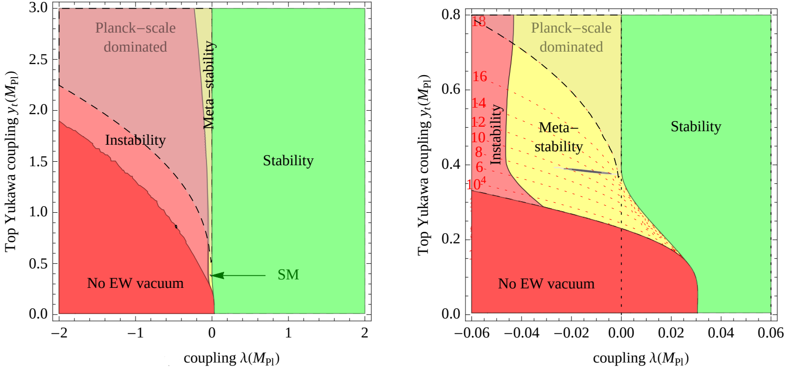
The SM phase diagram in terms of Planck scale couplings

$y_t(M_{\rm Pl})$ versus $\lambda(M_{\rm Pl})$

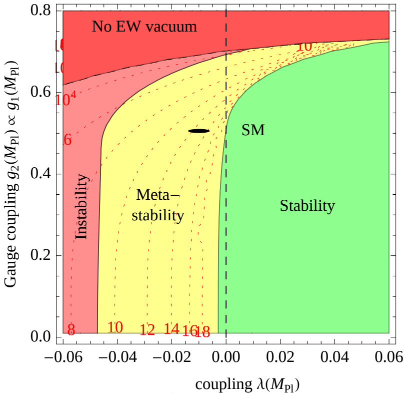


"Planck-scale dominated" corresponds to $\Lambda_I > 10^{18}$ GeV

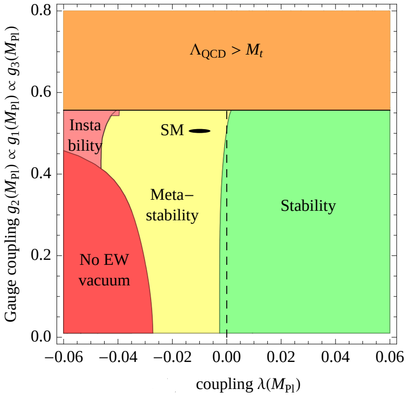
"No EW vacuum" corresponds to a situation in which λ is negative at the EW scale

The SM phase diagram in terms of Planck scale couplings

Gauge coupling g_2 at $M_{\rm Pl}$ versus $\lambda(M_{\rm Pl})$

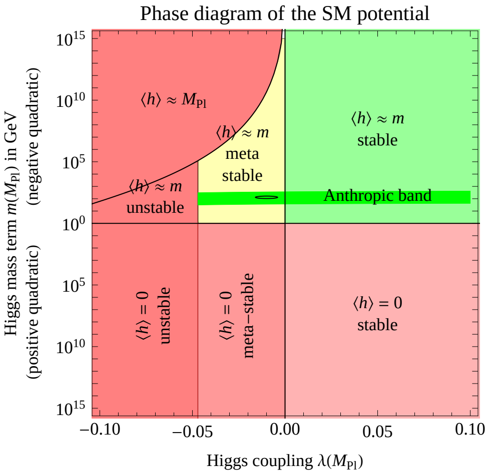


Left: $g_1(M_{\rm Pl})/g_2(M_{\rm Pl}) = 1.22, y_t(M_{\rm Pl})$ *and* $g_3(M_{\rm Pl})$ *are kept to the SM value*



Right: *a common rescaling factor is applied to* g_1, g_2, g_3. $y_t(M_{\rm Pl})$ *is kept to the SM value*

The SM phase diagram in terms of potential parameters



If $\lambda(M_{\rm Pl}) < 0$ there is an upper bound on m requiring $\langle h\rangle \neq 0$ at the EW scale.

This bound is, however, much weaker than the anthropic bound of
[Agrawal, Barr, Donoghue, Seckel (1997); Schellekens (2014)]

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Tunneling probability

The probability of creating a bubble of the absolute minimum in $dV\,dt$ was found by *[Kobzarev, Okun, Voloshin (1975); Coleman (1977); Callan, Coleman (1977)]*

$$d\wp = dt\,dV\;\Lambda_B^4\,e^{-S(\Lambda_B)}$$

$S(\Lambda_B) \equiv$ the action of the bounce of size $R = \Lambda_B^{-1}$, given by $S(\Lambda_B) = \dfrac{8\pi^2}{3|\lambda(\Lambda_B)|}$

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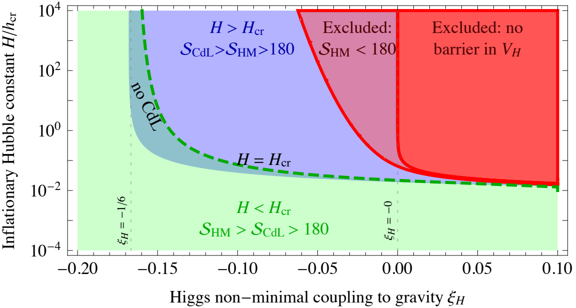
Upper bounds on the Hubble rate during inflation

The model:

$$\mathscr{L} = \mathscr{L}_{\rm EH} + \mathscr{L}_{\rm SM} + \xi_H |H|^2 R$$

The results:



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h inflation: definition

In the h inflation model the role of the inflaton is played by h

The model: *[Bezrukov, Shaposhnikov (2008)]*

$$\mathscr{L} = \mathscr{L}_{\mathrm{EH}} + \mathscr{L}_{\mathrm{SM}} + \xi |H|^2 R$$

h inflation: classical analysis

The part of S that depends on $g_{\mu\nu}$ and H *only* $\longrightarrow$

$$S_{gH} = \int d^4x\sqrt{-g}\left[\left(\frac{M_P^2}{2} + \xi|H|^2\right)R + |D_\mu H|^2 - V(H)\right]$$

The non-minimal coupling can be eliminated through a *conformal* transformation ...

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi|H|^2}{M_P^2}$$

In the unitary gauge, where the only scalar field is the radial mode $\phi \equiv \sqrt{2|H|^2}$

$$S_{gH} = \int d^4x\sqrt{-\hat{g}}\left[\frac{M_P^2}{2}\hat{R} + K\frac{(\partial\phi)^2}{2} - \frac{V}{\Omega^4}\right]$$

where $K \equiv (\Omega^2 + 6\xi^2\phi^2/M_P^2)/\Omega^4$ and we set the gauge fields to zero.

The ϕ kinetic term can be made canonical through $\phi = \phi(\chi)$ defined by

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}}$$

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$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}}$$

This is what we want in order to have slow-roll ...

Thus, χ feels a potential $$U \equiv \frac{V}{\Omega^4} = \frac{\lambda(\phi(\chi)^2 - v^2)^2}{4(1+\xi\phi(\chi)^2/M_P^2)^2} \underset{\phi > M_P/\sqrt{\xi}}{\simeq} \frac{\lambda}{4\xi^2}M_P^4$$

h inflation: classical analysis

All parameters can be fixed through experiments and observations ...

ξ can be fixed requiring the WMAP normalization *[WMAP Collaboration (2013)]*

$$\frac{U(\phi = \phi_{WMAP})}{\epsilon(\phi = \phi_{WMAP})} \simeq (0.02746 M_P)^4$$

ϕ_{WMAP} is fixed by requiring $\quad N = \int_{\phi_{\rm end}}^{\phi_{WMAP}} \frac{U}{M_P^2} \left(\frac{dU}{d\phi}\right)^{-1} \left(\frac{d\chi}{d\phi}\right)^2 d\phi \simeq 59$

[Bezrukov, Gorbunov, Shaposhnikov (2009); Garcia-Bellido, Figueroa, Rubio (2009)]

and $\phi_{\rm end}$ is the field value at the end of inflation: $\quad \epsilon(\phi_{\rm end}) \simeq 1$

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and $\phi_{\rm end}$ is the field value at the end of inflation: $\epsilon(\phi_{\rm end}) \simeq 1$

This leads to $\xi \simeq 4.7 \times 10^4 \sqrt{\lambda}$ and indicates that xi has to be large ...

h inflation: quantum analysis

Two regimes *[Bezrukov, Shaposhnikov, (2009)]*:

- small fields: $\phi \ll M_P/\xi$ (the SM is recovered)
- large fields: $\phi \gg M_P/\xi$ (chiral EW action with VEV set to $\phi/\Omega \simeq M_P/\sqrt{\xi}$) $\to$ decoupling of ϕ in the inflationary regime

h inflation: quantum analysis

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State-of-the-art calculation of the bound on M_h to have inflation:

- Two loop effective potential U_{eff} in the inflationary regime including the effect of ξ and the leading SM couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop SM RGE from the EW scale up to M_P/ξ for $\{\lambda, y_t, g_3, g_2, g_1\}$ ...
- Two loop RGE for the same SM couplings and one loop RGE for ξ in the chiral EW theory
- Two loop threshold corrections at the top mass, for these SM couplings

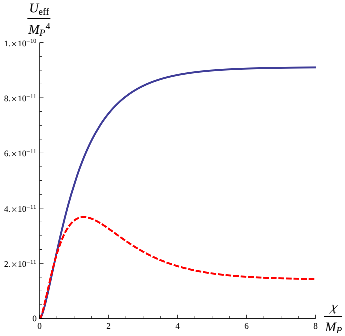
Previous calculations: *[Bezrukov, Magnin, Shaposhnikov (2009); Bezrukov, Shaposhnikov (2009); Allison (2013)]*

Bound on M_h to have h inflation

Derivation

1. We fix ξ as in the classical case, but with U replaced by $U_{\rm eff}$.
   ... this already gives $\xi_{\rm inf} \equiv \xi(M_P/\sqrt{\xi_t})$, where conventionally $\xi_t = \xi(M_t)$

2. If M_h is too small (or M_t is too large) we go from the blue behavior to the red one! When the slope is negative the Higgs cannot roll towards the EW vacuum



We set the th. errors to zero and the input parameters to the central values, except M_t:

- *Solid line:* $M_t = 171.43$*GeV* (*ξ fixed as described above*)
- *Dashed line:* $M_t = 171.437$*GeV* ($\xi_t = 300$)

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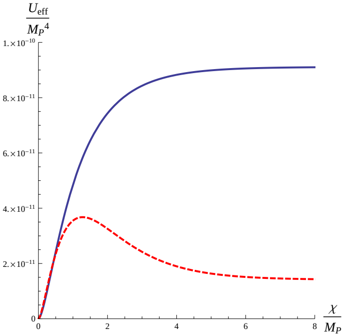

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- *Dashed line:* $M_t = 171.437$ *GeV* *($\xi_t = 300$)*

Result (bound to have h inflation):

$$M_h > 129.4\,{\rm GeV} + 2.0(M_t - 173.34\,{\rm GeV}) - 0.5\,{\rm GeV}\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\rm th}\,{\rm GeV}$$

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More details on right-handed neutrinos

$$Y = \frac{U_\nu^* D_{\sqrt{m}} \mathcal{R} D_{\sqrt{M}}}{v}$$

where

$$\begin{aligned} D_{\sqrt{m}} &\equiv \text{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}), \\ D_{\sqrt{M}} &\equiv \text{diag}(\sqrt{M_1}, \sqrt{M_2}, \sqrt{M_3}) \end{aligned}$$

and U_ν is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:
it can be decomposed as $U_\nu = V_\nu P_{12}$, where ($s_{ij} \equiv \sin(\theta_{ij})$, $c_{ij} \equiv \cos(\theta_{ij})$)

$$V_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

$$P_{12} = \begin{pmatrix} e^{i\beta_1} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathcal{R}$ is a generic complex orthogonal matrix. One can show that the simpler and realistic case of two right-handed neutrinos below M_{Pl} can be recovered by setting $m_1 = 0$ and

$$\mathcal{R} = \begin{pmatrix} 0 & 0 & 1 \\ \cos z & -\sin z & 0 \\ \xi \sin z & \xi \cos z & 0 \end{pmatrix}$$

where z is a complex parameter and $\xi = \pm 1$.
(In the plot ξ is irrelevant and we set $z = 0$)

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