Inflation in the Standard Model of the Universe

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The History of the Universe

It is a history of EXPANSION and cooling down.

EXPANSION: the space itself expands with the time.

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2 \quad , \quad a(t) =$ scale factor.

FRW: Homogeneous, isotropic and spatially flat geometry.

Cooling: temperature decreases as $1/a(t)$: $T(t) \sim 1/a(t)$.

The Universe underwent a succesion of phase transitions towards the less symmetric phases.

Wavelenghts redshift as $a(t) \quad : \quad \lambda(t) = a(t)\, \frac{\lambda(t_0)}{a(t_0)}$

Redshift z : $\quad z + 1 = \frac{a(\text{today})}{a(t)} \quad , \quad a(\text{today}) \equiv 1$

The deeper you go in the past, the larger is the redshift and the smaller is $a(t)$.

Standard Cosmological Model: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant

- Begins by the inflationary era. Slow-Roll inflation explains horizon and flatness.
- Gravity is described by Einstein's General Relativity.
- Particle Physics described by the Standard Model of Particle Physics: $SU(3) \otimes SU(2) \otimes U(1) =$ qcd+electroweak model.
- CDM: dark matter is cold (non-relativistic) during the matter dominated era where structure formation happens.
- Dark energy described by the cosmological constant Λ.

Standard Cosmological Model: ΛCDM

Explains the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances: BBN.
- Large Scale Structures (LSS) Observations. BAO.
- Acceleration of the Universe expansion:
  Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2a}{dt^2} > 0$.

During inflation the universe expands by at least 62 efolds in order to explain the present entropy of the universe (photons+neutrinos): $e^{62} \simeq 10^{27}$.

Inflation lasts $\simeq 10^{-36}$ sec and ends by $z \sim 10^{29}$ followed by a radiation dominated era.

Energy scale when inflation starts $\sim 10^{16}$ GeV ($\Longleftarrow$ CMB anisotropies) which **coincides** with the GUT scale.

Matter can be effectively described during inflation by a Scalar Field $\phi(t, \boldsymbol{x})$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\; a^2(t)} - V(\phi) \right]$.

plus General Relativity that describes the geometry of the Universe in the presence of the Inflaton field.

Plan of the Lectures

- Inflation as an effective theory in the Ginsburg-Landau sense. Fast-roll and slow-roll inflationary regimes.
- Quantum fluctuations of the geometry and the matter during inflation. Scalar and tensor primordial power of these inflationary fluctuations.
- Contrast with observed CMB anisotropies and LSS data through Monte Carlo Markov Chains (MCMC) analysis.
- The generic fast-roll stage before slow-roll can explain the observed CMB quadrupole suppression.
- Model independent approach to dark matter properties. The DM particle mass in the keV scale.
- Loop quantum corrections to observables in slow-roll inflation.
- Conclusions and future perspectives

Physics during Inflation

- Out of equilibrium evolution in a fastly expanding geometry. Vacuum energy DOMINATES:

  Friedmann equation: $H^2(t) = \frac{1}{3\,M_{Pl}^2}\,\rho\;,$ for constant ρ

  yields a de Sitter space-time:

  $a(t) = e^{H\,t} \quad , \quad H = \frac{1}{M_{Pl}}\,\sqrt{\frac{\rho}{3}}$. Natural solution!!

- Extremely high energy density: scale $\lesssim 10^{16}$GeV.
- Explosive particle production due to spinodal or parametric instabilities
- Quantum non-linear phenomena eventually shut-off the instabilities and stop inflation. RD era follows: $a(t) = \sqrt{t}$
- Huge redshift classicalizes the dynamics: an assembly of (superhorizon) quantum modes behave as a classical and homogeneous inflaton field. Inflaton slow-roll.

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AND REFERENCES THEREIN.

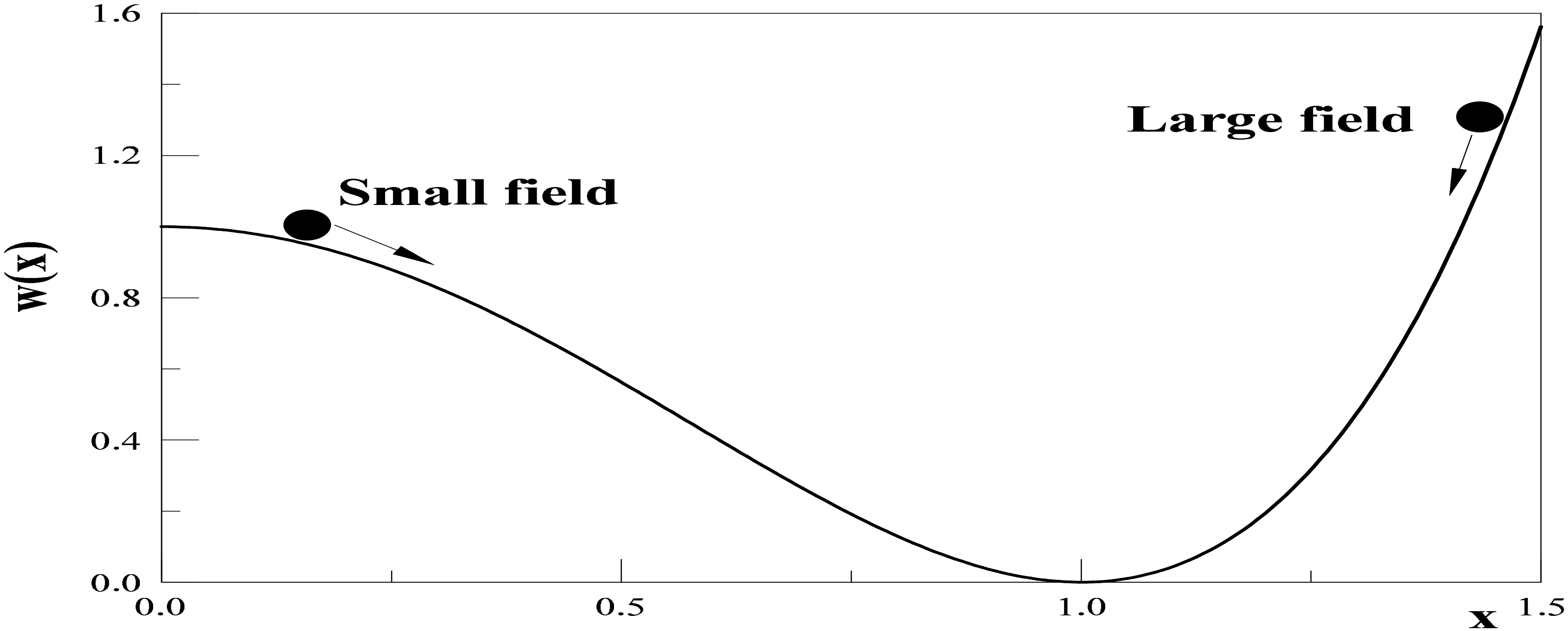
The Theory of Inflation

The inflaton is an effective field in the Ginsburg-Landau sense.

Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q \; , \; \sigma \simeq \bar{q}q$.
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)
- Fermi Theory of Weak Interactions (current-current).

Slow Roll Inflation



The field evolves towards the minimum of the potential.

$V(\mathrm{Min}) = V'(\mathrm{Min}) = 0$: inflation ends after a finite number of efolds.

Slow-roll is needed to produce enough efolds of inflation (≥ 62) to explain the entropy of the universe today
$\Longrightarrow$ the slope of the potential $V(\phi)$ must be small.

Inflation Evolution Equations

Evolution equation for the Inflaton:

$$\ddot{\phi} + 3\,H(t)\,\dot{\phi} - \frac{1}{a^2(t)}\,\nabla^2\phi + V'(\phi) = 0\ , \quad H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \text{Hubble.}$$

$$\text{energy density} = \rho = \frac{1}{2}\left[\dot{\phi}^2 + \frac{1}{a^2(t)}\,(\nabla\phi)^2\right] + V(\phi)$$

$$\text{pressure} = p = \frac{1}{2}\left[\dot{\phi}^2 - \frac{1}{3\,a^2(t)}\,(\nabla\phi)^2\right] - V(\phi)$$

The scale factor grows exponentially during inflation and suppresses spatial gradient terms.

The inflaton field is therefore homogeneous: $\phi = \phi(t)$.

$$\ddot{\phi} + 3\,H(t)\,\dot{\phi} + V'(\phi) = 0 \quad (1)$$

The Einstein equations reduce to a single equation: the Friedmann equation:

$$H^2(t) = \frac{1}{3\,M_{Pl}^2}\,\rho = \frac{1}{3\,M_{Pl}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right] \quad (2)$$

Slow-roll evolution of the Inflaton

During slow-roll the inflaton derivatives are small and the evolution equations (1) and (2) can be approximated by:

$$3\,H(t)\,\dot\phi + V'(\phi) = 0 \quad , \qquad H^2(t) = \frac{V(\phi)}{3M_{Pl}^2}$$

These first order equations can be solved in closed from as:

$$M_{Pl}^2\;N[\phi] = -\int_\phi^{\phi_{end}} \; V(\varphi)\;\frac{d\varphi}{dV}\;d\varphi \;, \qquad e^{N[\phi]} = a(\phi_{end})/a(\phi)\;,$$

$N[\phi] =$ the number of e-folds since the field ϕ exits the horizon till the end of inflation. $N \sim 60$.
$\phi_{end} =$ absolute minimum of $V(\phi)$.

Therefore, $\phi^2 =$ scales as $N\;M_{Pl}^2$. We define:

$\chi \equiv \frac{\phi}{\sqrt{N}\;M_{Pl}}$ dimensionless and slow field.

Universal form of the slow-roll inflaton potential:
$V(\phi) = N\;M^4\;w(\chi), M =$energy scale of inflation,$|w''(0)| = 1$

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N}\; M_{Pl}} \quad , \quad \tau = \frac{m\; t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m\;\sqrt{N}} \quad ,$$

$m \equiv \frac{M^2}{M_{Pl}} \quad , \quad |V''(0)| = m^2 =$ inflaton mass,

slow inflaton, slow time, slow Hubble.

χ and $w(\chi)$ are of order one.

Evolution Equations:

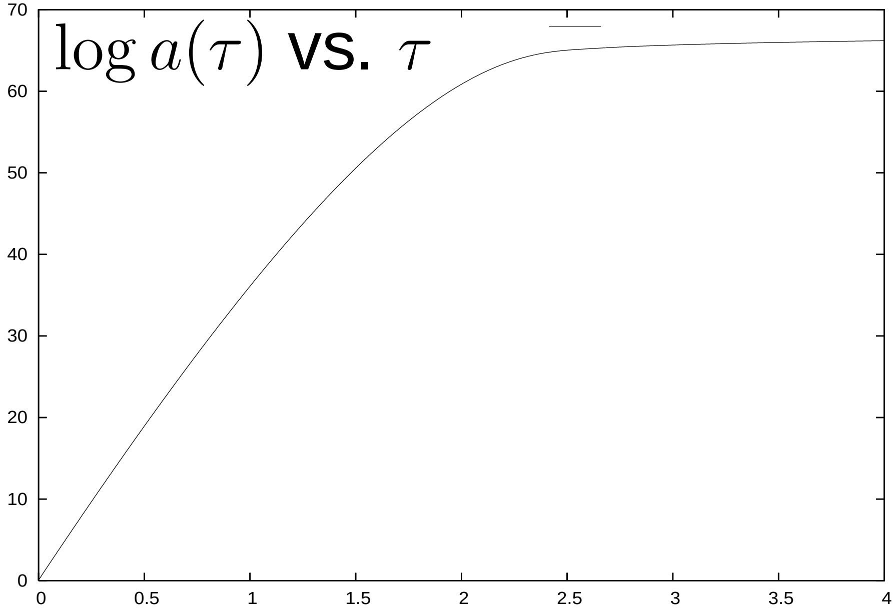
$$\mathcal{H}^2(\tau) = \frac{1}{3}\left[\frac{1}{2\;N}\left(\frac{d\chi}{d\tau}\right)^2 + w(\chi)\right] \quad ,$$

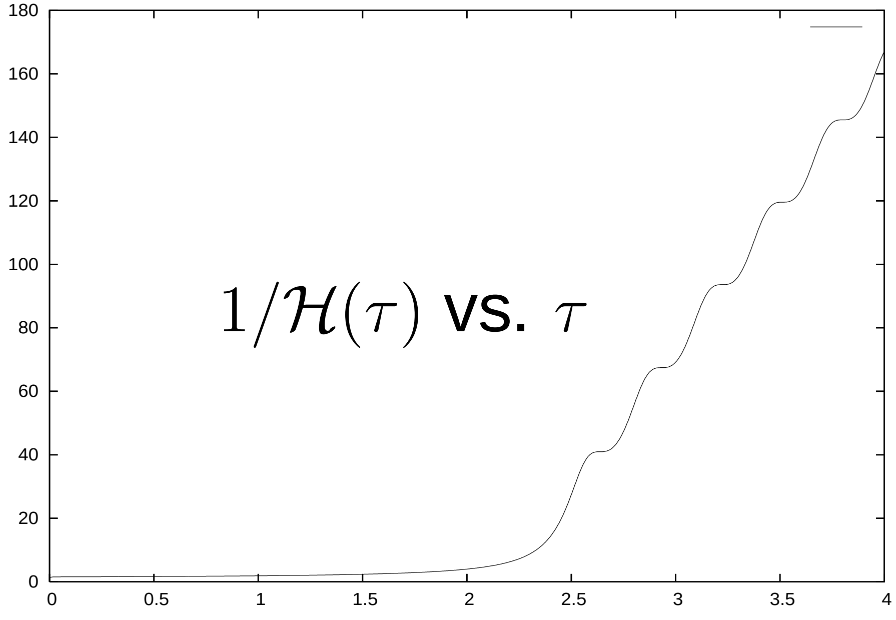
$$\frac{1}{N}\;\frac{d^2\chi}{d\tau^2} + 3\;\mathcal{H}\;\frac{d\chi}{d\tau} + w'(\chi) = 0 \quad .$$

$1/N$ terms: corrections to slow-roll

Higher orders in slow-roll are obtained systematically by expanding the solutions in $1/N$.

Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$





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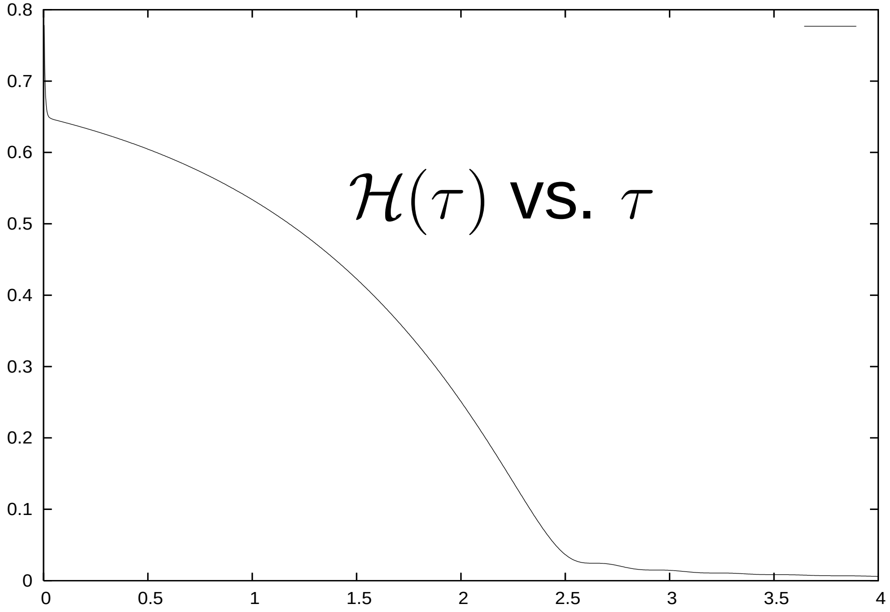
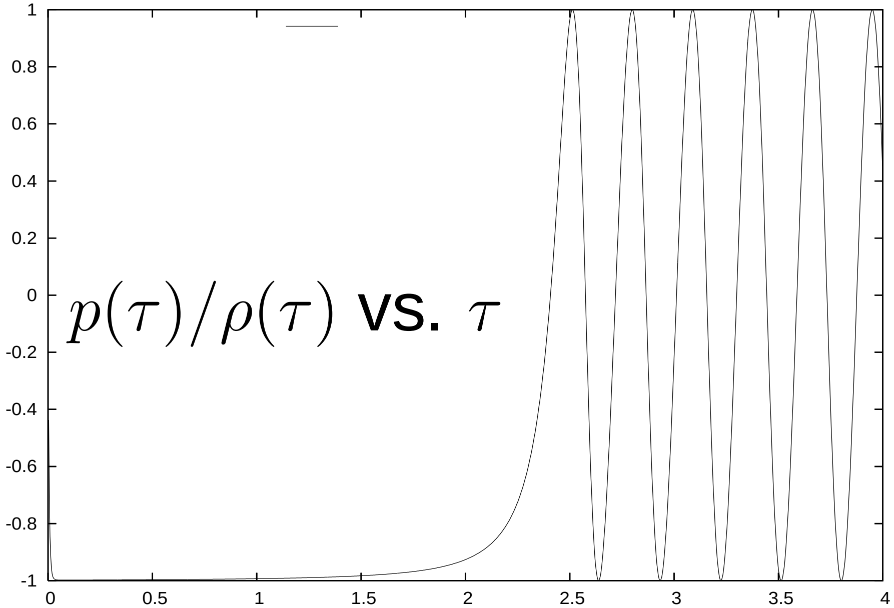


The Hubble parameter $H(t)$ slowly decreases during slow-roll:

$$\dot{H}(t) = -\frac{\dot{\phi}^2}{2\, M_{Pl}^2} < 0$$ and of the order $\mathcal{O}\left(\frac{1}{N}\right)$

$$H(\text{end}) \simeq \frac{H(\text{begin})}{\sqrt{N}} \simeq \frac{1}{8}\, H(\text{begin})$$

Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$

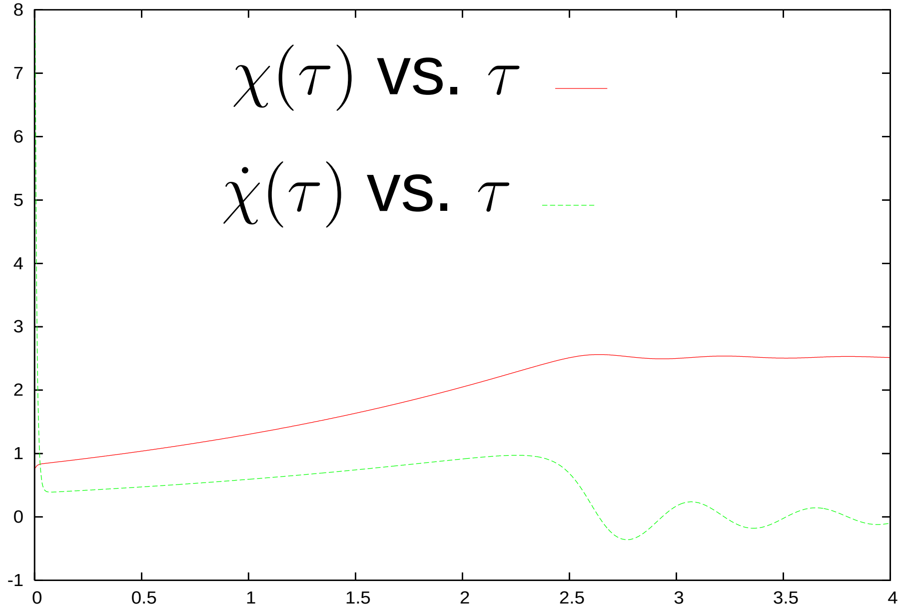


The vacuum energy transforms into particles and inflation is followed in this simplified approach by a matter dominated stage.

The equation of state is $p/e = -1$ during inflation.

p/e strongly oscillates between $+1$ and -1 during the matter dominated stage. We have in average $< p/e >= 0$.

Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$



In these plots $y = 1.26$ and $\chi_{min} = \sqrt{\frac{8}{y}} = 2.52$.

We choose $\chi(0) = 0.73587$, generic initial velocity
$\frac{1}{2\,N}\,\dot{\chi}^2(0) = w(\chi(0)) \implies \dot{\chi}(0) = 12.624$
which ensure $N_{tot} \simeq 66$. Notice fast-roll followed by slow-roll.

We have here neglected spatial gradient terms:

$\frac{(\nabla\phi)^2}{2\;a^2(t)}$ since $a(t)$ grows exponentially during inflation.

The entropy of the universe today

The entropy of the universe today is
dominated by photons (the CMB) and neutrinos:

$S \sim d_0^3 \, [s_\gamma + s_\nu] = 0.97 \times 10^{89} \; ,$ huge number !

$d_0 \simeq 3/H_0 =$ particle horizon today
(region in causal contact with us),

$s_\gamma, \; s_\nu =$ photon and the neutrinos entropies per unit
comoving volume: $s_\gamma = \frac{2\,\pi^2}{45} \, g_\gamma \, T_\gamma^3 \; , \quad s_\nu = \frac{7\,\pi^2}{180} \, g_\nu \, T_\nu^3 ,$

$g_\gamma = 2 =$ number of photon polarizations,

$g_\nu = 6 =$ number of neutrino states,

$T_\gamma = 2.725$ K $=$ CMB temperature today

$T_\nu = (4/11)^{\frac{1}{3}} \, T_\gamma =$ neutrino background temperature today.

The conservation of the entropy of the universe

Let us consider the energy $E(t)$ on a comoving volume V_c:

$E(t) = \rho(t)\ a^3(t)\ V_c$ (physical volume $= V(t) = a^3(t)\ V_c$)

Using energy-momentum conservation in FRW space-time:
$\dot\rho + 3\ H(t)\ \ (\rho + p) = 0$ we obtain:

$$V_c\, a^3(t)\ \ [\dot\rho + 3\ H(t)\ \ (\rho + p)] = \dot E(t) + p\, \dot V(t) = 0$$

Therefore, according to the first principle of thermodynamics: $0 = dE + p\, dV = T\, dS$

and entropy is conserved !!

Entropy remains constant according to the microscopic evolution equations both classical and quantum.

Entropy grows upon coarse-graining of degrees of freedom when quantum decoherence happens as it is the case during inflation. Inflation stretches the lengths by an enormous factor of at least $\sim e^{64} \sim 10^{28}$ making classical the quantum description of matter

Entropy is created during inflation and reheating

Reheating: transition period from out of equilibrium inflation to thermal equilibrium RD.

The horizon size by the end of inflation is $d(t_{end}) \sim e^{N_{tot}} / H$.

N_{tot} = total number of efolds of inflation.

By the end of reheating the horizon gets redshifted by a factor $\simeq \sqrt{H/H_{reh}}$ since $a \sim 1/\sqrt{H}$ during RD and then

$$d_{reh} \sim \sqrt{\frac{H}{H_{reh}}}\, d(t_{end}) \sim \frac{e^{N_{tot}}}{\sqrt{H\, H_{reh}}}$$

The Friedmann equation gives: $H^2_{reh} = \frac{\pi^2\, g_{reh}}{90\, M^2_{Pl}}\, T^4_{reh}$,

g_{reh} = number of ultrarelativistic degrees of freedom and we used that $\rho_{reh} = \frac{\pi^2}{30}\, g_{reh}\, T^4_{reh}$.

The entropy by the end of reheating is therefore,

$$S_{reh} \sim \frac{2\,\pi^2}{45}\, g_{rh}\, T^3_{reh}\, d^3_{reh} \quad \text{and} \quad T_{reh} = \sqrt{\frac{H_{reh}\, M_{Pl}}{\pi}}\, \left(\frac{90}{g_{reh}}\right)^{\frac{1}{4}}$$

Entropy bounds from below number of inflation efolds

Imposing that the entropy by the end of reheating accounts for the entropy today, $S \sim 10^{89}$ yields:

$$S_{reh} \sim 2^{\frac{3}{2}} \left(\frac{2\,\pi^2 \, g_{reh}}{45} \right)^{\frac{1}{4}} e^{3\,N_{tot}} \left(\frac{M_{Pl}}{H} \right)^{\frac{3}{2}} \geq 10^{89} \; .$$

Therefore,

$$N_{tot} \geq 62.4 - \frac{1}{2} \log \beta - \frac{1}{12} \log \frac{g_{reh}}{1000} \quad \text{where} \quad \beta \equiv \sqrt{\frac{10^{-4}\; M_{Pl}}{H}}$$

This number $\gtrsim 63$ of inflation efolds also solves the:

Horizon problem: the particle horizon when photons decoupled (last scattering surface) at $z \simeq 1100$ is subtended today by an angle of $0.03 \simeq 1.7^o$. Why is the CMB temperature isotropic up to 0.01% fluctuations?

Answer: modes horizon-sized today were in causal contact when they exited the horizon...

Inflation is the only available solution to the flatness, horizon and entropy problems in the universe.

Quantum Fluctuations During Inflation and after

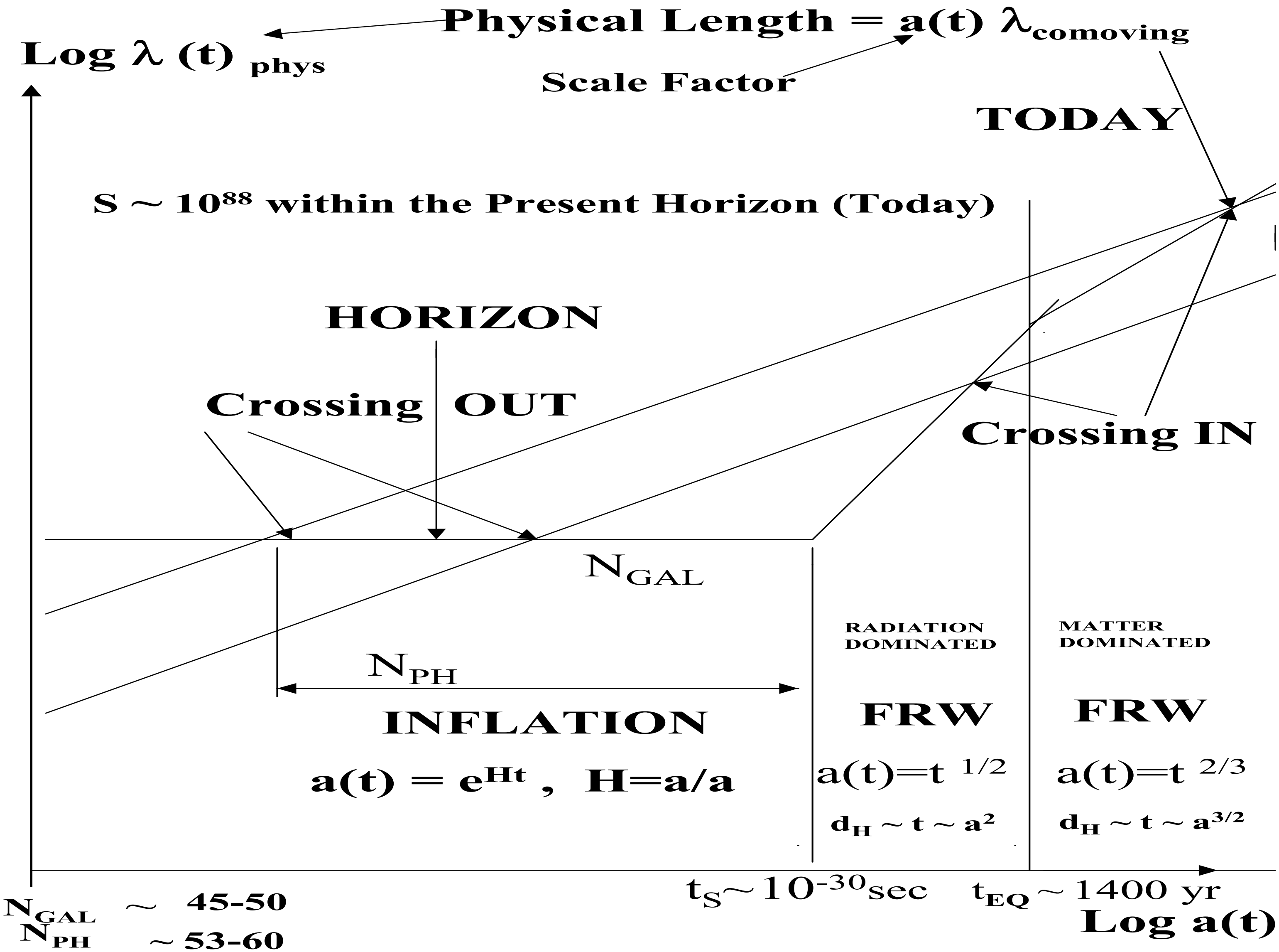
The Universe is homogeneous and isotropic after inflation thanks to the fast and gigantic expansion stretching lenghts by a factor $e^{62} \simeq 10^{27}$.
The universe by the end of inflation is a extraordinarily hot plasma at $T \sim 10^{14}$ GeV.

However, quantum fluctuations around the classical inflaton and FRW geometry were of course present.

These inflationary quantum fluctuations are the seeds of structure formation in the universe and the CMB anisotropies today: galaxies, clusters, stars, planets, ...

That is, our present universe was built out of inflationary quantum fluctuations.

Fluctuations inside and outside the Hubble Radius



*Scales CROSS OUT the Horizon and Later COME BACK : UNIQUE for INFLATION

**LARGER SCALES CROSS OUT FIRST and CROSS BACK LATER

Quantum Fluctuations

Geometry: small fluctuations around the homogeneous and isotropic FRW metric:

$$ds^2 = [1+2\,\widehat{\psi}(\vec{x},t)]\;dt^2 - a^2(t)\;[1-2\;\psi(\vec{x},t)]\;d\vec{x}^2 + a^2(t)\;h_{\mu\nu}\;dx^\mu\;dx^\nu\;.$$

Inflaton field Φ: small fluctuations around the homogeneous and isotropic expectation value $\phi(t) =< \Phi(\vec{x},t) >$

$$\Phi(\vec{x},t) = \phi(t) + \delta\varphi(\vec{x},t)$$

Gauge invariant scalar curvature perturbations:

$\mathcal{R} = -\psi - \frac{H}{\dot\phi}\,\delta\varphi$ where $\widehat{\psi} = \psi =$ Newtonian potential.

Gauge invariant tensor perturbations $h_{\mu\nu}$:

$$h_{00} = h^i_i = 0,\;\; h_{0i} = \partial^i h_{ij} = 0$$

Two independent polarization states called gravitons.

Scalar Curvature Fluctuations

It is convenient to introduce, $u(\boldsymbol{x},t) = -z(t)\,\mathcal{R}(\boldsymbol{x},t)$
where $z \equiv a(t)\,\frac{\dot{\phi}(t)}{H(t)}$.

The Einstein-Hilbert action for the gravitational field plus the action for the inflaton in the cosmological space-time yields to quadratic order and in conformal time $\eta \equiv \int dt/a(t)$

$$S = \frac{1}{2}\int d\eta\, d^3x \left[(\partial_\eta u)^2 - (\nabla u)^2 + \frac{\partial_\eta^2 z}{z}\, u^2\right]$$

Fourier expanding $u(\boldsymbol{x},\eta)$ in creation and annihilation operators yields, $u(\boldsymbol{x},\eta) =$

$$\int \frac{d^3k}{(2\,\pi)^{\frac{3}{2}}}\left[\alpha_{\mathcal{R}}(\boldsymbol{k})\, S_{\mathcal{R}}(k;\eta)\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \alpha^\dagger_{\mathcal{R}}(\boldsymbol{k})\, S^*_{\mathcal{R}}(k;\eta)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right] ,$$

The operators obey canonical commutation relations:
$\left[\alpha_{\mathcal{R}}(\boldsymbol{k}),\ \alpha^\dagger_{\mathcal{R}}(\boldsymbol{k}')\right] = \delta(\boldsymbol{k}-\boldsymbol{k}')$, and the $\alpha_{\mathcal{R}}(k)$ annihilate the vacuum state $|0>$ where $\phi(t) =< \Phi(\vec{x},t) >$.

Evolution of the Scalar Curvature Fluctuations

The mode functions obey the Schrödinger-type differential order equation

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_{\mathcal{R}}(\eta)\right] S_{\mathcal{R}}(k;\eta) = 0 \quad , \quad W_{\mathcal{R}}(\eta) \equiv \frac{1}{z}\,\frac{d^2 z}{d\eta^2}$$

Canonical commutation relations for the field $u(\boldsymbol{x},t)$ entail that the $S_{\mathcal{R}}(k;\eta)$ are normalized by their Wronskian as

$$W[S_{\mathcal{R}}(k;\eta), S^*_{\mathcal{R}}(k;\eta)] = S_{\mathcal{R}}(k;\eta) S^{*\,\prime}_{\mathcal{R}}(k;\eta) - S'_{\mathcal{R}}(k;\eta) S^*_{\mathcal{R}}(k;\eta) = i$$

This Wronskian normalization entails that the field $u(\boldsymbol{x},t)$ obeys canonical commutation relations.

The potential $W_{\mathcal{R}}(\eta)$ felt by the fluctuations takes the form

$$W_{\mathcal{R}}(\eta) = a^2(\eta)\; H^2(\eta)\left[2 - 7\,\epsilon_v + 2\,\epsilon_v^2 - \frac{\sqrt{8\;\epsilon_v}\;V'(\phi)}{M_{Pl}\;H^2} - \eta_v(3-\epsilon_v)\right]$$

where $\epsilon_v \equiv \frac{1}{2\;M_{Pl}^2}\,\frac{\dot{\phi}^2}{H^2} \quad , \quad \eta_v \equiv M_{Pl}^2\,\frac{V''(\phi)}{V(\phi)}$

Evolution of the Fluctuations during Slow-Roll

In the slow and dimensionless variables $\chi = \phi/[\sqrt{N}\; M_{Pl}]$,

$$\epsilon_v = \frac{1}{2\,N}\,\frac{1}{\mathcal{H}^2}\,\left(\frac{d\chi}{d\tau}\right)^2 = \frac{1}{2\,N}\,\left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \mathcal{O}\left(\frac{1}{N^2}\right)\;,\qquad \eta_v = \frac{1}{N}\,\frac{w''(\chi)}{w(\chi)}$$

ϵ_v and η_v are always of order $1/N \sim 0.02$ for all slow-roll models. ϵ_v and η_v are called slow-roll parameters.

During slow-roll $a(\eta) = -\frac{1}{\eta\; H\,\left[1-\epsilon_v+\mathcal{O}\left(\frac{1}{N^2}\right)\right]}$ and

$$W_{\mathcal{R}}(\eta) = \frac{2}{\eta^2}\left[1+\frac{3}{2}\,(3\,\epsilon_v - \eta_v) + \mathcal{O}\left(\frac{1}{N^2}\right)\right] = \frac{\nu_{\mathcal{R}}^2-\frac{1}{4}}{\eta^2}$$

$$\nu_{\mathcal{R}} = \frac{3}{2} + 3\,\epsilon_v - \eta_v + \mathcal{O}\left(\frac{1}{N^2}\right)$$

General solution of the scalar fluctuations equation:

$$S_{\mathcal{R}}(k;\eta) = A_{\mathcal{R}}(k)\; g_{\nu_{\mathcal{R}}}(\eta) + B_{\mathcal{R}}(k)\; g^*_{\nu_{\mathcal{R}}}(\eta)\;,$$

$$g_\nu(k;\eta) = \frac{1}{2}\, i^{\nu+\frac{1}{2}}\,\sqrt{-\pi\eta}\; H^{(1)}_\nu(-k\,\eta),\;\; g_{\frac{3}{2}}(k;\eta) = \frac{e^{-i\,k\,\eta}}{\sqrt{2k}}\left[1-\frac{i}{k\,\eta}\right]$$

$H^{(1)}_\nu(z) =$ Hankel function, $\nu_{\mathcal{R}} = 3/2 + \mathcal{O}(1/N)$

Initial Conditions for the Fluctuations

Deep inside the Hubble radius

$|k_{phys}/H| = |k/(a\ H)| \simeq |k\,\eta| \gg 1$:

$$g_\nu(k;\eta) \overset{\eta \to -\infty}{=} \frac{1}{\sqrt{2\,k}}\; e^{-i\,k\,\eta} \;.$$

These are Vacuum or Bunch-Davies initial conditions.

Primordial Scalar Curvature Power $=< \mathcal{R}^2(\boldsymbol{x},\eta) >$.

Using $\alpha_{\mathcal{R}}(\boldsymbol{k})|0 >= 0$,

$$< \mathcal{R}^2(\boldsymbol{x},\eta) >=< \frac{u^2(\boldsymbol{x},\eta)}{z^2(\eta)} >= \int_0^\infty \frac{|S_{\mathcal{R}}(k;\eta)|^2}{z^2(\eta)}\; \frac{k^2\,dk}{2\,\pi^2}.$$

Define the power per unit logarithmic interval in $\log k$:

$$P_{\mathcal{R}}(k,\eta) \equiv \frac{k^3}{2\;\pi^2}\;\frac{|S_{\mathcal{R}}(k;\eta)|^2}{z^2(\eta)} \Rightarrow < \mathcal{R}^2(\boldsymbol{x},\eta) >= \int_0^\infty \frac{dk}{k}\; P_{\mathcal{R}}(k,\eta)$$

General Solution: $S_{\mathcal{R}}(k;\eta) = A_{\mathcal{R}}(k)\; g_{\nu_{\mathcal{R}}}(\eta) + B_{\mathcal{R}}(k)\; g^*_{\nu_{\mathcal{R}}}(\eta)$

Constancy of the Wronskian implies: $|A_{\mathcal{R}}(k)|^2 - |B_{\mathcal{R}}(k)|^2 = 1$

Primordial Scalar Power

By the end of inflation $\eta \to 0^-$:

$$g_\nu(k;\eta) \overset{\eta\to 0^-}{=} \frac{\Gamma(\nu)}{\sqrt{2\,\pi\,k}} \left(\frac{2}{i\,k\,\eta}\right)^{\nu-\frac{1}{2}} .$$

Then, the scalar power by the end of inflation:

$$P_{\mathcal{R}}(k) = \lim_{\eta\to 0^-} P_{\mathcal{R}}(k,\eta) = \lim_{\eta\to 0^-} \frac{k^3}{2\,\pi^2} \frac{|S_{\mathcal{R}}(k;\eta)|^2}{z^2(\eta)} =$$

$$= P_{\mathcal{R}}^{BD}(k)\,[1 + D_{\mathcal{R}}(k)] \;,$$

$$D_{\mathcal{R}}(k) = 2\,|B_{\mathcal{R}}(k)|^2 - 2\,\mathrm{Re}\left[A_{\mathcal{R}}(k)\;B^*_{\mathcal{R}}(k)\,i^{1-n_s}\right]$$

is the transfer function and

$$P_{\mathcal{R}}^{BD}(k) = |\Delta_{k\;ad}^{\mathcal{R}}|^2 \left(\frac{k}{k_0}\right)^{n_s-1} ,\; n_s - 1 = 2\,\eta_v - 6\,\epsilon_v = \mathcal{O}\left(\frac{1}{N}\right)$$

$n_s =$ spectral index, $\quad k_0 =$ pivot scale and

$$|\Delta_{k\;ad}^{\mathcal{R}}|^2 = \frac{1}{8\,\pi^2\,\epsilon_v} \left(\frac{H}{M_{Pl}}\right)^2$$ to leading order in $1/N$

Primordial Power Spectrum for BD initial conditions

We used that during slow-roll,

$$W_{\mathcal{R}}(\eta) = \frac{1}{z} \frac{d^2 z}{d\eta^2} = \frac{\nu_{\mathcal{R}}^2 - \frac{1}{4}}{\eta^2} \Rightarrow \; z(\eta) = \left(a \, \frac{\dot{\phi}}{H}\right)_{exit} (-k_0 \, \eta)^{\frac{1}{2} - \nu_R} \, .$$

since $-k_0 \, \eta = k_0/[H \; a] = 1$ at horizon exit.

$P_{\mathcal{R}}^{BD}(k)$ corresponds to BD initial conditions on $S_{\mathcal{R}}(k;\eta)$ imposed asymptotically for $\eta \to -\infty$:

$$A_{\mathcal{R}}(k) = 1, \; B_{\mathcal{R}}(k) = 0, \; S_{\mathcal{R}}(k;\eta) = g_{\nu_{\mathcal{R}}}(k;\eta) \; \Rightarrow D_{\mathcal{R}}(k) = 0$$

Only BD initial conditions reproduce the data!!

To leading order in $1/N$:

$$|\Delta_{k\;ad}^{\mathcal{R}}|^2 = \frac{1}{8\,\pi^2 \; \epsilon_v} \; \left(\frac{H}{M_{Pl}}\right)^2 = \frac{N^2}{12\,\pi^2} \; \left(\frac{M}{M_{Pl}}\right)^4 \; \frac{w^3(\chi)}{w'^2(\chi)}$$

χ stands for the inflaton field at horizon exit.

Energy Scale of Inflation from CMB anisotropy amplitude

For all slow-roll inflation models $w(\chi) = \mathcal{O}(1) = w'(\chi)$:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}} \left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP5 result: $|\Delta_{k\ ad}^{(S)}| = (0.494 \pm 0.1) \times 10^{-4}$

determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 0.85 \times 10^{-5} \longrightarrow M = 0.70 \times 10^{16} \text{ GeV}$$

The inflation energy scale turns to be the grand unification energy scale !!

The scale M is independent of the shape of $w(\chi)$.

We find the scale of inflation without knowing the tensor/scalar ratio r !!

Tensor Fluctuations

The Einstein-Hilbert action for the gravitational field yields to quadratic order for the tensor fluctuations in conformal time:

$$S = \frac{1}{2} \left(\frac{M_{Pl}}{2}\right)^2 \int d\eta \; d^3x \; a^2(\eta) \; \partial_\mu h^i_j \; \partial^\mu h^j_i$$

Fourier expanding $h^i_j(\vec{x}, \eta)$ in creation and annihilation operators yields,

$$h^i_j(\vec{x}, \eta) = \frac{2}{a(\eta) \; M_{Pl}} \sum_{\lambda=\times,+} \int \frac{d^3k}{(2\,\pi)^{\frac{3}{2}}} \; \epsilon^i_j(\lambda, \vec{k}) \Big[$$

$$e^{i\vec{k}\cdot\vec{x}} \; \alpha_{T,\lambda}(\boldsymbol{k}) \;\; S_T(k, \eta) + e^{-i\vec{k}\cdot\vec{x}} \; \alpha^\dagger_{T,\lambda}(\boldsymbol{k}) \;\; S^*_T(k, \eta)\Big]$$

where λ labels the two standard transverse and traceless polarizations $\times$ and $+$.

The mode functions $S_T(k;\eta)$ obey the differential equation:

$$S''_T(k;\eta) + \left[k^2 - \frac{a''(\eta)}{a(\eta)}\right] S_T(k;\eta) = 0.$$

In the slow-roll regime: $\frac{a''(\eta)}{a(\eta)} = \frac{\nu_T^2 - \frac{1}{4}}{\eta^2} \; , \qquad \nu_T = \frac{3}{2} + \epsilon_v + \mathcal{O}\left(\frac{1}{N^2}\right)$

Tensor power

Tensor power by the end of inflation:

$$P_T(k) = \lim_{\eta\to 0^-} P_T(k,\eta) = \lim_{\eta\to 0^-} \frac{4\,k^3}{M_{Pl}^2\,\pi^2} \left| \frac{S_T(k;\eta)}{a(\eta)} \right|^2$$

$$= P_T^{BD}(k)\,[1 + D_T(k)] \ .$$

$$D_T(k) = 2\,|B_T(k)|^2 - 2\,\mathrm{Re}\left[A_T(k)\ B_T^*(k)\,i^{-n_T}\right]$$

is the transfer function and

$$P_T^{BD}(k) = |\Delta_k^T|^2 \ \left(\frac{k}{k_0}\right)^{n_T} \ , \ \ n_T = -2\,\epsilon_v = \mathcal{O}\left(\frac{1}{N}\right).$$

$$|\Delta_k^T|^2 = \frac{2}{\pi^2}\ \frac{H^2}{M_{Pl}^2}\left[1 + \mathcal{O}\left(\frac{1}{N}\right)\right] \ , \ n_T =$$ tensor spectral index,

$P_T^{BD}(k)$ corresponds to BD initial conditions on $S_T(k;\eta)$:

$A_T(k) = 1,\ B_T(k) = 0,\ S_T(k;\eta) = g_{\nu_T}(k;\eta) \Rightarrow D_T(k) = 0$.

Ratio of tensor to scalar fluctuations r:

$$r = \frac{|\Delta_k^T|^2}{|\Delta_{k\ ad}^{\mathcal{R}}|^2} = 16\,\epsilon_v + \mathcal{O}\left(\frac{1}{N^2}\right) = \mathcal{O}\left(\frac{1}{N}\right) \ .$$

Tensor fluctuations = primordial gravitons.

Spectral index n_s, its running $dn_s/d\ln k$ and the ratio r

$$n_s - 1 = 3 - 2\,\nu_{\mathcal{R}} = 2\,\eta_v - 6\,\epsilon_v = -\frac{3}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N}\,\frac{w''(\chi)}{w(\chi)}$$

$$r = 16\,\epsilon_v = \frac{8}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2$$. The HZ point is: $n_s = 1\ ,\ r = 0$.

$$\frac{dn_s}{d\ln k} = -\frac{2}{N^2}\,\frac{w'(\chi)\,w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2}\,\frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2}\,\frac{[w'(\chi)]^2\,w''(\chi)}{w^3(\chi)}$$

χ is the inflaton field at horizon exit $k = a(\chi)\,H(\chi)$.

$n_s - 1$ and r are always of order $1/N \sim 0.02$ (model indep.)

The k dependence of n_s is subleading in slow-roll ($1/N$): running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

Tensor fluctuations suppressed with respect to scalar because scalar fluctuations are quantum fluctuations around the classical inflaton while tensor fluctuations are quantum zero-point fluctuations.

Matter distribution (inflaton) homogeneous and isotropic can only produce scalar fluctuations and not tensor ones

Ginsburg-Landau Approach

Ginsburg-Landau potentials:
polynomials in the field starting by a constant term.

Linear terms can always be eliminated by a constant shift of the inflaton field.

The quadratic term can have a positive or a negative sign:

$$\begin{cases} w''(0) > 0 \to \text{single well potential} \to \text{large field (chaotic) inflation} \\ w''(0) < 0 \to \text{double well potential} \to \text{small field (new) inflation} \end{cases}$$

The inflaton potential must be bounded from below $\Longrightarrow$ highest order term must be even with a positive coefficient.

Renormalizability $\Longrightarrow$ degree of the inflaton potential ≤ 4.

The theory of inflation is an effective theory $\Longrightarrow$
higher degree potentials are acceptable

Fourth order Ginsburg-Landau inflationary models

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3\,\chi^3 + G_4\,\chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N\;M^4\;w\left(\frac{\phi}{\sqrt{N}\;M_{Pl}}\right) = V_o \pm \frac{m^2}{2}\,\phi^2 + g\,\phi^3 + \lambda\,\phi^4\;.$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}}\left(\frac{M}{M_{Pl}}\right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N}\;\left(\frac{M}{M_{Pl}}\right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 60\;.$$

- Small couplings arise naturally as ratio of two energy scales: inflation and Planck.
- The inflaton is a light particle:

  $m = \frac{M^2}{M_{Pl}} \simeq 0.003\;M\;, \quad m = 2.5 \times 10^{13}$ GeV

  $H \sim \sqrt{N}\;m \simeq 2 \times 10^{14}$ GeV.

The Fourth order Double Well inflationary potential

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$$

produces inflation with $0 < \sqrt{y}\;\chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

This is small field inflation.

The number of e-folds $N[\chi]$ since the field χ exits the horizon [when $k/a(\chi) = H(\chi)$] till the end of inflation is:

$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)}\, d\chi$. We choose then $N = N[\chi]$.

Computing the above integral: $\quad y = z - 1 - \log z$

where $\quad z \equiv y\,\chi^2/8\quad$ and

we have $0 < z < 1$ for $0 < \chi < \chi_{end} = \sqrt{\frac{8}{y}}$.

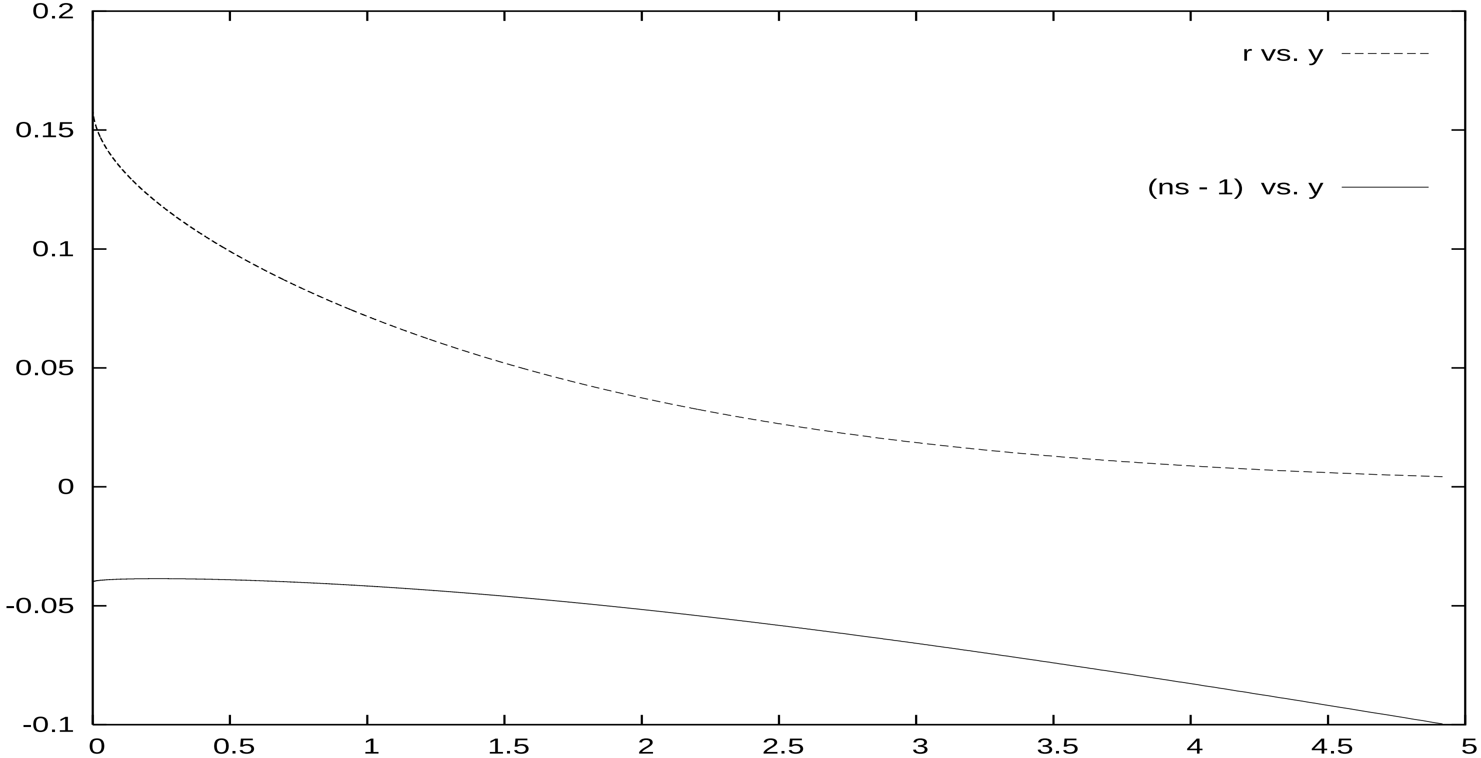
Spectral index n_s and the ratio r as functions of z:

$$n_s = 1 - \frac{1}{N}\,(z - 1 - \log z)\,\frac{3\,z+1}{(z-1)^2}\;, \qquad r = \frac{16}{N}\,(z - 1 - \log z)\,\frac{z}{(z-1)^2}$$

Fourth order Double Well Inflation: (y = coupling).

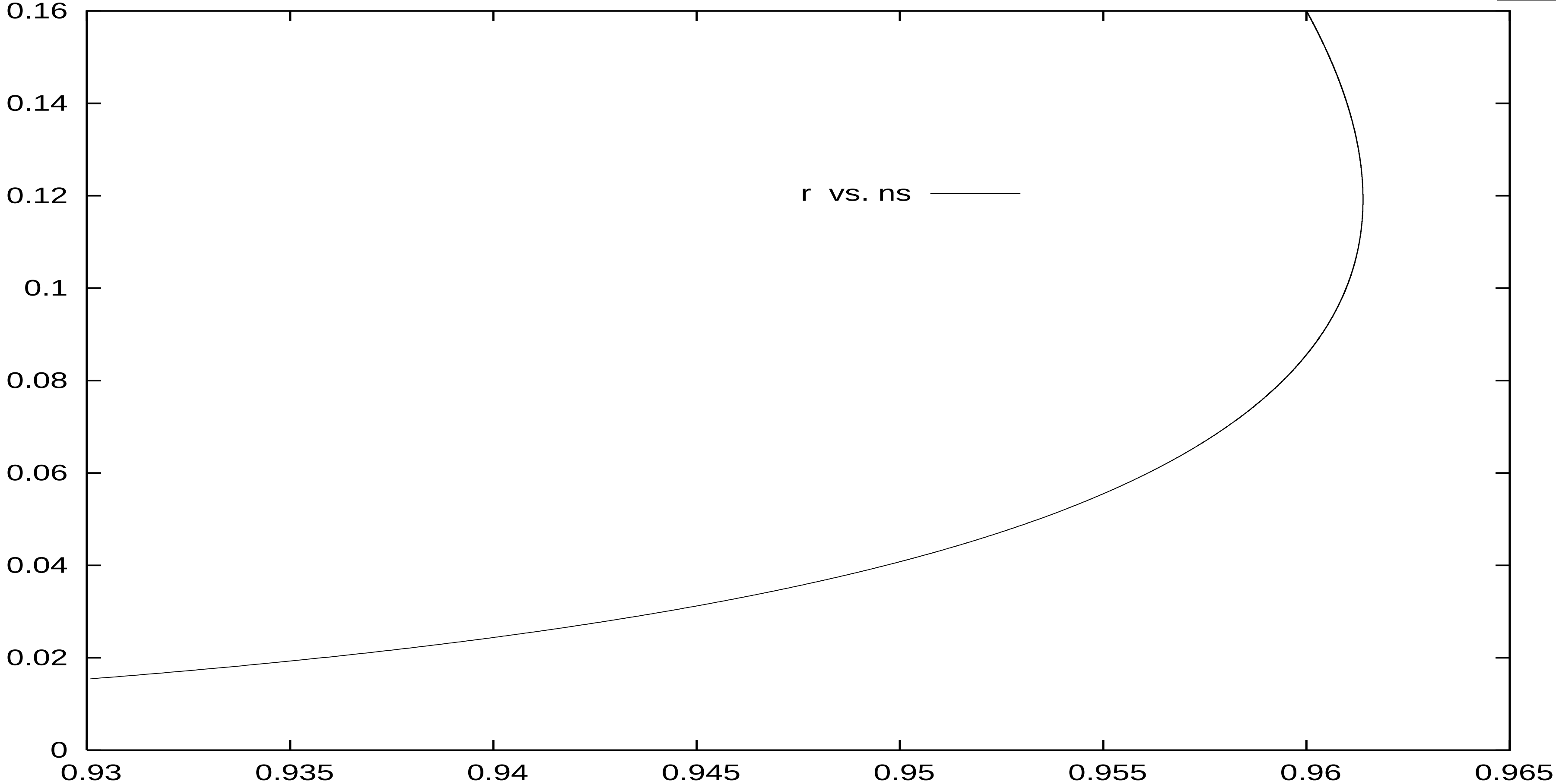
r decreases monotonically with y :

(strong coupling) $0 < r < \frac{8}{N} = 0.133\ldots$ (zero coupling).



n_s first grows with y, reaches a maximum value $n_{s,maximum} = 0.96139\ldots$ at $y = 0.2387\ldots$ and then n_s decreases monotonically with y. $n_s(y=0) - 1 = -2/N$.

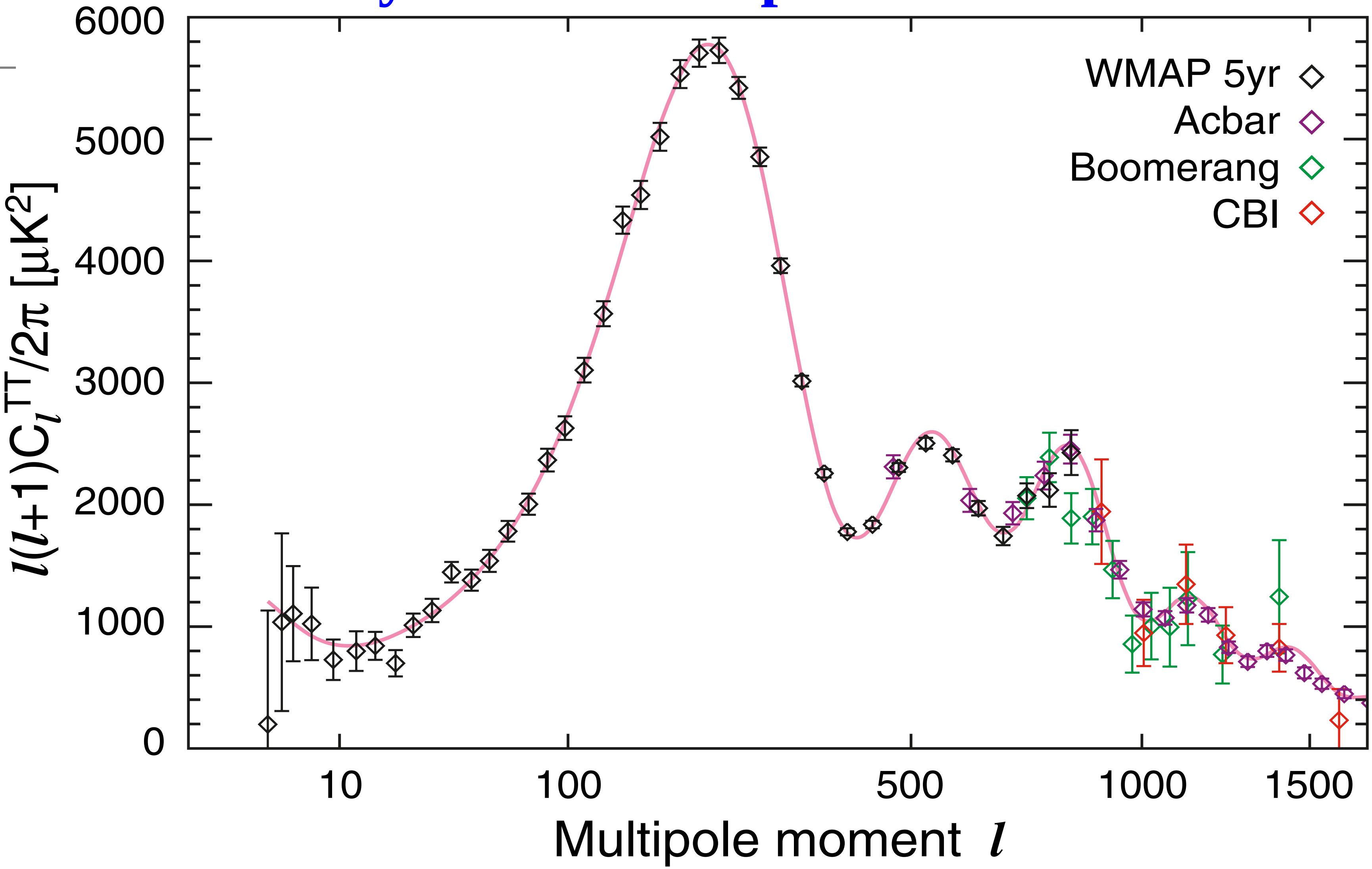
Fourth order Double Well New Inflation



$r = \frac{8}{N} = 0.133\ldots$ and $n_s = 1 - \frac{2}{N} = 0.966\ldots$ at $y = 0$.

r is a double valued function of n_s.

WMAP 5 years data set plus other CMB data



Theory and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

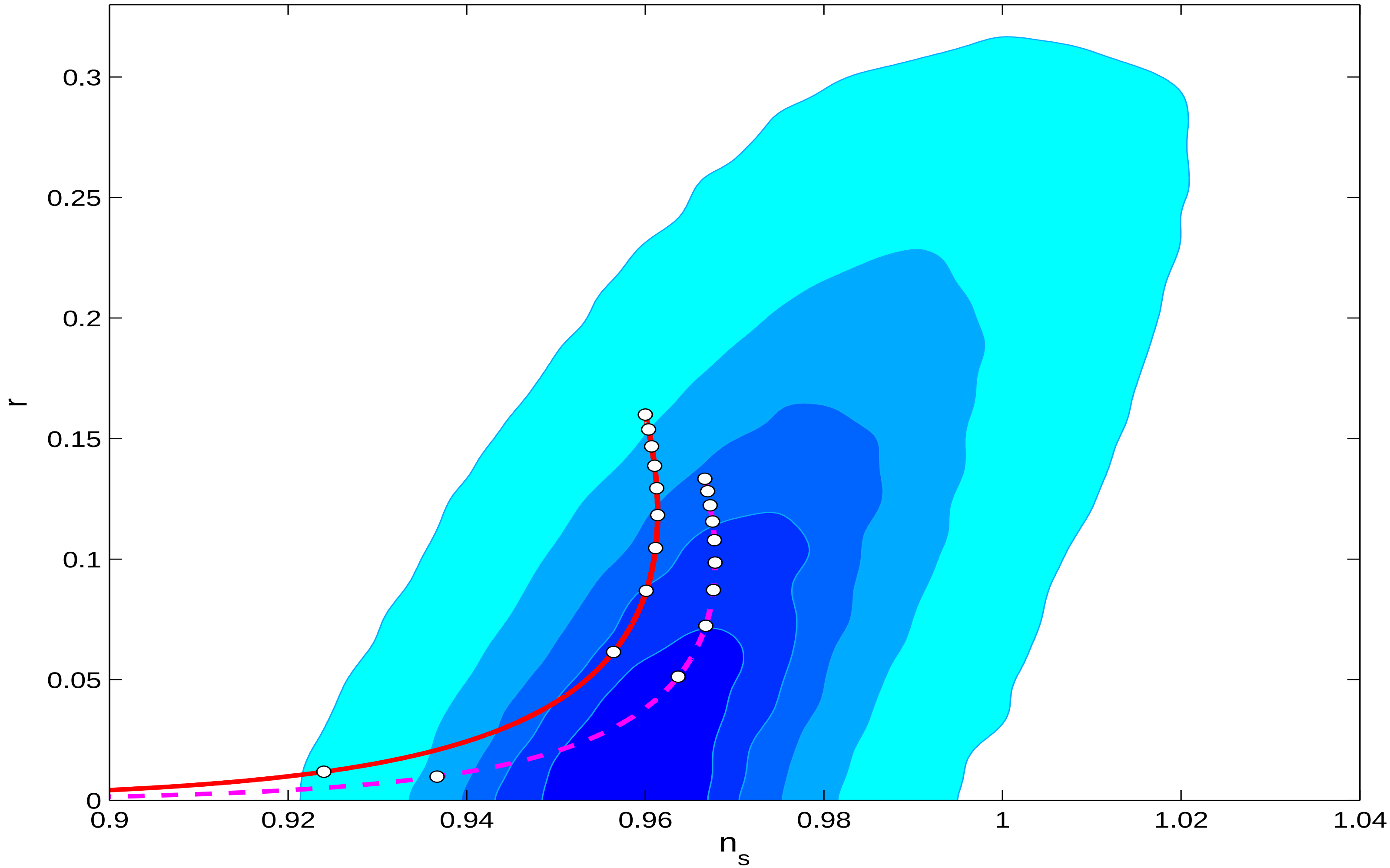
Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

We found n_s and r and the couplings y and h by MCMC.
NEW: We imposed as a hard constraint that r and n_s are given by the inflaton potential.
Our analysis differs in this crucial aspect from previous MCMC studies of the WMAP data.

The color–filled areas correspond to $12\%,\ 27\%,\ 45\%,\ 68\%$ and 95% confidence levels according to the WMAP3 and Sloan data.
The color of the areas goes from the darker to the lighter for increasing CL.

MCMC Results for the double–well inflaton potential



Solid line for $N = 50$ and dashed line for $N = 60$

White dots: $z = 0.01 + 0.11 * n, \ n = 0, 1, \ldots, 9$,

y increases from the uppermost dot $y = 0, \ z = 1$.

MCMC Results for double–well inflaton potential

Bounds: $r > 0.023 \; (95\% \; \mathrm{CL}) \quad , \quad r > 0.046 \; (68\% \; \mathrm{CL})$
Most probable values: $n_s \simeq 0.964, \; r \simeq 0.051 \Leftarrow$measurable!!
The most probable double–well inflaton potential has a moderate nonlinearity with the quartic coupling $y \simeq 1.26 \ldots$.
The $\chi \to -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$$

MCMC analysis calls for $w''(\chi) < 0$ at horizon exit
$\Longrightarrow$ double well potential favoured.
C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.
Similar results from WMAP5 data.
Acbar08 data slightly increases $n_s < 1$ and r.

Higher Order Inflaton Potentials

Till here we considered fourth degree inflaton potentials. Can higher order terms modify the physical results and the observable predictions?

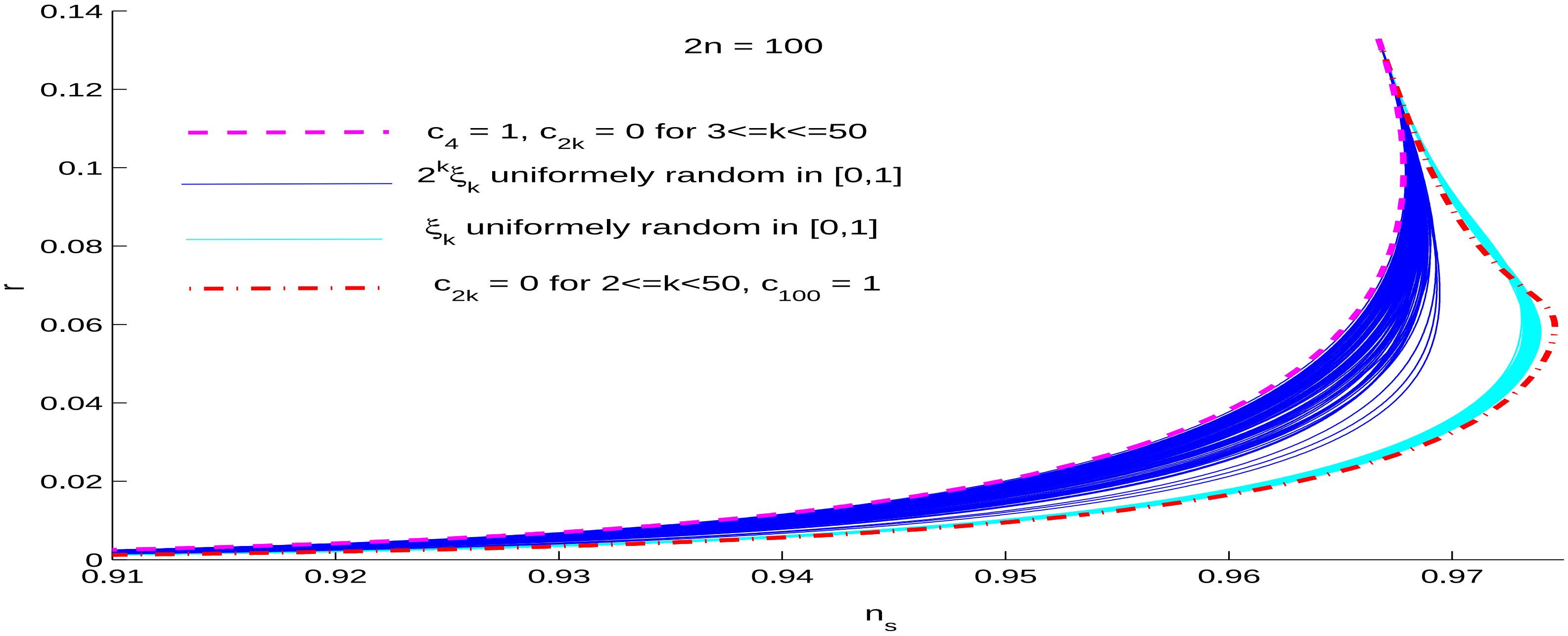
We systematically study the effects produced by higher order terms ($n > 4$) in the inflationary potential on the observables n_s and r.
All coefficients in the potential w become order one using the field χ within the Ginsburg-Landau approach:

$$w(\chi) = c_0 - \frac{1}{2}\,\chi^2 + \sum_{n=2}^{\infty} \frac{c_n}{n}\,\chi^{2\,n} \quad , \quad c_n = \mathcal{O}(1)\,.$$

All $r = r(n_s)$ curves for double–well potentials of arbitrary high order fall **inside** a universal banana-shaped region $\mathcal{B}$.
Moreover, the $r = r(n_s)$ curves for double–well potentials even for arbitrary positive higher order terms lie inside the banana region $\mathcal{B}$.

C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0906.4102.

The 100th degree polynomial inflaton potential



$$w(\chi) = \frac{4}{y} - \frac{1}{2}\,\chi^2 + \frac{4}{y}\sum_{k=2}^{n} \frac{c_{2k}}{k}\,\left(\frac{y^k}{8^k}\,\chi^{2k} - 1\right)$$

The coefficients c_{2k} were extracted at random.

The lower border of the banana-shaped region is given by the potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\,\chi^2 + \frac{4}{n\,y}\,\left(\frac{y^n}{8^n}\,\chi^{2n} - 1\right) \text{ with } n = 50.$$

The inflaton potential from a fermion condensate

Inflaton coupled to Dirac fermions Ψ during inflation:

$$\mathcal{L} = \overline{\Psi} \left[i \, \gamma^\mu \, \mathcal{D}_\mu - m_f - g_Y \, \phi \right] \Psi$$

$g_Y =$ Yukawa coupling, $\gamma^\mu =$ curved space-time γ-matrices.

Hubble parameter $H =$ constant. Effective potential $\equiv$ fermions energy for a constant inflaton ϕ during inflation.

Dynamically generated inflaton potential:

$V_f(\phi) = V_0 + \frac{1}{2} \, \mu^2 \, \phi^2 + \frac{1}{4} \, \lambda \, \phi^4 + H^4 \, Q\left(g_Y \, \frac{\phi}{H} \right)$, where

$\mu^2 = -m^2 < 0$ mass squared, $\lambda =$ quartic coupling,

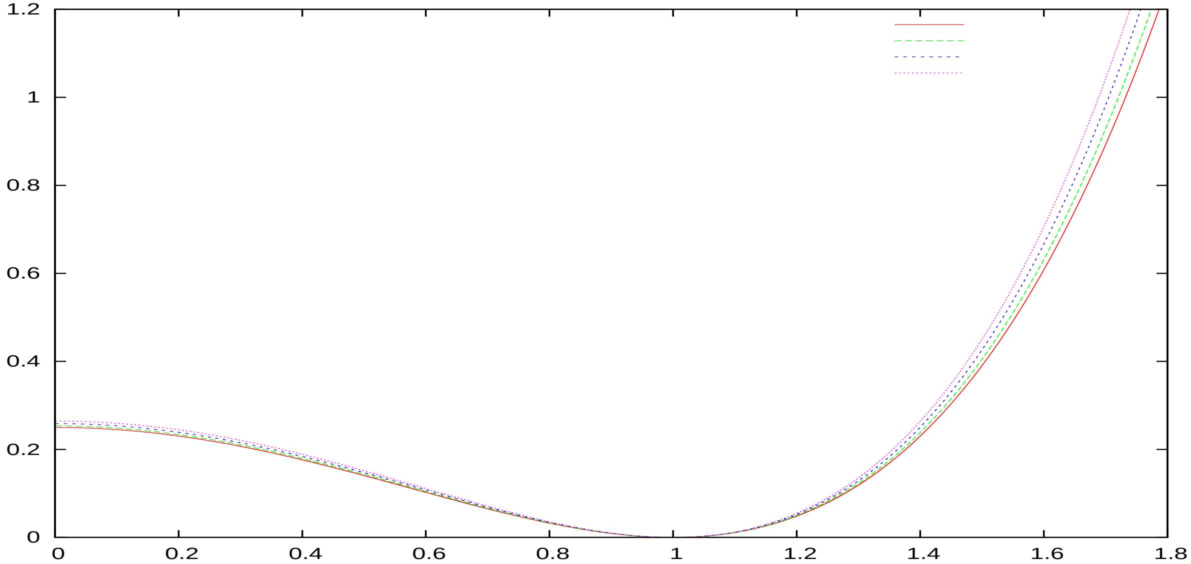
$$Q(x) = \frac{x^2}{8\,\pi^2} \left\{ (1+x^2) \left[\gamma + \mathrm{Re}\, \psi(1 + i\, x) \right] - \zeta(3)\, x^2 \right\} =$$

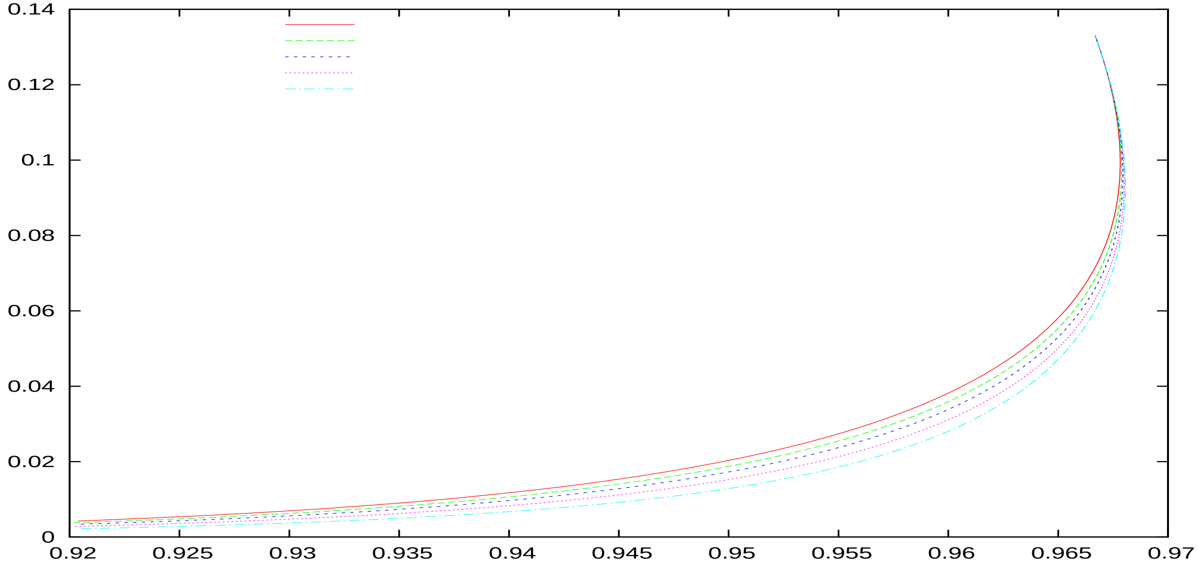
$$= \frac{x^4}{8\,\pi^2} \left[(1+x^2) \sum_{n=1}^{\infty} \frac{1}{n\,(n^2+x^2)} - \zeta(3) \right] \;, \quad x \equiv g_Y \, \frac{\phi}{H}$$

$$Q(x) \overset{x\to\infty}{=} \frac{x^4}{8\,\pi^2} \left[\log x + \gamma - \zeta(3) + \mathcal{O}\left(\frac{1}{x}\right) \right]$$

Minkowski limit (Coleman-Weinberg potential)

Effective fermionic inflaton potential and r vs. n_s

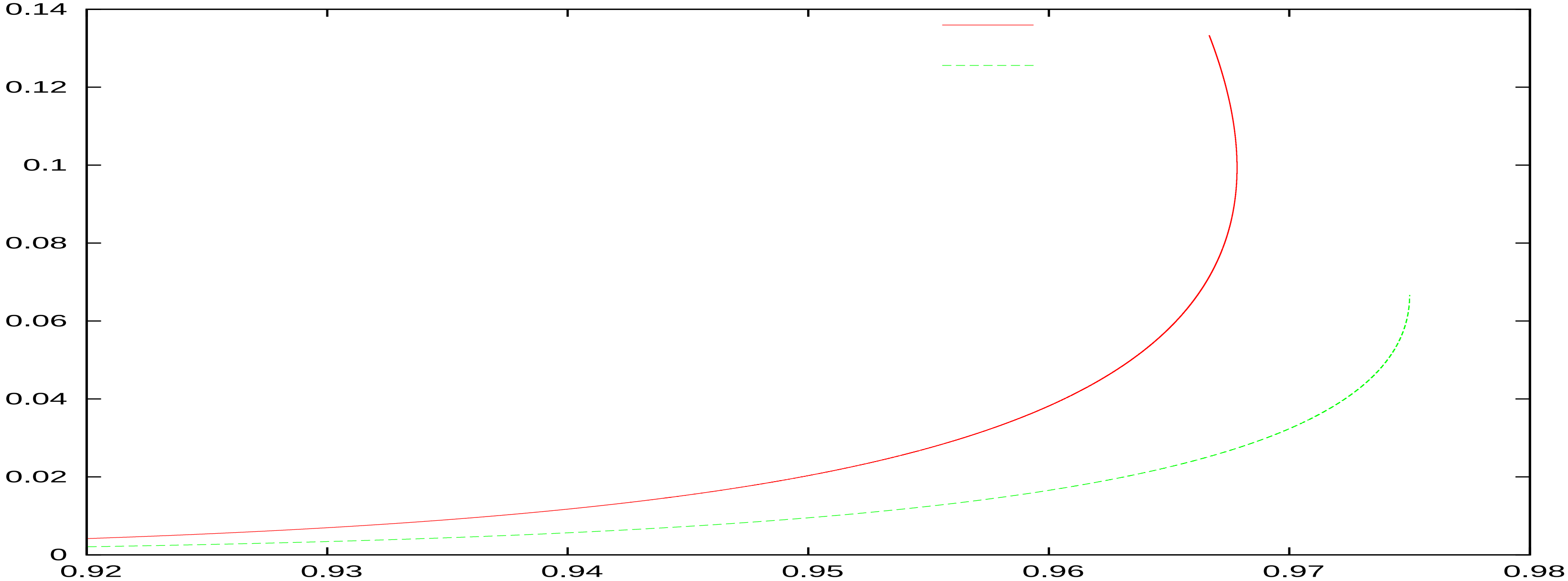




$y\, V(\phi)/[8\, N\; M^4]$ vs. ϕ/ϕ_{min} for $0 < g_Y < 500\; H/\phi_{min}$

r vs. n_s for $0 < g_Y < 800\; H/\phi_{min}$

The universal banana region



We find that all $r = r(n_s)$ curves for double–well inflaton potentials in the Ginsburg-Landau spirit fall inside the universal banana region $\mathcal{B}$.

The lower border of $\mathcal{B}$ corresponds to the limiting potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\,\chi^2 \quad \text{for } \chi < \sqrt{\frac{8}{y}} \quad , \quad w(\chi) = +\infty \quad \text{for } \chi > \sqrt{\frac{8}{y}}$$

This gives a lower bound for r for all potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value of the spectral index $n_s = 0.964$.

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they all meet at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the see-saw mechanism: $m_\nu \sim \frac{M^2_{\rm Fermi}}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least three independent ways.

Moreover, moduli potentials: $V_{moduli} = M^4_{\rm SUSY}\, v\left(\frac{\phi}{M_{Pl}}\right)$

ressemble inflation potentials provided $M_{\rm SUSY} \sim 10^{16}$ GeV.
First observation of SUSY in nature??

Quadrupole suppression and Fast-roll Inflation

The observed CMB-quadrupole (COBE,WMAP5) is six times smaller than the ΛCDM-slowroll model value.

In the best ΛCDM fit the probability that the quadrupole is as low or lower than the observed value is only 3%.

It is hence relevant to find a cosmological explanation of the quadrupole suppression.

Generically, the classical evolution of the inflaton has a brief fast-roll stage where $\dot{\phi}^2 \sim V(\phi)$ before the slow-roll regime. Fast-roll typically lasts 1 efold.

The slow-roll regime is an attractor with a large basin of attraction.
In case the quadrupole CMB mode ($\sim$ Hubble radius today) leaves the horizon by the end of fast roll (beginning of slow-roll), then the quadrupole modes get suppresed in agreement with the CMB observations.

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I. Initial conditions of inflationary perturbations,
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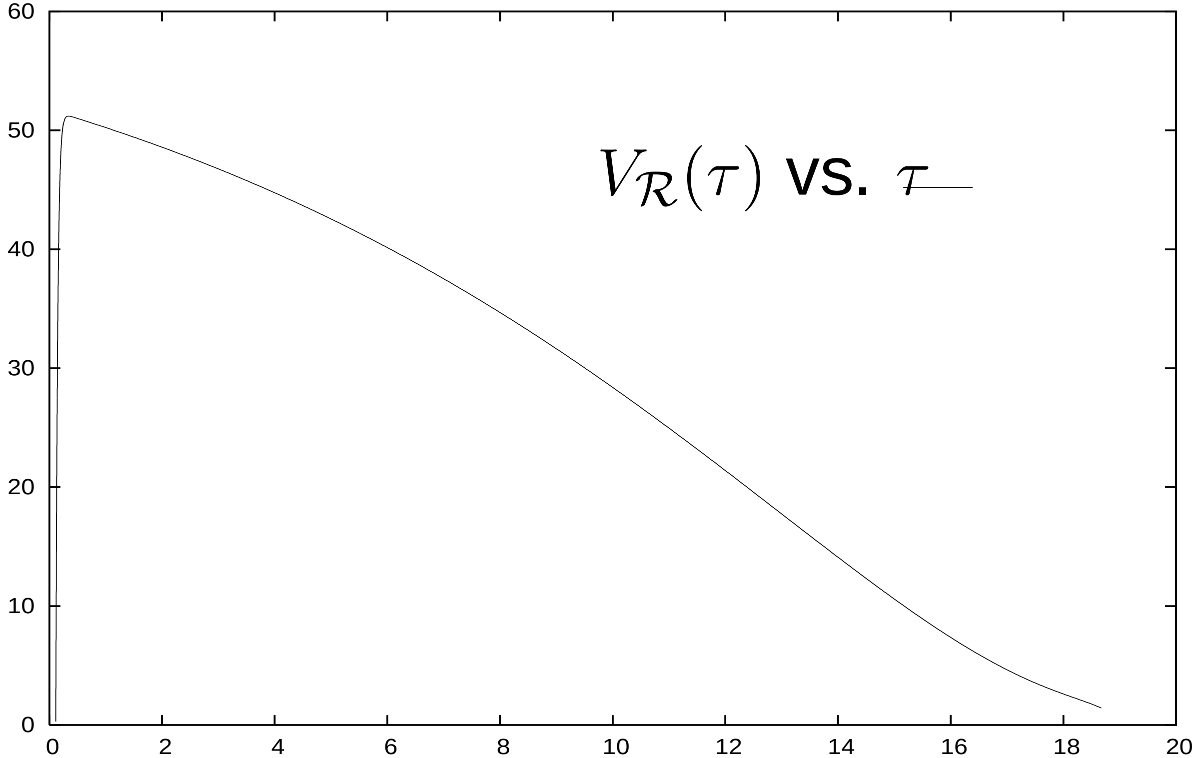
astro-ph/0607508 and astro-ph/0607487.

C. Destri, H. J. de Vega, N. G. Sanchez,
The CMB Quadrupole depression produced by early
fast-roll inflation: MCMC analysis of WMAP and SDSS data.
Phys. Rev. **D78**, 023013 (2008), arXiv:0804.2387.

and references therein.

Evolution of Curvature Fluctuations during Fast-Roll



$V_{\mathcal{R}}(\tau) \equiv \frac{W_{\mathcal{R}}(\eta)}{a^2(\eta)}$ slowly decreases during slow-roll as

$V_{\mathcal{R}}(\tau) \simeq 2\; N\; \mathcal{H}^2(\tau) + 1 + O(1/N)$

$V_{\mathcal{R}}(\tau) < 0$, attractive for earlier times τ.

The fast-roll transfer function $D(k)$

Two essential effects:

- The inflaton evolution starts at a finite time in the past. (Not at $\eta = -\infty$!). Initial conditions on the fluctuations can be imposed at a finite time in the past too.
- The potential felt by the fluctuations $V_{\mathcal{R}}(\tau)$ is not constant neither positive during fast-roll.

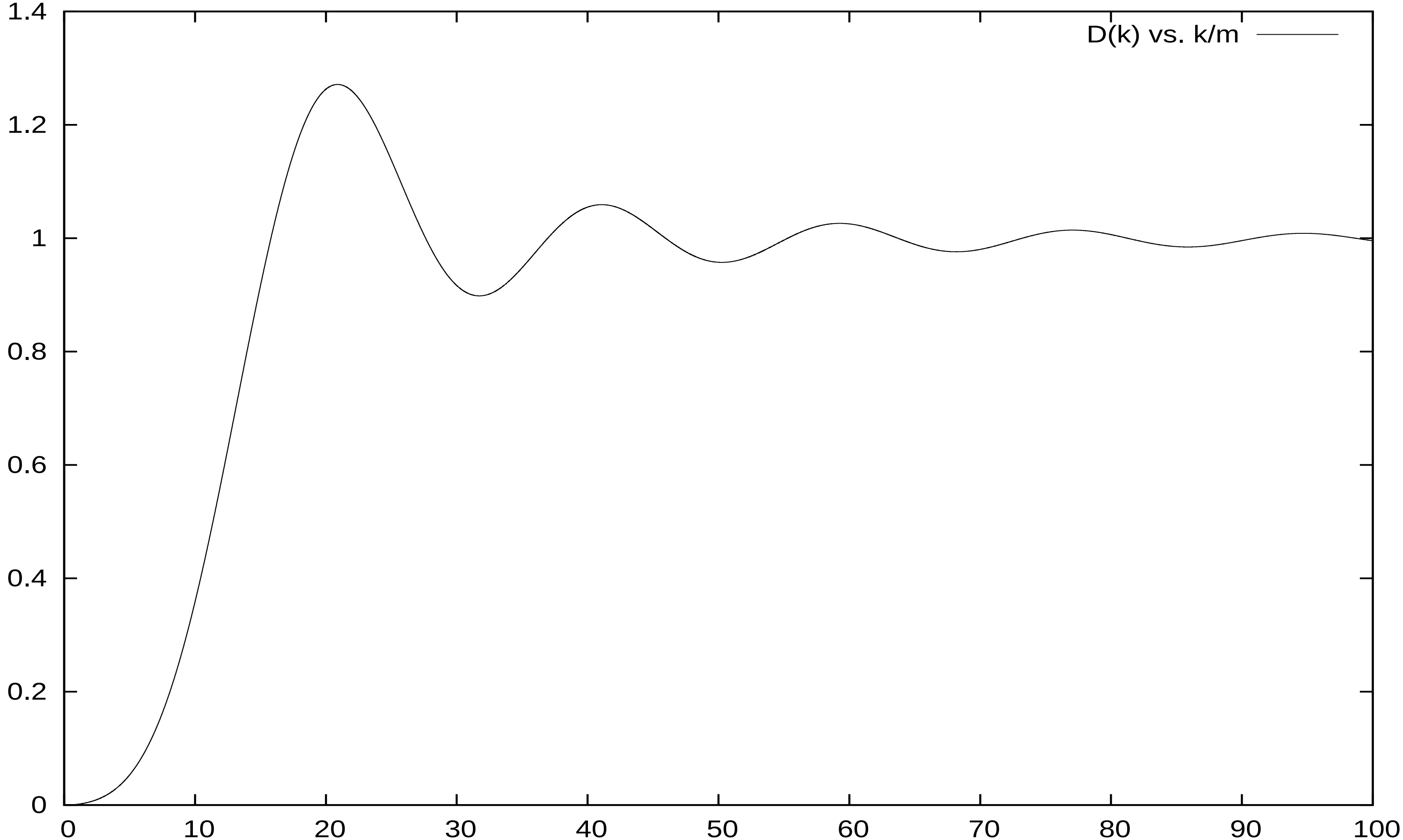
All this produces as primordial power:

$$P_{\mathcal{R}}(k) = \lim_{\eta \to 0^-} \frac{k^3}{2\,\pi^2} \, \frac{|S_{\mathcal{R}}(k;\eta)|^2}{z^2(\eta)} = P_{\mathcal{R}}^{BD}(k) \left[1 + D_{\mathcal{R}}(k)\right]$$

where the transfer function $D_{\mathcal{R}}(k)$ is determined by the evolution during fast-roll and

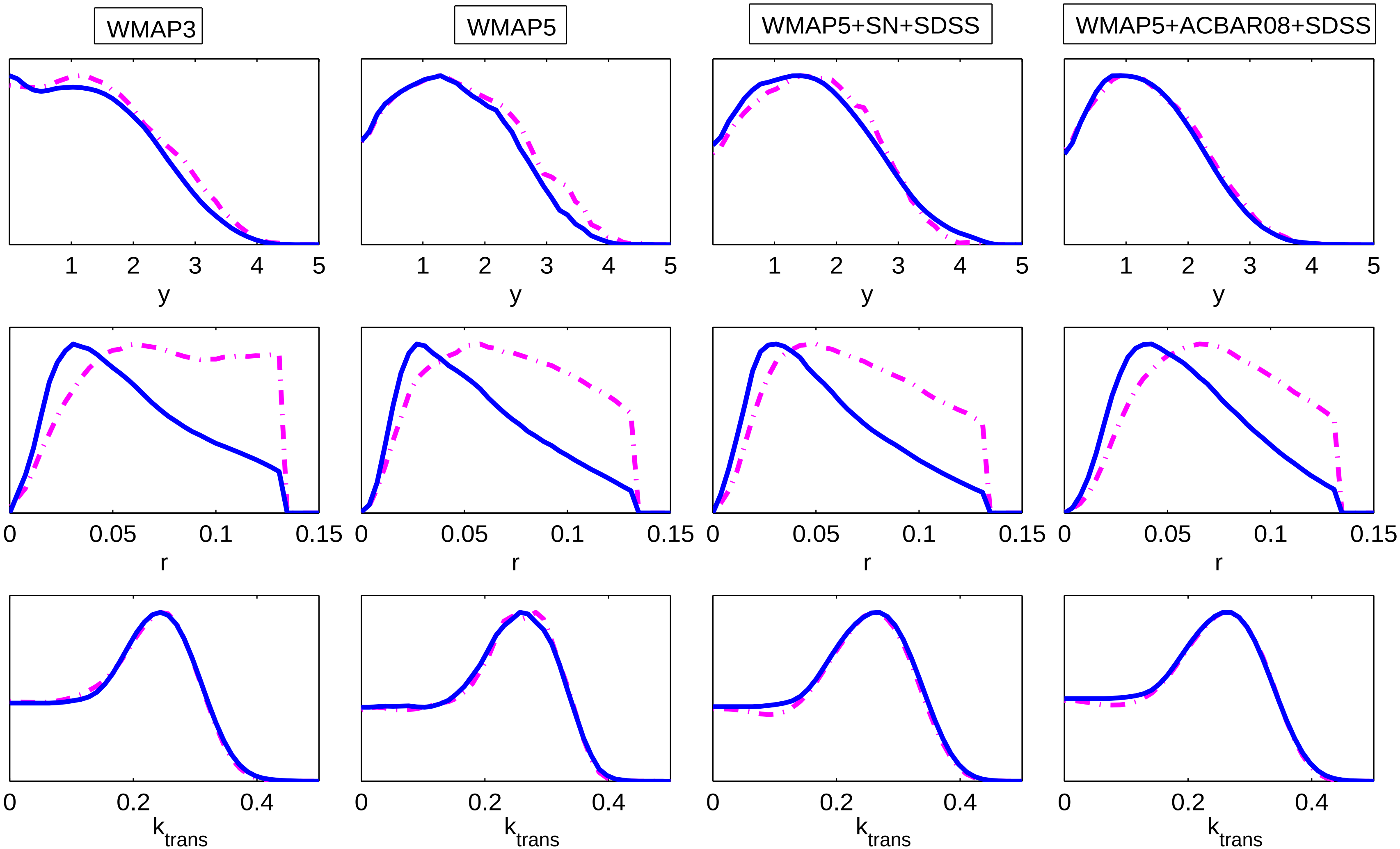
$$P_{\mathcal{R}}^{BD}(k) = |\Delta_{k\;ad}^{\mathcal{R}}|^2 \left(\frac{k}{k_0}\right)^{n_s - 1} , \quad 1 + D_{\mathcal{R}}(0) = 0 \; , \;\; D_{\mathcal{R}}(\infty) = 0$$

The Fast-Roll Transfer Function



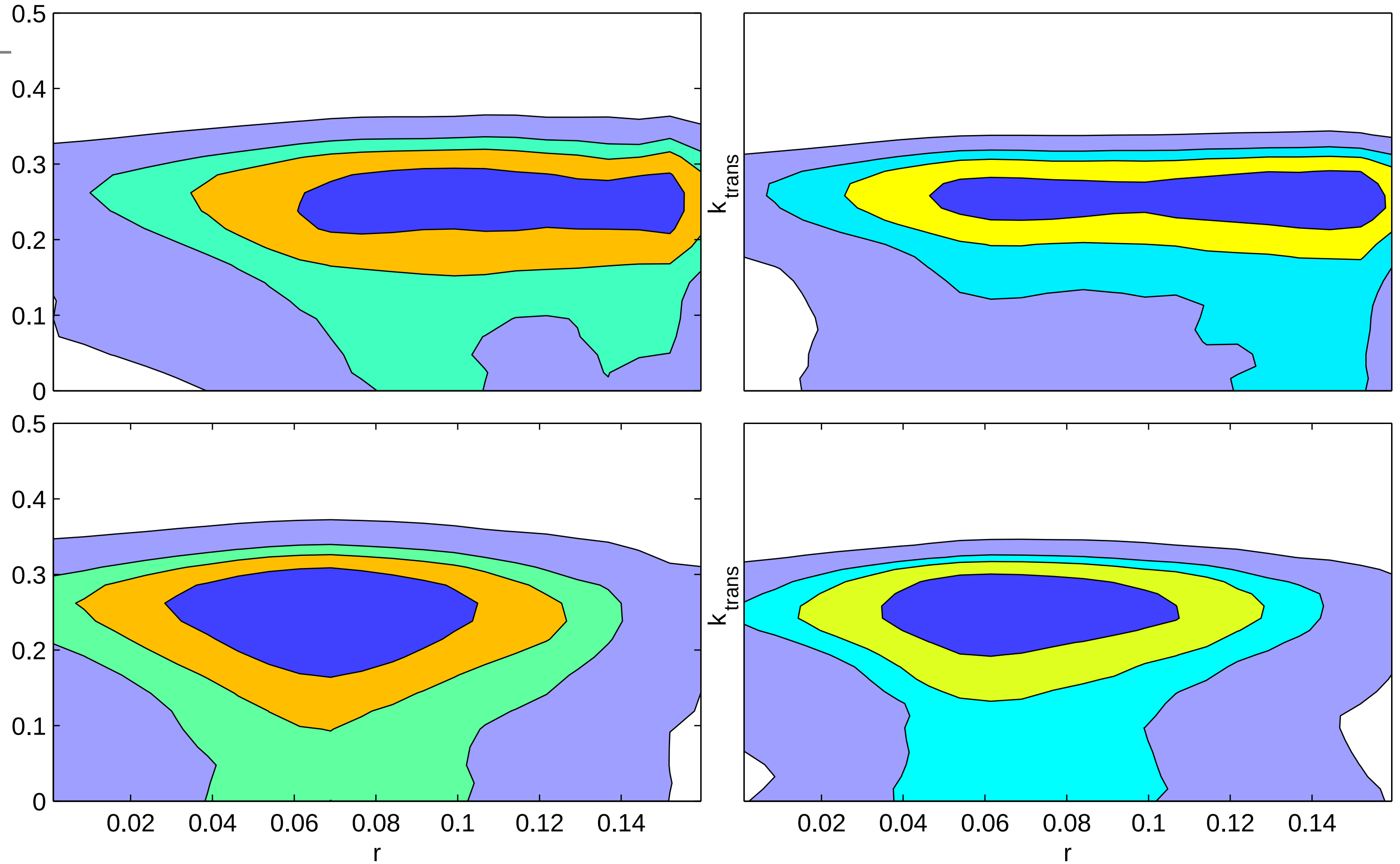
$k_Q = 11.5\, m, \; k_{trans} = 14\, m, \; k_0 = k_{pivot} = 96.7\, m,$
$m = 1.21\, 10^{13}\text{GeV}, k_Q^{today} = 0.238 \text{ Gpc}^{-1} \implies$ redshift at the beginning of inflation $= 0.9 \times 10^{56} \simeq e^{129} = (1 + z_r)\; e^{N_{tot}}$.

MCMC analysis of WMAP+SDSS data for ΛCDM+fast-roll



Marginalized distributions (blue) and mean likelihoods (magenta) for the ΛCDM+fast-roll model and the shown datasets.

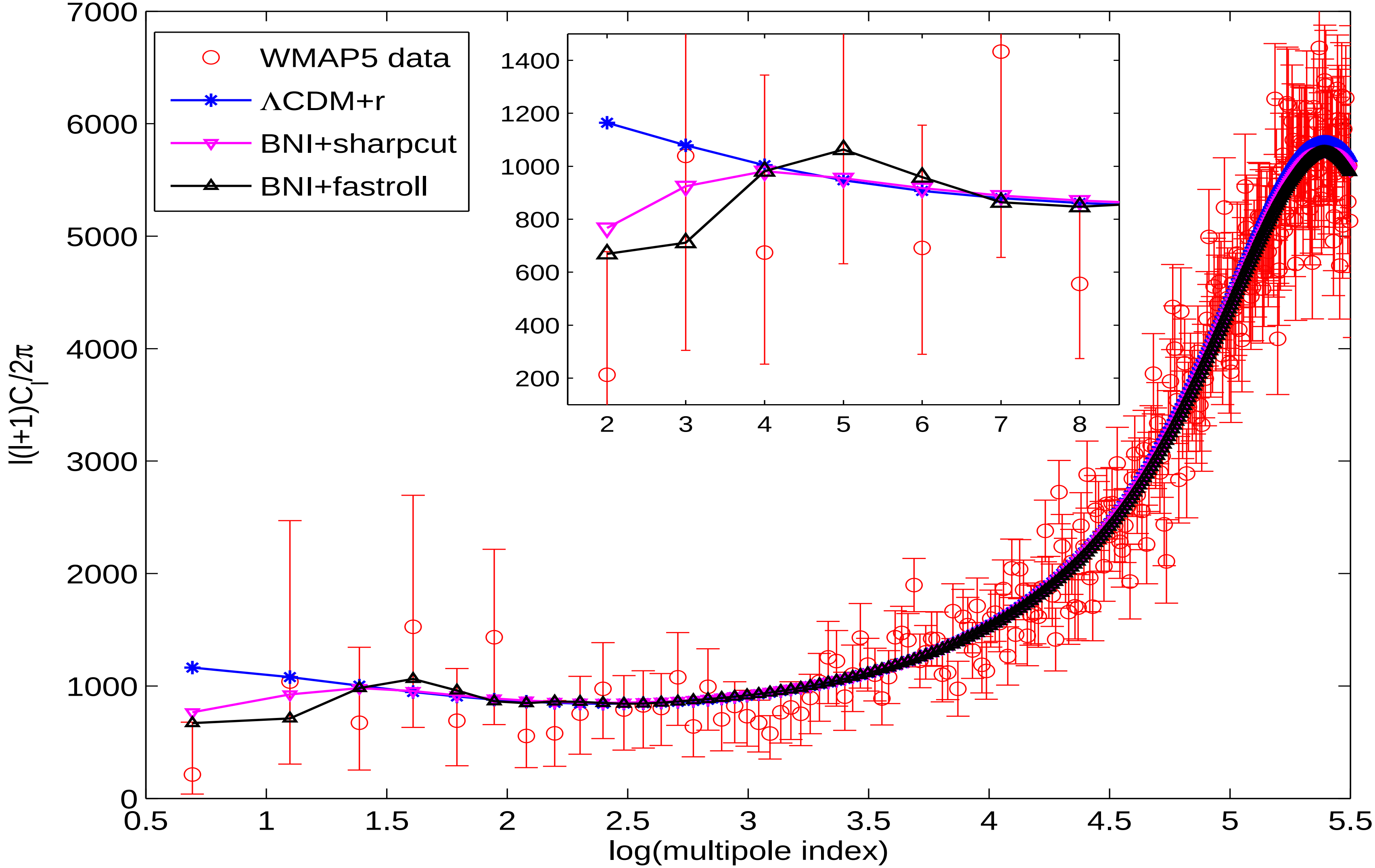
Marginalized distributions for ΛCDM+fast-roll



Marginalized pair distributions in the $(r,\, k_{trans})$ plane, at 20%, 41%, 68% and 95% CL.CMB data (above) and CMB+SDSS data (below) for ΛCDM quartic double well fastroll (right) and with cutoff (left). k_{trans} is in Gpc^{-1}.

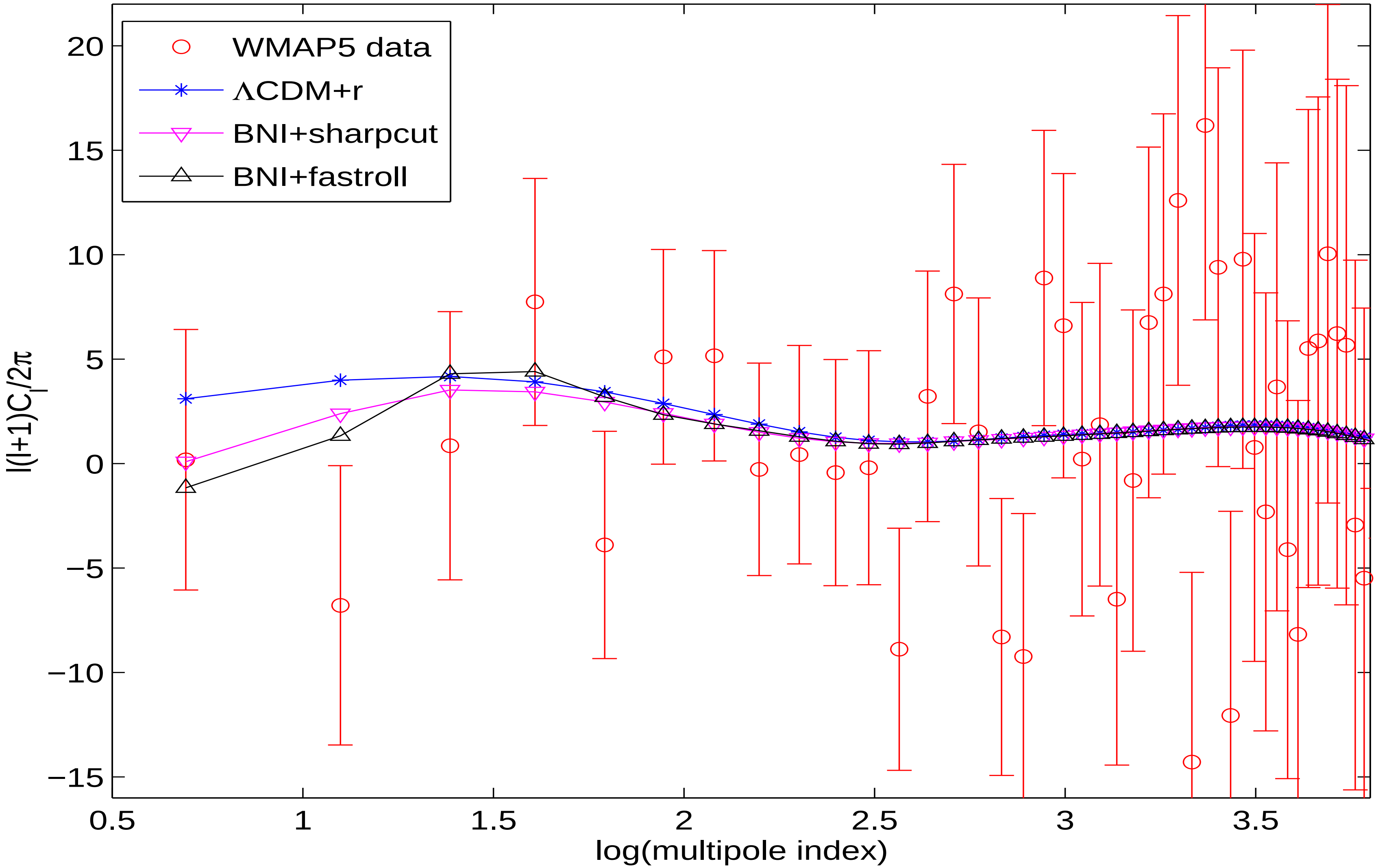
Comparison, with the experimental WMAP5 data

of the theoretical C_ℓ^{TT} multipoles



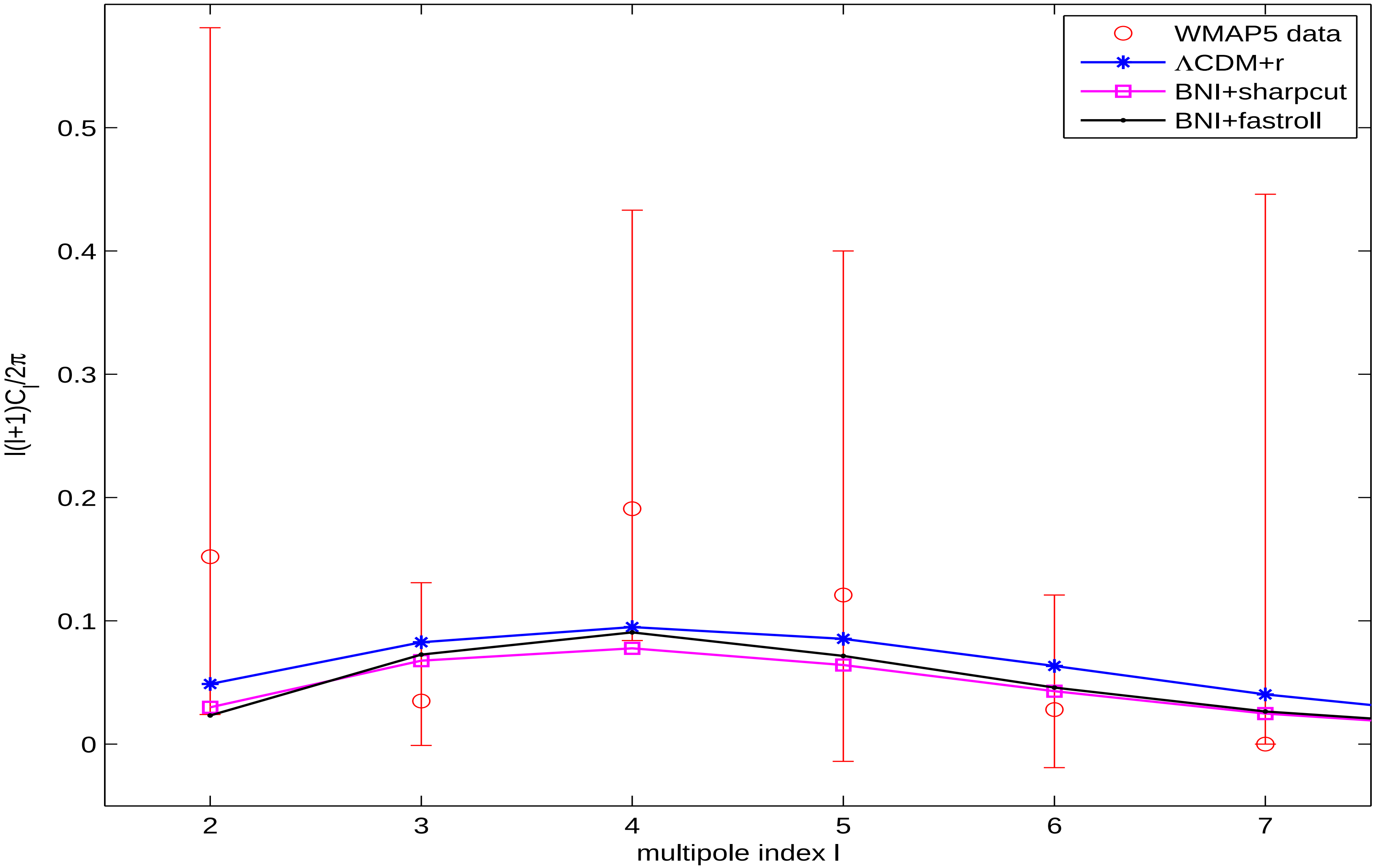
Comparison, with the experimental WMAP5 data

of the theoretical C_ℓ^{TE} multipoles

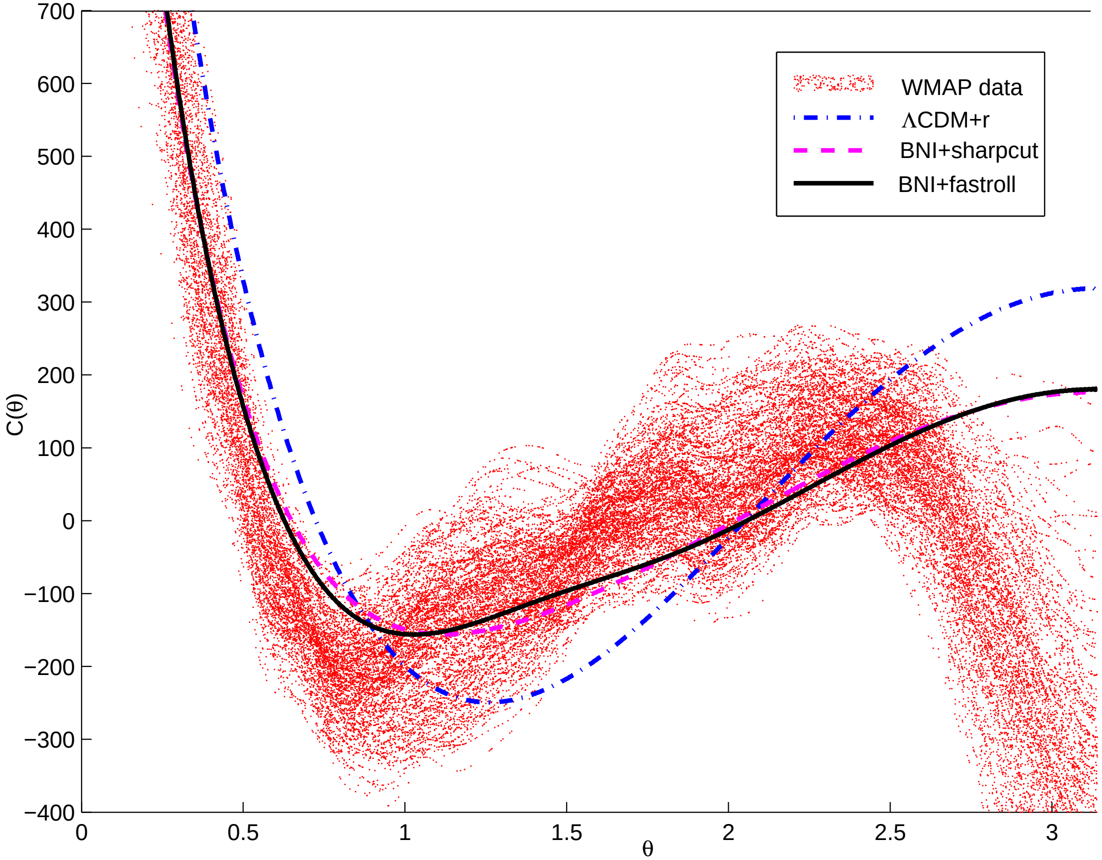


Comparison, with the experimental WMAP-5 data

of the theoretical C_ℓ^{EE} multipoles



Real space TT correlation function $C^{TT}(\theta)$



$C^{TT}(\theta)$ for ΛCDM, sharp cutoff and fast-roll models vs. the angle θ. The ΛCDM correlator differs from the two others only for large angles $\theta \gtrsim 1$.

Low multipoles dominate large angle correlations.

Fixing the Total Number of Inflation e-folds

The CMB Quadrupole suppression is explained if the quadrupole modes (horizon size today) exit the horizon by the end of fast-roll.

That is, by redshift $0.9 \times 10^{56} \simeq e^{129} = (1 + z_r)\, e^{N_{tot}}$

We can compute $1 + z_r =$ redshift by the beginning of RD.

In RD $a(t) = \sqrt{t/t_r}$, $H(t) = 1/(2\,t)$. In inflation: $H_r = H/\sqrt{N}$

Thus, $a_r = a_{eq} \sqrt{\dfrac{H_{eq}}{H_r}}$, r = beginning of RD era,
$eq =$ equilibration = RD $\to$ MD eras.

$$H_{eq}^2 = \frac{2}{3\, M_{Pl}^2}\, \rho_M(eq) \ , \ \rho_M(eq) = \rho_M(today)\, (1 + z_{eq})^3 \ ,$$

$\rho_M(today) = \Omega_M\, \rho_c = 3\, \Omega_M\, H_0^2\, M_{Pl}^2$, $H_0 =$ Hubble today.

$$a_r = \left(\frac{2\, \Omega_M\, N}{1+z_{eq}}\right)^{\frac{1}{4}} \sqrt{\frac{H_0}{H}}$$

Fixing the Total Number of Inflation e-folds

It is convenient to define $\beta \equiv \sqrt{\frac{10^{-4}\; M_{Pl}}{H}} \sim 1$,
(recall that $H \sim 10^{14}$ GeV).

Therefore, $a_r = 100\;\beta\;\left(\frac{2\;\Omega_M\;N}{1+z_{eq}}\right)^{\frac{1}{4}}\;\sqrt{\frac{H_0}{M_{Pl}}}$

and $1 + z_r = \frac{1}{a_r} = 4 \times 10^{28}\;\frac{1}{\beta}$.

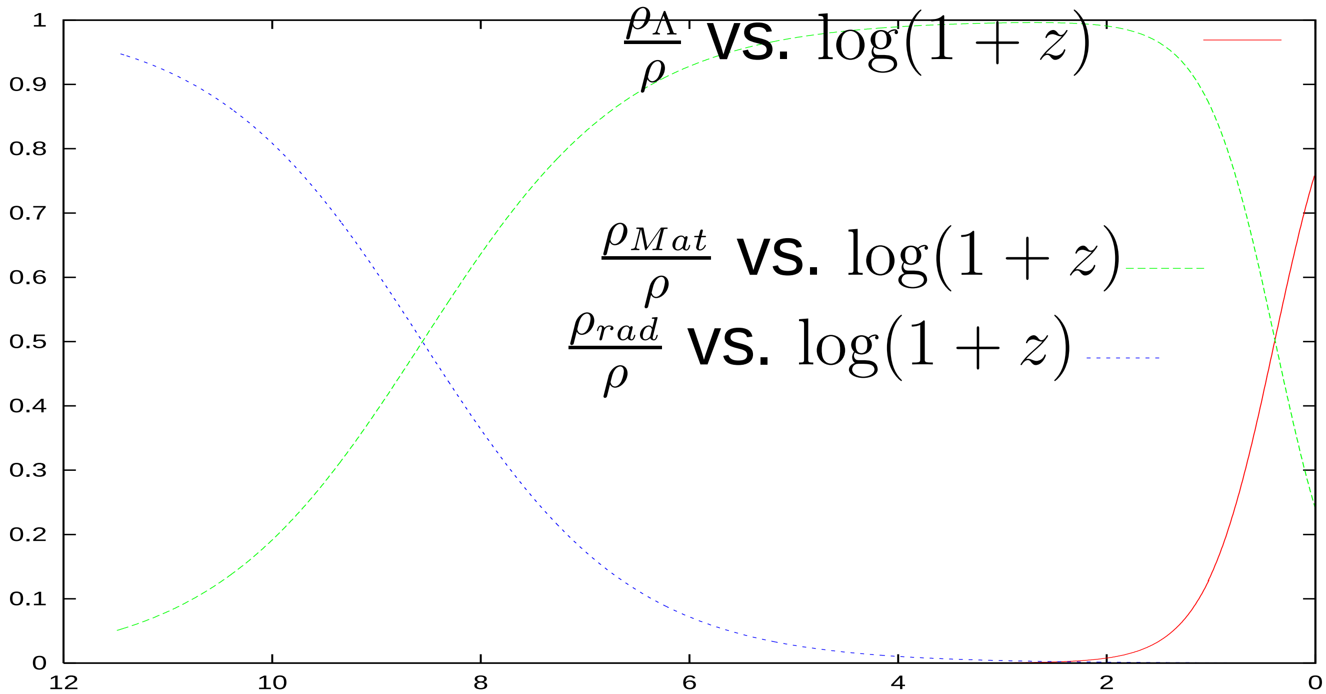
Since $(1 + z_r)\; e^{N_{tot}} = 0.9 \times 10^{56} \Longrightarrow\; e^{N_{tot}} = 0.229\; 10^{28}\;\beta$

and $N_{tot} = 63 + \log\beta$.

Recall our result for the quartic double–well using cmb+lss data: $\beta \simeq 2\;, \log\beta \simeq 0.7$

This value for N_{tot} is remarkably close to the lower bound on N_{tot} obtained from the entropy of the universe today.

The Universe is made of radiation, matter and dark energy



End of inflation: $z \sim 10^{29}, \; T_{reh} \lesssim 10^{16}$ GeV, $t \sim 10^{-36}$ sec.

E-W phase transition: $z \sim 10^{15}, T_{\rm EW} \sim 100$ GeV, $t \sim 10^{-11}$ s.

QCD conf. transition: $z \sim 10^{12}, T_{\rm QCD} \sim 170$ MeV, $t \sim 10^{-5}$ s.

BBN: $z \sim 10^9 \;, \quad T \simeq 0.1$ MeV, $t \sim 20$ sec.

Rad-Mat equality: $z \simeq 3200, \; T \simeq 0.7$ eV, $t \sim 57000$ yr.

CMB last scattering: $z \simeq 1100, \; T \simeq 0.25$ eV , $t \sim 370000$ yr.

Mat-DE equality: $z \simeq 0.47, \; T \simeq 0.345$ meV , $t \sim 8.9$ Gyr.

Today: $z = 0, \; T = 2.725$K $= 0.2348$ meV $t = 13.72$ Gyr.

Dark Matter

DM must be non-relativistic by structure formation ($z < 30$) in order to reproduce the observed small structures at $\sim 2-3$ kpc.

DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.

Consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).

Distribution function:

$F_d[a(t)\ P_f(t)] = F_d[p_c]$ freezes out at decoupling.
$P_f(t) = p_c/a(t) =$ Physical momentum.
$p_c =$ comoving momentum.

Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \frac{\int \frac{d^3P_f}{(2\pi)^3}\ \frac{\vec{P}_f^2}{m^2}\ F_d[a(t)\ P_f]}{\int \frac{d^3P_f}{(2\pi)^3}\ F_d[a(t)\ P_f]} = \left[\frac{T_d}{m\ a(t)}\right]^2\ \frac{\int_0^\infty y^4 F_d(y)dy}{\int_0^\infty y^2 F_d(y)dy}\ .$$

Dark Matter density and DM velocity dispersion

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2} \; F_d[a(t)\, P_f]$

g : # of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$. For $z \lesssim 30 \;\Rightarrow$ DM particles are non-relativistic:

$$\rho_{DM}(t) = \frac{m\ g}{2\pi^2} \frac{T_d^3}{a^3(t)} \int_0^\infty y^2 \; F_d(y) \; dy \; ,$$

Using entropy conservation: $\quad T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} \; T_{CMB}$,

$g_d =$ effective # of UR degrees of freedom at decoupling, $T_{CMB} = 0.2348 \; 10^{-3}$ eV, and

$$\rho_{DM}(\text{today}) = \frac{m\ g}{\pi^2\ g_d} \; T_{CMB}^3 \int_0^\infty y^2 \; F_d(y) \; dy = 1.107 \; \frac{\text{keV}}{\text{cm}^3} \quad (1)$$

We obtain for the primordial velocity dispersion:

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3} \, \langle \vec{V}^2 \rangle(z)} = 0.05124 \; \frac{1+z}{g_d^{\frac{1}{3}}} \left[\frac{\int_0^\infty y^4 \; F_d(y) \; dy}{\int_0^\infty y^2 \; F_d(y) \; dy}\right]^{\frac{1}{2}} \frac{\text{keV}}{m} \, \frac{\text{km}}{\text{s}}$$

Goal: determine m and g_d. We need TWO constraints.

The Phase-space density ρ/σ^3 and its decrease factor Z

The phase-space density $\frac{\rho}{\sigma^3}$ is invariant under the cosmological expansion and can only decrease under self-gravity interactions (gravitational clustering).

The phase-space density today follows observing dwarf spheroidal satellite galaxies of the Milky Way (dSphs)

$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \, \frac{\mathrm{keV/cm^3}}{(\mathrm{km/s})^3} = (0.18 \ \mathrm{keV})^4$ Gilmore et al. 07 and 08.

During structure formation $(z \lesssim 30)$, ρ/σ^3 decreases by a factor that we call Z.

$$\frac{\rho_s}{\sigma_s^3} = \frac{1}{Z} \, \frac{\rho_{DM}}{\sigma_{DM}^3} \quad (2)$$

N-body simulations results: $1000 > Z > 1$.

Constraints: First $\rho_{DM}(\mathrm{today})$, Second $\rho/\sigma^3(\mathrm{today}) = \rho_s/\sigma_s^3$

Mass Estimates for DM particles

Combining the previous expressions lead to general formulas for m and g_d:

$$m = 0.2504\,\mathrm{keV}\ \left(\frac{Z}{g}\right)^{\frac{1}{4}} \frac{\left[\int_0^\infty y^4\ F_d(y)\ dy\right]^{\frac{3}{8}}}{\left[\int_0^\infty y^2\ F_d(y)\ dy\right]^{\frac{5}{8}}}$$

$$g_d = 35.96\, Z^{\frac{1}{4}}\ g^{\frac{3}{4}}\ \left[\int_0^\infty y^4\ F_d(y)\ dy\ \int_0^\infty y^2\ F_d(y)\ dy\right]^{\frac{3}{8}}$$

These formulas yield for relics decoupling UR at LTE:

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}}\ \mathrm{keV} \left\{ \begin{array}{l} 0.568 \\ 0.484 \end{array} \right. ,\ g_d = g^{\frac{3}{4}}\ Z^{\frac{1}{4}} \left\{ \begin{array}{ll} 155 & \mathrm{Fermions} \\ 180 & \mathrm{Bosons} \end{array} \right. .$$

Since $g = 1 - 4$, we see that $g_d > 100 \Rightarrow T_d > 100$ GeV.

$1 < Z^{\frac{1}{4}} < 5.6$ for $1 < Z < 1000$. Example: for DM Majorana fermions $(g = 2)$ $m \simeq 0.85$ keV.

Out of thermal equilibrium decoupling

Results for m and g_d on the same scales for DM particles decoupling UR out of thermal equilibrium.

Particle physics candidates for UR decoupling in the keV scale: sterile neutrinos, gravitinos, ...

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

H. J. de Vega, N. G. Sanchez,
arXiv:0901.0922 and arXiv:0907.0006

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac52}\,\pi^{\frac72}}{45}\, g_d\, Y_\infty\, \left(\frac{T_d}{m}\right)^{\frac32}\, e^{-\frac{p_c^2}{2\,m\,T_d}} = \frac{2^{\frac52}\,\pi^{\frac72}}{45}\, \frac{g_d\, Y_\infty}{x^{\frac32}}\, e^{-\frac{y^2}{2\,x}}$$

$Y(t) = n(t)/s(t)$, $n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d$, $T_d < m$.

$$Y_\infty = \frac{1}{\pi}\sqrt{\frac{45}{8}}\,\frac{1}{\sqrt{g_d}\,T_d\,\sigma_0\,M_{Pl}}$$ late time limit of Boltzmann.

σ_0: thermally averaged total annihilation cross-section times the velocity.

From our previous general equations for m and g_d:

$$m = \frac{45}{4\,\pi^2}\,\frac{\Omega_{DM}\,\rho_c}{g\,T_\gamma^3\,Y_\infty} = \frac{0.748}{g\,Y_\infty}\ \text{eV} \quad \text{and} \quad m^{\frac52}\,T_d^{\frac32} = \frac{45}{2\,\pi^2}\,\frac{1}{g\,g_d\,Y_\infty}\,Z\,\frac{\rho_s}{\sigma_s^3}$$

Finally: $$\sqrt{m\,T_d} = 1.47\,\left(\frac{Z}{g_d}\right)^{\frac13}\,\text{keV}$$

We used ρ_{DM} today and the decrease of the phase space density by a factor Z.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d.

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \; 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV} \; \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3 \quad \text{for} \quad 1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV} \quad , \quad T_d < 10.2$ keV.

Only using ρ_{DM} today (ignoring the phase space density information) gives one equation with three unknowns: $m, \; T_d$ and σ_0,

$\sigma_0 = 0.16 \text{ pbarn} \dfrac{g}{\sqrt{g_d}} \dfrac{m}{T_d}$ http://pdg.lbl.gov

WIMPS with $m = 100$ GeV and $T_d = 5$ GeV require $Z \sim 10^{23}$.

The constant surface density in dark matter galaxies

Surface density of dark matter (DM) halos $\mu_{0D} \equiv r_0 \; \rho_0$,
$r_0 =$ halo core radius, $\rho_0 =$ central density

$\mu_{0D} \simeq 140 \; \frac{M_\odot}{\text{pc}^2} = 6400 \; \text{MeV}^3 = (18.6 \; \text{Mev})^3$ Donato et al.09

Universal value for μ_{0D}: independent of galaxy luminosity for a large number of galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and of different Hubble types.

Similar values $\mu_{0D} \simeq 80 \; \frac{M_\odot}{\text{pc}^2}$ in interstellar molecular clouds of size r_0 of different type and composition over scales $0.001 \, \text{pc} < r_0 < 100$ pc (Larson laws, 1981).

Density profile in Galaxies: $\rho(r) = \rho_0 \; F\left(\frac{r}{r_0}\right) \; , \; F(0) = 1.$

Profiles: $F_{Burkert}(x) = \frac{1}{(1+x)(1+x^2)} \; , \; F_{Sersic}(x) = e^{-x^{\frac{1}{n}}} \; , \; x \equiv \frac{r}{r_0}$

Same $1/r^3$ tail as cuspy NFW profile $F_{NFW}(x) = \frac{4}{x \; (1+x)^2}$

Virial theorem plus extensivity of energy $\Longrightarrow \mu_{0D} =$ constan

Virial theorem for self-gravitating systems:

$E = \frac{1}{2} \langle U \rangle = -\langle K \rangle$, $\quad E =$ total energy,

$U =$ potential energy, $K =$ kinetic energy. Therefore,

$$E = -\frac{G}{4} \int \frac{d^3r \, d^3r'}{|\vec{r}-\vec{r}'|} \langle \rho(r) \; \rho(r') \rangle = -\frac{G}{4} \rho_0^2 \, r_0^5 \int \frac{d^3x \, d^3x'}{|\boldsymbol{x}-\boldsymbol{x}'|} \langle F(x) \; F(x') \rangle$$

Energy divided by the characteristic volume r_0^3 goes as

$$\frac{-E}{r_0^3} \sim G \, \rho_0^2 \, r_0^2 = G \, \mu_{0D}^2$$

Energy extensivity requires E/r_0^3 fixed for large values of r_0
$\Longrightarrow \mu_{0D}$ must take the same constant value for all r_0

Estimating $\langle K \rangle$ yields $\langle K \rangle = \frac{1}{2} \int d^3r \, \langle \rho(r) \rangle \, \langle v^2 \rangle =$

$$= \frac{1}{2} \, \rho_0 \, r_0^3 \, \langle v^2 \rangle \int d^3x \, \langle F(x) \rangle \sim \rho_0 \, r_0^3 \, \langle v^2 \rangle \Longrightarrow \langle v^2 \rangle \sim G \, \mu_{0D} \, r_0$$

This is true both for molecular clouds and for galaxies.

DM surface density from linear Boltzmann-Vlasov eq

The distribution function of the decoupled DM particles:

$$f(\vec{x}, \vec{p}; t) = g \; f_0(p) + F_1(\vec{x}, \vec{p}; t)$$

$f_0(p) =$ thermal equilibrium function at temperature T_d

$$m \; g \int \frac{d^3p}{(2\pi)^3} \; f_0(p) = \rho_{DM} = \Omega_M \; \rho_c = 3 \, \Omega_M \; M_{Pl}^2 \; H_0^2$$

The linearized Boltzmann-Vlasov equation in the MD era can be recasted as the Gilbert integral equation (Volterra equation of 2nd kind) for

$$\Delta(k, t) \equiv m \; \int \frac{d^3p}{(2\pi)^3} \; \int d^3x \; e^{-i \, \vec{x} \cdot \vec{k}} \; F_1(\vec{x}, \vec{p}; t)$$

We evolve the fluctuations during the MD era using as initial conditions the density fluctuations by the end of the RD era,

$\Delta(k, t_{eq}) = \Omega_M \; \rho_c \; V \; \delta(k, t_{eq}) \;, \; t_{eq} =$ equilibration time,

$$V \sim 1/k_{eq}^3 \simeq \frac{f}{H_0^3}, \; k_{eq} \simeq 42.04 \; H_0 = 9.88 \; \mathrm{Gpc}^{-1}, \; f \simeq 1.35 \, 10^{-5}$$

Fluctuations $k > k_{eq}$ inside the horizon by t_{eq} are relevant

Density Profiles from the Gilbert equation

At the end of the RD era $t = t_{eq}$:

$$\delta(k, t_{eq}) = 24 \, |\phi_k| \, \log\left(0.116 \, \frac{k}{k_{eq}}\right)$$

[W. Hu and N. Sugiyama (1996).]

$|\phi_k| =$ primordial inflationary fluctuations:

$$|\phi_k| = \sqrt{2} \, \pi \, |\Delta_0| \, \left(\frac{k}{k_0}\right)^{n_s/2-2} \, ,$$

where $|\Delta_0| \simeq 4.94 \; 10^{-5}, \; n_s \simeq 0.964, \; k_0 = 2 \text{ Gpc}^{-1}$.

Density profile today in the linear approximation:

$$\rho_{lin}(r) = \frac{1}{2\,\pi^2\, r} \int_0^\infty k \; dk \; \sin(k\, r) \; \Delta(k, t_{\text{today}})$$

H. J. de Vega, N. G. Sanchez,
On the constant surface density in dark matter galaxies and interstellar molecular clouds, arXiv:0907.0006

The Gilbert equation

Define: $\widehat{\Delta}(k,t) \equiv \Delta(k,t)/\Delta(k,t_{eq})$.

The Gilbert equation takes the form:

$$\widehat{\Delta}(k,u) - \frac{6}{\alpha}\int_0^u \Pi[\alpha\,(u-u')]\,\frac{\widehat{\Delta}(k,u')}{[1-u']^2}\,du' = I[\alpha\,u]$$

where,

$$\Pi[z] = \frac{1}{I_2}\int_0^\infty dy\,y\,f_0(y)\,\sin(y\,z),\;\; I[z] = \frac{1}{I_2}\int_0^\infty dy\,y\,f_0(y)\,\frac{\sin(y\,z)}{z}$$

$$y \equiv \frac{p}{T_d}, \quad z \equiv \alpha\,u, \quad \alpha \equiv \frac{2\,k}{H_0}\sqrt{\frac{1+z_{eq}}{\Omega_M}}\,\frac{T_d}{m},$$

$$I_2 = \int_0^\infty dy\,y^2\,f_0(y), \quad 1+z_{eq} = \frac{1}{a_{eq}} \simeq 3200,$$

$u =$ dimensionless time variable,

$$u = 1-\sqrt{\frac{a_{eq}}{a}}\,,\; 0 \le u \le u_{\rm today} = 1-\sqrt{a_{eq}} \simeq 0.982$$

$$a(u) = \frac{a_{eq}}{(1-u)^2}\,,\; a({\rm today}) = 1\,.$$

$$\widehat{\Delta}(k,t) \overset{t\to t_{\rm today}}{=} \frac{3}{5}\,T(k)\,(1+z_{eq}), \quad T(k) =$$ transfer function.

The solution of the Gilbert equation today

Transfer function: $T(0) = 1$ and $T(k \to \infty) = 0$.

The solution of the Gilbert equation $\widehat{\Delta}(k,t)$ for $k < k_{fs}$ grows proportional to the scale factor.

k_{fs} = free-streaming (Jeans) comoving wavenumber.

k_{fs} = characteristic scale for the decreasing of $T(k)$ with k
$\Longrightarrow$ the natural variable here is $\gamma \equiv k \; r_{lin}$

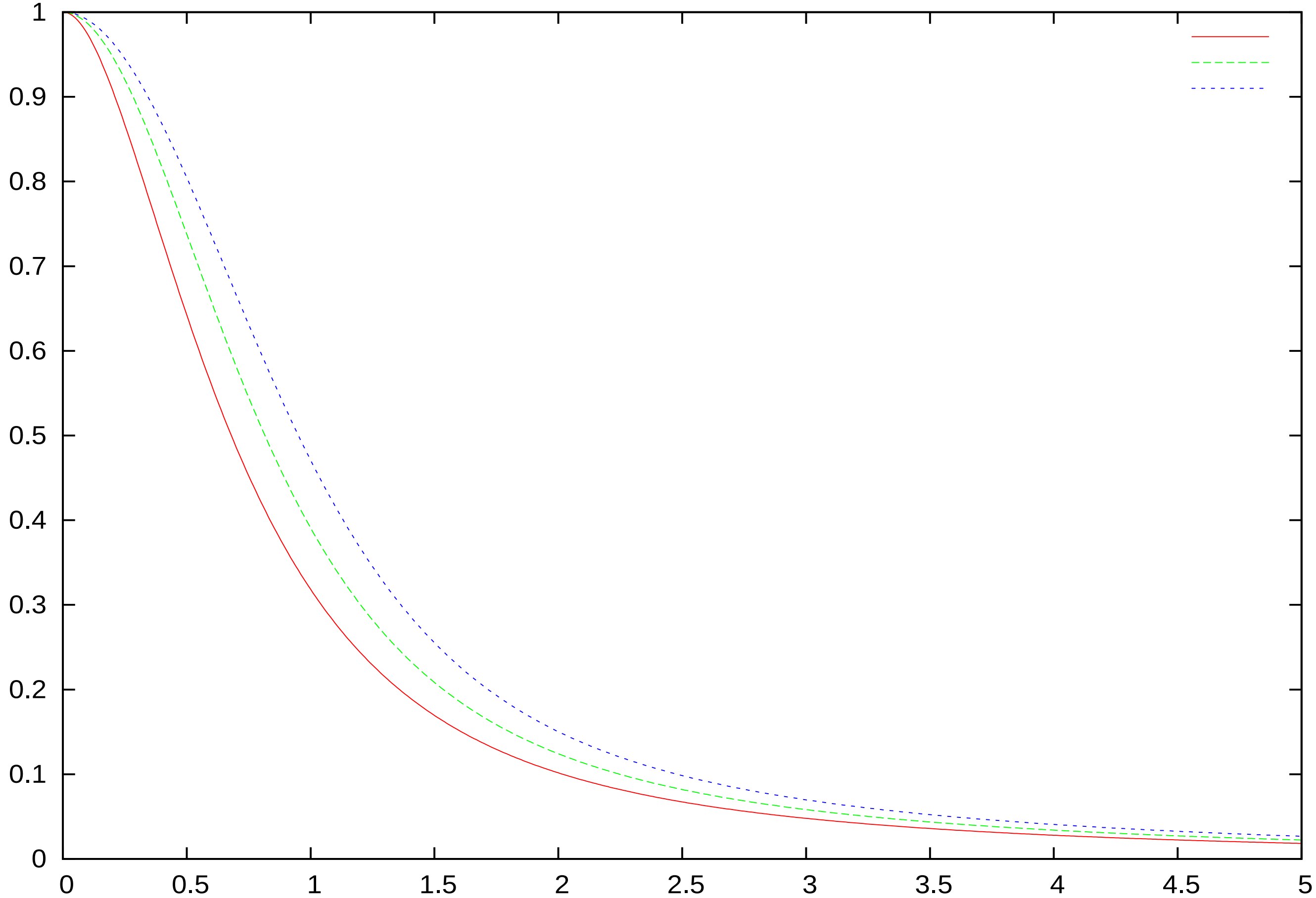
$r_{lin} \equiv \frac{\sqrt{2}}{k_{fs}} = \frac{2}{H_0} \, \sigma_{DM} \, \sqrt{\frac{1+z_{eq}}{\Omega_M}}$ and

$$\sigma_{DM} = \left(3 \;\; M_{Pl}^2 \; H_0^2 \; \Omega_{DM} \; \frac{1}{Z} \; \frac{\sigma_s^3}{\rho_s}\right)^{\frac{1}{3}} \Longrightarrow r_{lin} = 125.1 \;\; \left(\frac{10}{Z}\right)^{\frac{1}{3}} \text{ kpc}$$

Collecting all formulas we obtain for the fluctuations today

$$\Delta(k, t_{\text{today}}) = 1.926 \, \frac{M_{Pl}^2}{H_0} \, |\Delta_0| \; T(k) \;\; \left(\frac{k}{k_0}\right)^{n_s/2-2} \;\; \log\left(0.116 \, \frac{k}{k_{eq}}\right)$$

Density profiles in the linear approximation



Profiles $\rho_{lin}(r)/\rho_{lin}(0)$ vs. $x \equiv r/r_{lin}$

Fermions and Bosons decoupling ultrarelativistically and particles decoupling non-relativistically (Maxwell-Boltzmann statistics)

Density profiles in the linear approximation

The Fourier transform of the fluctuations today yield

$$\rho_{lin}(r) = (5.826 \text{ Mev})^3 \, \frac{Z^{n_s/6}}{r} \times$$

$$\times \int_0^\infty \gamma^{n_s/2-1} \, \log\left(\widehat{c}\, Z^{\frac{1}{3}} \, \gamma\right) \, \sin\left(\gamma \, \frac{r}{r_{lin}}\right) \, T(\gamma) \, d\gamma \; ,$$

$$\mu_{0D} = r_{lin} \; \rho_{lin}(0) =$$

$$= (5.826 \text{ Mev})^3 \; Z^{n_s/6} \; \int_0^\infty \gamma^{n_s/2} \; \log\left(\widehat{c}\, Z^{\frac{1}{3}} \, \gamma\right) \; T(\gamma) \; d\gamma \; ,$$

where:

$n_s/2 - 1 = -0.518, \; n_s/2 = 0.482, \; n_s/6 = 0.160$ and $\widehat{c} = 43.6$

Particle Statistics	$\mu_{0D} = r_{lin}\, \rho_{lin}(0)$
Bose-Einstein	$(16.71 \text{ Mev})^3 \; (Z/10)^{0.16}$
Fermi-Dirac	$(15.65 \text{ Mev})^3 \; (Z/10)^{0.16}$
Maxwell-Boltzmann	$(14.73 \text{ Mev})^3 \; (Z/10)^{0.16}$

Observed value: $\mu_{0D} \simeq (18.6 \text{ Mev})^3 \Rightarrow Z \sim 10 - 100$

Linear results for μ_{0D} and the profile vs. observations

Since the surface density $r_0 \; \rho(0)$ should be universal, we can identify $r_{lin} \, \rho_{lin}(0)$ from a spherically symmetric solution of the linearized Boltzmann-Vlasov equation.

The linear profiles obtained are cored since $T(k)$ decays for $k > k_{fs} \sim 1/r_{lin} \sim 0.008 \; (Z/10)^{\frac{1}{3}} \; (\mathrm{kpc})^{-1}$.

$\rho_{lin}(r)$ scales with the primordial spectral index n_s:

$\rho_{lin}(r) \overset{r \gg r_{lin}}{=} r^{-1-n_s/2} = r^{-1.482}$,

in agreement with the universal empirical behaviour $r^{-1.6\pm0.4}$, M. G. Walker et al., (2009).

For larger scales nonlinear effects from small k should give the customary r^{-3} tail.

The agreement between the linear theory and the observations is remarkable.

The comparison of our theoretical values for μ_{0D} and the observational value indicates that $Z \sim 10 - 100$.

This implies that the DM particle mass is in the keV range.

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark !
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \; \rho_c = (2.39 \text{ meV})^4 \quad , \quad 1 \text{ meV} = 10^{-3} \text{ eV}$.
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x},t) = \Phi_0(t) + \varphi(\vec{x},t), \quad \Phi_0(t) \equiv <\phi(\vec{x},t)>, \quad <\varphi(\vec{x},t)> = 0$$

$$\varphi(\vec{x},t) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} \left[a_{\vec{k}} \, \chi_k(\eta) \, e^{i\vec{k}\cdot\vec{x}} + \text{h.c.} \right],$$

$a^\dagger_{\vec{k}}, \; a_{\vec{k}}$ are creation/annihilation operators,
$\chi_k(\eta)$ are mode functions. $\eta =$ conformal time.
To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3\,H\,\dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0)\,\langle[\varphi(\boldsymbol{x},t)]^2\rangle = 0$$

where $g(\Phi_0) = \frac{1}{2}\,V'''(\Phi_0)$.
The mode functions obey:

$$\chi''_k(\eta) + \left[k^2 + M^2(\Phi_0)\,a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3\,H_0^2\,\eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi''_k(\eta) + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] \chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0 \, \eta \, (1 - \epsilon_V)}$.

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V = \mathcal{O}(1/N)$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$\chi_k(\eta) = \frac{1}{2} \sqrt{-\pi\eta} \; i^{\nu+\frac{1}{2}} \; H^{(1)}_\nu(-k\,\eta)$, $H^{(1)}_\nu(z) =$ Hankel function

Quantum fluctuations: $\langle [\varphi(\boldsymbol{x}, t)]^2 \rangle = \frac{1}{a^2(\eta)} \int \frac{d^3k}{(2\pi)^3} \, |\chi_k(\eta)|^2$

$$\frac{1}{2}\langle [\varphi(\boldsymbol{x}, t)]^2 \rangle = \left(\frac{H_0}{4\,\pi}\right)^2 \left[\Lambda^2 + \ln(4\,\Lambda^2) + \frac{1}{\Delta} + 2\,\gamma - 4 + \mathcal{O}(\Delta)\right]$$

UV cutoff $\Lambda = \text{physical cutoff}/H_0$, $\quad \frac{1}{\Delta} =$ infrared pole.

$\langle \dot{\varphi}^2 \rangle \quad , \quad \left\langle (\nabla\varphi)^2 \right\rangle$ are infrared finite

The one-loop calculation near $\Delta = 0$

$$\langle[\varphi(\boldsymbol{x},t)]^2\rangle = \frac{H_0^2\,(-\eta)^3}{8\,\pi} \int k^2\,dk\,|H_\nu^{(1)}(-k\eta)|^2$$

where we used that $a = -1/[H_0\,\eta]$. New integration variable:

$$q \equiv -k\,\eta = \frac{k}{H_0\,a} = \frac{k_{phys}}{H_0}, \quad \langle[\varphi(\boldsymbol{x},t)]^2\rangle = \frac{H_0^2}{8\,\pi} \int_0^\Lambda q^2\,dq\,|H_\nu^{(1)}(q)|^2 \;.$$

Exactly at the scale invariant point $\Delta = 0 \;, \;\; \nu = \frac{3}{2}$:

$q^2 \left|H_{\frac{3}{2}}^{(1)}(q)\right|^2 = \frac{2}{\pi}[\frac{1}{q} + q]$. Near the origin at arbitrary Δ:

$$q^2 \left|H_\nu^{(1)}(q)\right|^2 \overset{q\to 0}{=} \left[\frac{2^\nu\;\Gamma(\nu)}{\pi}\right]^2 q^{2\,\Delta-1} \;, \;\; \nu = \frac{3}{2} - \Delta \simeq \frac{3}{2}$$

We now split the integral $(0,\Lambda)$ into the integral $(0,\mu)$ plus the integral (μ,Λ) where $\mu \ll 1$:

$$\int_0^\mu dq\; q^2\,\left|H_\nu^{(1)}(q)\right|^2 = \frac{1}{\pi}\left[\frac{1}{\Delta} + \mu^2 + 2\,\gamma - 4 + 2\,\ln(2\,\mu) + \mathcal{O}(\Delta)\right]$$

$$\int_\mu^\Lambda dq\; q^2\,\left|H_\nu^{(1)}(q)\right|^2 = \frac{1}{\pi}\left[\Lambda^2 - \mu^2 + \log\frac{\Lambda^2}{\mu^2} + \mathcal{O}(\Delta)\right]$$

Summing up both integrals gives a μ independent result.

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3\,M_{Pl}^2}\left[\frac{1}{2}\,\dot{\Phi}_0^{\;2} + V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

Quantum corrections are proportional to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$!!

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0)\left[1 + \left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V}\right]$$

in terms of slow-roll parameters

Very DIFFERENT from the one-loop effective potential in Minkowski space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R^{''}(\Phi_0)]^2}{64\,\pi^2}\ln\frac{V_R^{''}(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4}\, g_{\mu\nu} \; < T_\alpha^\alpha >$

Hence, $V_{eff} = V_R + < T_0^0 >= V_R + \frac{1}{4} < T_\alpha^\alpha >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle.
Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0)\left[1 + \frac{H_0^2}{3\,(4\pi)^2\, M_{Pl}^2}\left(\frac{\eta_v - 4\,\epsilon_v}{\eta_v - 3\,\epsilon_v} + \frac{3\,\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T}\right)\right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30},\ \mathcal{T}_t = -\frac{717}{5},\ \mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_k|^2 \left\{1+ \right.$$
$$\left. +\frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1+\frac{\frac{3}{8}\,r\,(n_s-1)+2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{40}\right]\right\}$$
$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_k|^2 \left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

The anomaly contribution $-\frac{2903}{20} = -145.15$ DOMINATES as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta^{(S)}_k|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_k|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Summary and Conclusions

- We formulate inflation as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$. Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$. Infrared regime !!
- For all slow-roll models $n_s - 1$ and r are $1/N, \; N \sim 60$.
- MCMC analysis of WMAP+LSS data plus this theory input indicates a spontaneously broken inflaton potential: $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2 \; , \; y \simeq 1.26$.
- Lower Bounds: $r > 0.023 \; (95\% \text{ CL}) \; , \; r > 0.046 \; (68\% \text{ CL})$. The most probable values are $r \simeq 0.051 (\Leftarrow$ measurable !!) $n_s \simeq 0.964$.
- Model independent analysis of dark matter points to a particle mass at the keV scale. T_d may be > 100 GeV. DM is cold.

Summary and Conclusions 2

- CMB quadrupole suppression can be explained by the effect of fast-roll inflation provided the today's horizon size modes exited by the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

A wealth of data from WMAP (7 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy and Star formation. DM properties from astronomical observations. Better bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

The Dark Ages...Reionisation...the 21cm line...

Nature of Dark Energy? 76% of the energy of the universe.

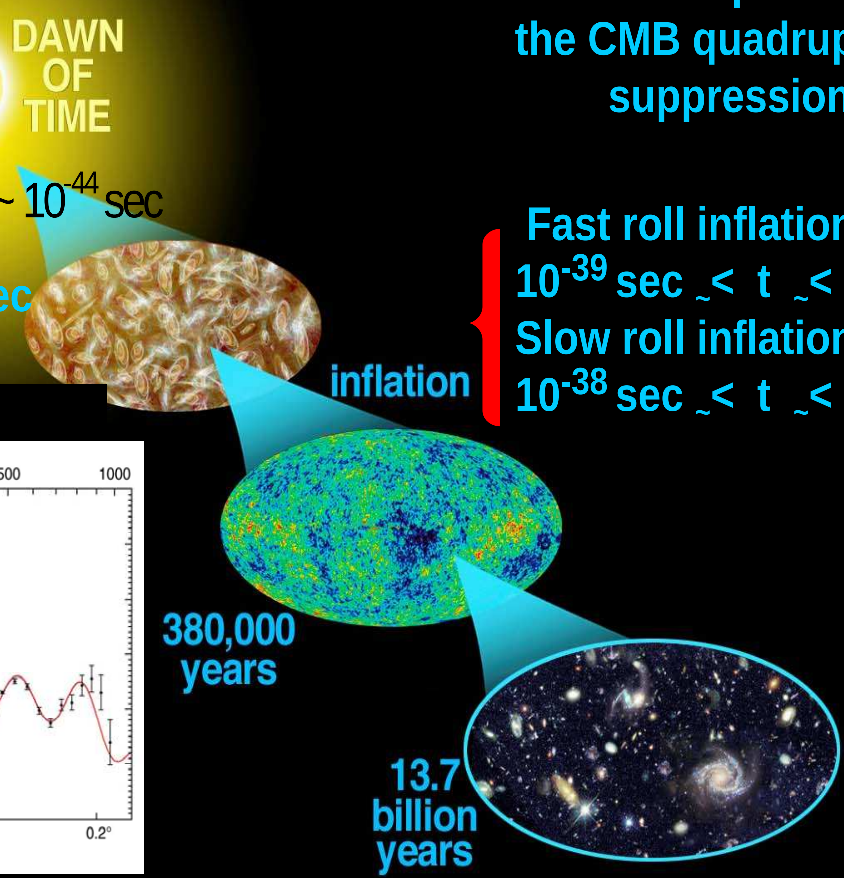
Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim$ keV.

Sterile neutrinos? Some unknown light particle ??

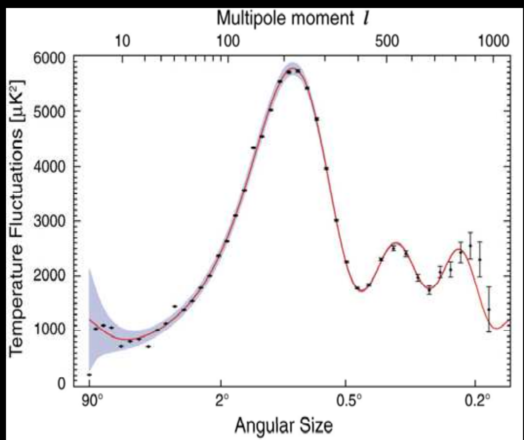
Need to learn about the physics of light particles (< 1 MeV).

COSMIC HISTORY AND CMB QUADRUPOLE SUPPRESSION



Fast roll inflation produces the CMB quadrupole suppression

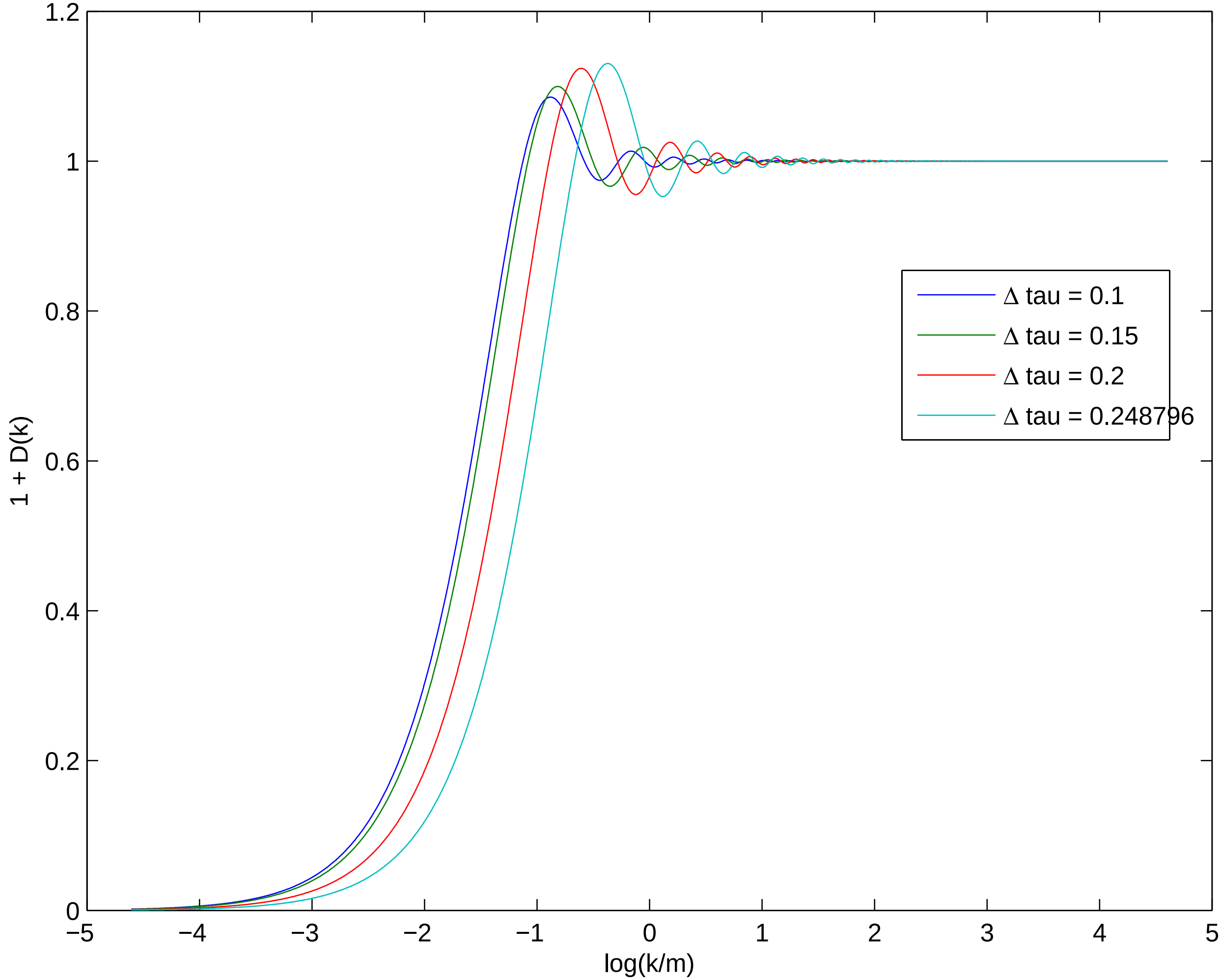
Fast roll inflation
10^{-39} sec $\lesssim t \lesssim 10^{-38}$ sec
Slow roll inflation
10^{-38} sec $\lesssim t \lesssim 10^{-36}$ sec



THANK YOU VERY MUCH

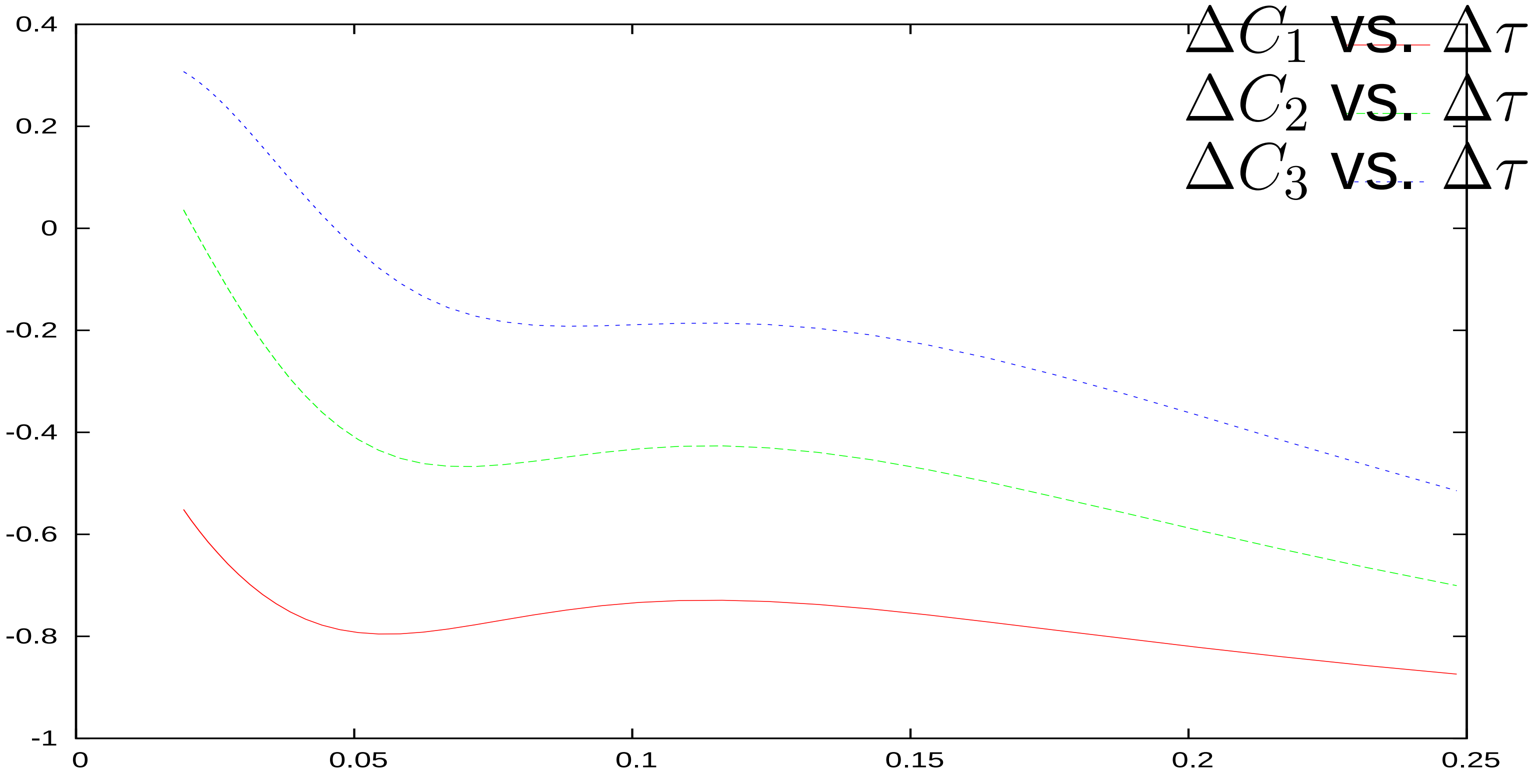
FOR YOUR ATTENTION!!

Transfer Function for different initial times of fluctuations



Transfer function $1 + D(k)$ for different initial times of fluctuations: $\Delta\tau$ from the begining of fast-roll. BD initial conditions. $\Delta\tau = 0.25$: begining of slow-roll.

$\Delta C_\ell^{\rm TT}$ vs. initial time of fluctuations



Changes on the dipole, quadrupole and octupole amplitudes according to the starting time $\Delta\tau$ chosen for the fluctuations from the begining of fast-roll. BD initial conditions. $\Delta\tau = 0.25$: begining of slow-roll.