COSMOLOGICAL MAGNETIC FIELDS AND CMBR POLARIZATION

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8-9 December, 2004

8th Paris Cosmology Colloquium 2004

WMAP AND THE EARLY UNIVERSE

CONTENT

1. Description of polarization of photons.
2. Polarization field of CMBR
3. Faraday effect.
4. Cosmic magnetic fields.
5. Faraday rotation of CMBR polarization

Description of polarization of photons.

Polarization density matrix:

$$\rho_{ij} = \langle E_i E_j^* \rangle$$

ρ_{ij} is 2nd rank tensor in 2D (x, y)-space if photon propagates along z.

INVARIANTS:

1. Trace = intensity of radiation:

$$T = \delta_{ij}\rho_{ij} = |E_x|^2 + |E_y|^2$$

2. Helicity:

$$\mathbf{V} = \epsilon_{\mathbf{ij}}\rho_{\mathbf{ij}}$$

Stokes parameters:

$$\rho_{ij} = T\,(I/2 + \xi_k \sigma_k)$$

I =unit matrix,
σ_k=Pauli matrices, $k = 1, 2, 3$.

Harmonic photons:

$$E_x = E_0 e_x \exp[-i\omega t + i\beta_x]$$
$$E_y = E_0 e_y \exp[-i\omega t + i\beta_y]$$
$$e_x^2 + e_y^2 = 1$$

Stokes parameters:

$$\xi_2 = e_x e_y \sin(\beta_x - \beta_y)$$

ξ_2 is invariant and describes circular polarization, i.e. photon helicity (pseudoscalar):

$$\lambda = \mathbf{sk}/\omega$$

ξ_1 and ξ_3 describe linear polarization:

$$\xi_3 = \left(e_x^2 - e_y^2\right)/2$$
$$\xi_1 = e_x e_y \cos(\beta_x - \beta_y)$$

They transform under coordinate rotation in (x, y)-plane by angle ϕ as:

$$\xi_1' = \xi_1 \cos 2\phi - \xi_3 \sin 2\phi$$
$$\xi_3' = \xi_1 \sin 2\phi + \xi_3 \cos 2\phi \xi_3$$

One can always make by rotation

$$\xi_1 = 0$$

Eigen-functions of rotation:

$$\xi_3 \pm i\xi_1 \to \exp\left[\pm 2i\phi\right]\left(\xi_3 \pm i\xi_1\right)$$

POLARIZATION BY THOMSON SCATTERING

Unpolarized photons on non-relativistic electrons:

$$\gamma + e \longrightarrow \gamma' + e'$$

produce polarized photons. If the reactrion amplitude is

$$A = e'_i \mathcal{A}_i$$

then

$$\rho_{ij} \sim \mathcal{A}_i \mathcal{A}_j^*$$

Coordinates: z is γ' direction, x is in the reaction plane, θ is the scattering angle:

$$\xi_3 = \frac{\sin^2\theta}{\omega/\omega' + \omega'/\omega - \sin^2\theta} \approx \frac{\sin^2\theta}{1+\cos^2\theta}$$

Thomson cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{3}{16\pi}\frac{8\pi\alpha^2}{3m_e^2}(1 + \cos^2\theta)$$

The only non-vanishing combination in the amplitude:

$$\mathbf{e}'\,\mathbf{k} \sim \sin\theta$$

Hence,

$$\xi_3 \sim \sin\theta^2$$

By the choice of coordinate direction:

$$\xi_1 = 0$$

Due to PARITY CONSERVATION:

$$\xi_2 = 0$$

POLARIZATION OF CMBR

Polarization vanishes in homogeneous and isotropic world.
Assumed vanishing circular polarization.
In this case intensity of polarization is described by two functions, $Q = T\xi_3$ and $U = T\xi_1$, where T is total intensity of radiation with frequency ω:

$$\bar{\rho} = \begin{bmatrix} Q & U \\ U & -Q \end{bmatrix}$$

Total polarization as a result of Thomson scatterring should be obtained by integration over all angles $d\Omega = d\cos\theta d\phi$ with rotation around z to the common coordinate system:

$$Q - iU = \frac{\sigma_T}{\sigma_N} \int d\omega \sin^2\theta \exp[2i\phi] T'(\theta, \phi)$$

where σ_N is a normalization area.

Scattered polarization proportional to quadrupole moment of incoming radiation.

PROPERTIES OF CMBR POLARIZATION FIELD

Two more (differential) invariants:
1. Scalar: $S = \partial_i \partial_j \rho_{ij}$
2. Pseudoscalar: $P = \epsilon_{ik} \partial_i \partial_j \rho_{jk}$ If density

pertubations are purely scalar, then:

$$\rho_{ij} = \left(2\partial_i \partial_j - \delta_{ij} \partial^2 \right) \Psi$$

For scalar perturbations

P=0

Non-zero P is an indication for something beyond scalar perturbations

Could be:

1. **Vector perturbations, e.g. magnetic field**

$$\rho_{ij} = \partial_i V_j - \partial_j V_i$$

$$P = \epsilon_{ij} \partial^2 \partial_i V_j$$

2. **Tensor perturbations, gravitational waves**

$$\rho_{ij} \sim \partial_i h_{3j} - \partial_j h_{3i}$$

3. **Second order perturbations** (S. Matarrese, yesterday)

$$\rho_{ij} \sim \partial_i \Psi_1 \partial_j \Psi_2 - \partial_i \Psi_2 \, \partial_j \Psi_1$$

(e.g. $\Psi_2 = \partial_t \Psi_1$)

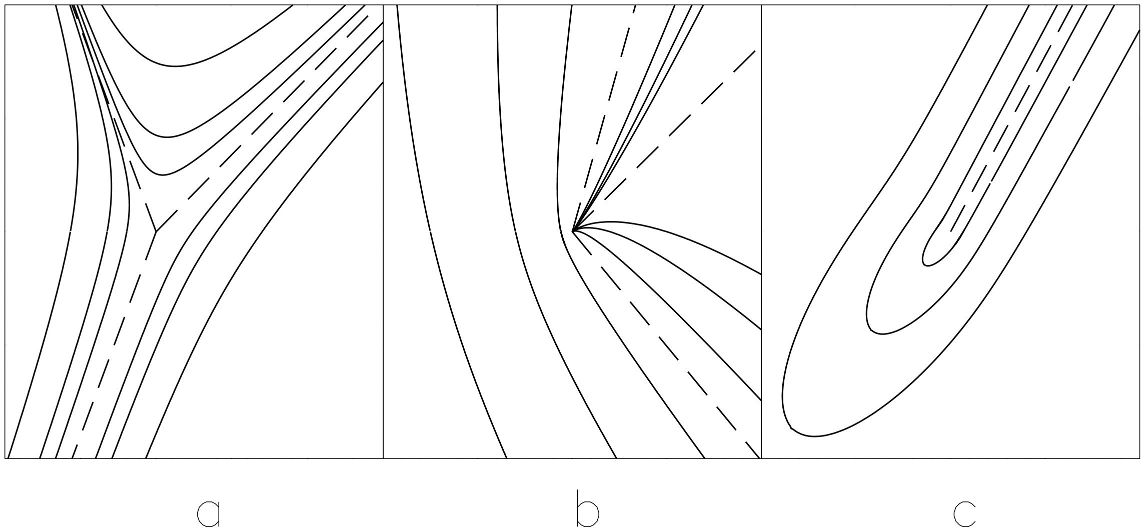
“Direction” of polarization by “vector”

$$v = (Q, U)$$

v is not a vector but some mixture of 2nd rank tensor components. Polarization map changes under rotation.
Singular points of v: usual saddles, foci, knots.

Different singular points transform into each other under rotation!

Real vectors indicating direction of polarization field are eigenvectors of ρ_{ij}.
Nonanalitic at zero polarization points - new types of singulariteis.



TYPES OF SINGULAR POINTS OF THE EIGENVECTORS OF POLARIZATION MATRIX: SADDLE, BEAK, COMET

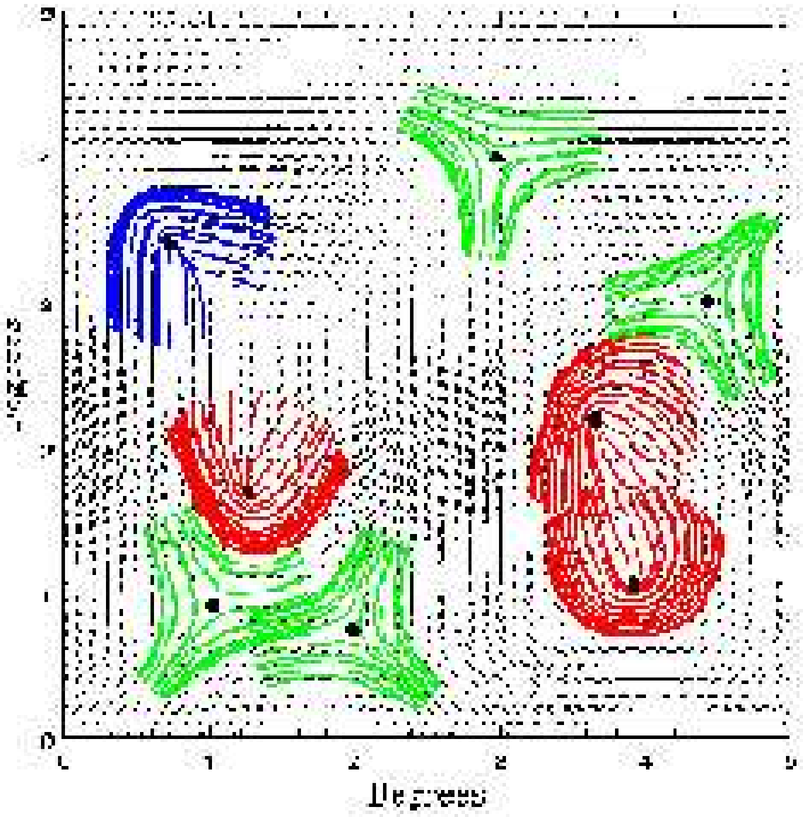


Figure 1: Simulated map of CMB polarization vector field $\vec{n}^{(+)}$ Solid lines show the flux line behavior near singular points where polarization vanishes.

FARADAY EFFECT

(rotation of polarization plane of linearly polarized photons in magnetic field)

In medium without reflection symmetry refration index for left- and right-handed photons are different, $n_+ \neq n_-$.

Linearly polarized wave can be decomposed into two rotationally polarized ones rotating in opposite directions:

$$1 = (1+i)/2 + (1-i)/2,$$

or

$$E^{(in)} = E_x = (E_+ + E_-)/2$$

Each helicity state propagates independently:

$$E_\pm^{(fin)} = \exp[ik_\pm l] E_\pm^{(in)}$$

If $k_{\pm} = k_0 \pm \Delta k$, then

$$E_x^{(fin)} = E_x^{(in)} \exp[ik_0 l] \cos(\Delta k l)$$
$$E_y^{(fin)} = E_x^{(in)} \exp[ik_0 l] \sin(\Delta k l)$$

Relative phase remains zero and the rotation angle is

$$\Phi = \arctan\left[E_y^{(fin)}/E_x^{(fin)}\right] = \Delta k l$$

REFRACTION INDEX OF IONIZED GAZ

Equation of motion of electrons, with charge $(-e)$, in external magnetic field $\mathbf{B_0}$ and electromagnetic wave $\mathbf{E}\exp[i\omega t]$:

$$\ddot{\mathbf{r}} = e\mathbf{B_0} \times \dot{\mathbf{r}} - e\mathbf{E}\exp[i\omega t]$$

Decompose propagating wave in terms of helicity states

$$\mathbf{E} = C_+\left(\mathbf{n}_x + i\mathbf{n}_y\right) + C_-\left(\mathbf{n}_x - i\mathbf{n}_y\right)$$

for which equation diagonalizes and solves as

$$\mathbf{x}_\pm = \frac{e\mathbf{E}_\pm}{m\omega\left(\omega \mp \omega_B\right)}$$

where $\omega_B = eB_0/m$.

Electric polarization moment:

$$\mathcal{P}_\pm = -N_e e\mathbf{x}_\pm$$

Dielectric constant

$$\epsilon_\pm = 1 + 4\pi\mathcal{P}/E = 1 + \frac{4\pi e^2 N_e}{m\omega(\omega \mp \omega_B)}$$

Refraction index $n = \sqrt{\epsilon}$ and thus differential Faraday rotation is

$$\frac{d\phi}{dl} = \frac{2\pi N_e e^3 B_0}{m^2 \omega^2}$$

where m is electron mass, $e^2 = \alpha = 1/137$, N_e is number density of electrons.

Usually the resuls is presented in terms of frequency $\nu = \omega/(2\pi)$ or wave length $\lambda = 1/\nu$.

COSMIC MAGNETIC FIELDS

Observed in galaxies

$$B_{gal} = \text{a few } \mu\text{G},$$

with coherence scale a few kpc.
Intergalactic fields $B_{ig} \sim 10^{-3} B_{gal}$, scale: $\sim (0.1 - 1)$ Mpc.

Adiabatic compression: $B \sim 1/l^2$:

$$l_{gal}^{(in)}/l_{gal} \sim 10^2,$$
$$l_{ig}^{(in)}/l_{ig} \sim 3,$$

Expect $B_{gal} \sim 10^3 B_{ig}$, if common origin and no galactic dynamo amplification.

Possible galactic dynamo amplifies by $10^{15 \pm 5}$! If this is the cased then primordial magnetic fields wold not influence CMBR polarization. Otherwise, if $B \sim 10^{-9}$ Gauss, the effect may be noticeable.

POSSIBLE MECHANISMS OF FIELD GENERATION.

1. Galactic processes, stellar phenomena and reconnection of field lines.
2. Processes during sructure formation.
3. -”- recombination epoch; vorticity, $\nabla \times V$ may be generated in the second order.
4. -”- in the early universe:
a) inflation $\longrightarrow$ small fields but large scale;
b) phase transitions $\longrightarrow$ large fields but small scales.

2,3,4 might create noticeable fields at CMBR decoupling potentially observable by Faraday rotation.

Dependence on cosmic scale factor:

$$d\Phi \sim \lambda^2 N_e B\, a\, d\eta \sim a^2 \frac{1}{a^3}\frac{1}{a^2}\, a \sim \frac{1}{a^2}$$

η is conformal time; $B \sim 1/a^2$ (assumed!). Rotation is dominated by early epoch, around recombination. Before recombination l_{free} is small and $\langle\Phi\rangle = 0$. After, N_e drops down. Reionization epokh?

ESTIMATE OF ROTATION ANGLE

Differential rotation angle:

$$\frac{d\Phi}{d\eta} = \frac{x_e N_e e^3 a}{2\pi m^2 \nu^2} \mathbf{Bn}$$

where x_e is ionization fraction and $\mathbf{n}$ is the unit vector in the direction of propagation of radiation.

By assumption $Ba^2 = const = B_0 a_0^2$ is equal to the present day value.

Optical depth:

$$\frac{d\tau}{d\eta} = N_e \sigma_T a$$

Total rotation angle (for homogeneous field along photon propagation):

$$\Phi = \frac{3\lambda_0^2 \mathbf{B}_0 \cdot \mathbf{n}}{16\pi^2 e} \int d\tau \exp\left(-\tau\right) = \frac{3\lambda_0^2 \mathbf{B}_0 \cdot \mathbf{n}}{16\pi^2 e}$$

(remember that $e^2 = \alpha$). Numerically:

$$\boldsymbol{\Phi \approx 2^o \left(\frac{B_0}{10^{-9} Gauss}\right) \left(\frac{30\,GHz}{\nu_0}\right)^2}$$

(1 Gauss $= 6.9 \cdot 10^{-14}$ MeV^2).

STATISTICAL PROPERTIES OF MAGNETIC FIELD

All in present day values:

$$\mathbf{B}_0(\mathbf{x}) = a^2(\eta)\,\mathbf{B}(\mathbf{x},\eta)$$

Fourier modes:

$$\mathbf{B}_0(\mathbf{x}) = \frac{1}{(2\pi)^3}\int d^3k e^{-i\mathbf{kx}}\,\mathbf{b_0}(\mathbf{k})$$

Sub-zero is omitted below.

Correlator:

$$\langle B_i(\mathbf{x_1})B_j(\mathbf{x_2})\rangle = C_{ij}(|\mathbf{x_1}-\mathbf{x_2}|)$$

because of homogeneity and isotropy on the average.

$$\Pi_{ij} = \langle b_i(\mathbf{k_1})b_j^*(\mathbf{k_2})\rangle = 2\,(2\pi)^3\delta\,(\mathbf{k}_1-\mathbf{k}_2)$$
$$\left[\left(\delta_{ij}-\kappa_i\kappa_j\right)\,S(k)+i\epsilon_{ijl}\kappa_l A(k)\right]$$

$\kappa_i = k_i/|\mathbf{k}|$.

Energy of magnetic field

$$\int d^3x B_j^2 = (2/\pi^2)\int dk k^2 S(k)$$

CORRELATOR OF ROTATION ANGLES

$$\langle \Phi(\mathbf{n})\Phi(\mathbf{m})\rangle = \left(\frac{3}{16\pi^2 e}\right)^2 \int d\eta\, g(\eta)$$

$$\int d\eta' g(\eta') \langle \left[\mathbf{B}_0(\Delta\eta\ \mathbf{n})\cdot\mathbf{n}\right)(\mathbf{B}_0(\Delta\eta'\ \mathbf{m})\cdot\mathbf{m}] \rangle$$

where $g(\eta) = (d\tau/d\eta)\exp[-\tau(\eta)]$
and $\delta\eta = \eta - \eta_0$.

$$\langle(\mathbf{B}\cdot\mathbf{n})(\mathbf{B}\cdot\mathbf{m})\rangle =$$

$$\frac{1}{2(2\pi)^3}\int d^3k\, \{[(\mathbf{nm}) - (\mathbf{n}\cdot\kappa)(\mathbf{m}\cdot\kappa)]\, S(k) +$$

$$i\,[(\mathbf{n}\times\mathbf{m})\cdot\kappa]\, A(k)\}\, \exp\left[-i\mathbf{k}\left(\mathbf{n}\Delta\eta - \mathbf{m}\Delta\eta'\right)\right]$$

Term $\sim A$ vanishes.

Expressions through Fourier spectrum of B:

$$\langle(\mathbf{Bn})(\mathbf{Bm})\rangle =$$
$$\left[(\mathbf{nm})C_\perp(r) + (\mathbf{nr}/r)(\mathbf{mr}/r)(C_\parallel(r) - C_\perp(r))\right]$$

where $\mathbf{r} = \mathbf{n}\Delta\eta - \mathbf{m}\Delta\eta'$, and

$$C_\perp(r) = \frac{2}{3(2\pi)^3}\int_0^\infty dk\mathcal{E}_\mathrm{B}(k)\left[j_0(kr) - \frac{j_2(kr)}{2}\right]$$
$$C_\parallel(r) = \frac{2}{3(2\pi)^3}\int_0^\infty dk\mathcal{E}_\mathrm{B}(k)\left[j_0(kr) + j_2(kr)\right]$$

where $j_i(x)$ are the spherical Bessel functions of the i^{th} order.

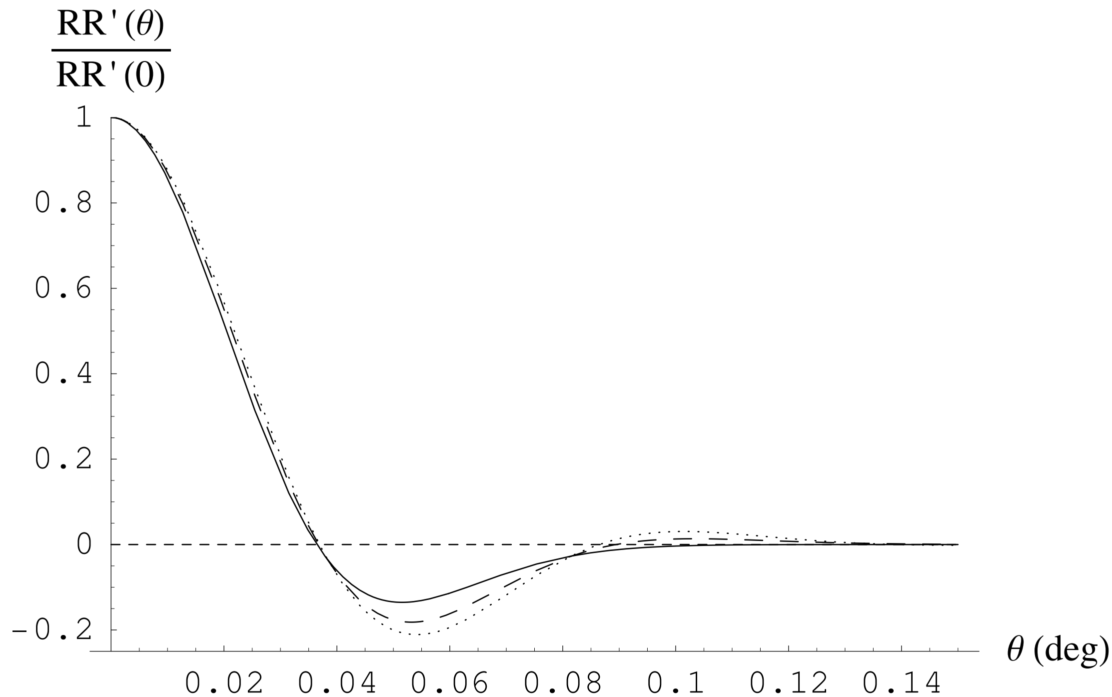


Figure 2: Faraday rotation measure correlation $RR'(\theta)$ as a function of the separation angle θ. The three lines correspond to the magnetic field spectral index $n_S = 2$ (*solid line*), $n_S = 4$ (*dashed line*) and $n_S = 6$ (*dotted line*). The correlation length of the magnetic field is $\xi = 20$ Mpc.

CONCLUSION

1. We do not understand how large scale cosmic magnetic fields have been formed. If B_{gal} and B_{ig} have the same origin and galactic dynamo did not operate, impact of primordial fields would be observable in CMBR polarization.

2. P-type (or B-type) polarization may mimick GW but different frequency dependence.

3. Eigenvector description may be useful (?). Their statistics?