Forecast for the Planck precision on the primordial amount of gravitational waves and other cosmological parameters

---

The standard model of the Universe (also known as "concordance" model) provides a realistic context to analyze the cosmic microwave background (CMB), large scale structure (LSS) and other cosmological/astrophysical data; inflation (quasi exponential accelerated expansion of the early universe) is part of this model. Recently, an european team (from Paris Observatory, UPMC-Paris, INFN-Milano Bicocca, IASF-Bologna and ASI-Rome-Frascati), forecasted the Planck satellite precision and detection probability for the crucial parameter r-accounting for the primordial amount of (still undetected) gravitationnal waves-and other cosmological parameters. With a theoretical framework developed at Paris Observatory and previously applied to the CMB + LSS data set (including WMAP + Sloan, OP news April 2008 http://www.obspm.fr/actual/nouvelle/apr08/wmap.en.shtml), the team determines $r = 0.0427$ as fiducial value to asses the Planck capability for r. Combining analytical and numerical (Monte Carlo Markov Chain) computations of the CMB sky (mock data) and including appropriate modelling of foreground residuals and systematic effects of instrumental and/or astrophysical origin, the team finds $0.028 < r < 0.116$ at 95 % CL with the best value $r = 0.04$ and the scalar spectral index $n_s = 0.9608$. The whole study is based on publically available information. The r detection probability is border line (r recovery will depend on the the data analysis quality and interpretation (and also of a question of luck)). Interestingly enough, improvements in the measurements of n_s will immediately give an improvement on the prediction of (non-vanishing) r from the (n_s, r) " banana-shaped" region found by the team, even if a secure detection of r will be still lacking. The range (and most probable value) infered for r supports the searching of CMB tensor-mode polarization in the current data as well as the planned CMB polarization missions under study by ESA and NASA.

Typical observables of the cosmic microwave background (CMB) anisotropies are the Temperature-Temperature correlations in different points or angles of the sky, the Temperature-polarization correlations, and the polarization-polarization correlations. Angular distance correlations in the sky are translated into ℓ multipole decompositions of amplitude C_ℓ. The CMB polarization is composed by two modes: an "electric" vector part (denoted E) and a "magnetic" or primordial tensor part (called B). The *Planck* satellite http://www.rssd.esa.int/planck, successfully launched on May 14th 2009 to measure with unprecedented accuracy the CMB temperature and polarization anisotropies, is expected to considerably constrain the primordial parameter r, (through the tensor B mode polarization accounting for the amount of primordial gravitational waves), if not obtain a detection of it. In this respect, the way of extracting and physically interpreting cosmological parameters (once the CMB data cleaned from the different astrophysical foregrounds) will be very important. It is then important and timely to make precise forecasts for the Planck determination of r and other cosmological parameters taking into account the modeling and interpretation progress made in the field, in particular from the WMAP results (see for example OP news April 2008).

The Standard Model of the Universe (or "concordance" model) provides a realistic context to analyze the CMB and other cosmological/astrophysical data. Inflation (quasi-exponential accelerated expansion) of the early Universe is part of this model and one important goal of CMB experiments is probing the physical and cosmological predictions of it. Inflation solves the shortcomings of the decelerated expanding cosmology (horizon problem, flatness, Universe entropy) and explains the observed CMB anisotropies providing the mechanism for the generation of scalar and tensor fluctuations seeding the large scale structures (LSS) and primordial (still undetected) gravitational waves (B mode polarization). The current CMB + LSS data support the standard inflationary predictions of a nearly spatially flat Universe with adiabatic and nearly scale invariant initial density perturbations. In the effective theory approach to inflation (based on the Ginzburg-Landau (G-L) approach to superconductivity and phase transitions) the potential of inflation is a polynomial in the field. The fourth degree double–well inflaton potential gives an excellent fit of the present CMB + LSS data. Adding higher order terms with additional parameters does not improve significantly the fits. In this framework, analytic formulae for n_s, r are obtained and the variation (running) of the scalar spectral index $dn_s/d\ln k$ turns to be of order of $\frac{1}{(60)^2} \sim 3 \times 10^{-4}$, very small and can be neglected.

We evaluate the accuracy in the recovery of the cosmological parameters expected from the Planck data, and include the degradation in accuracy that could come from various sources of systematic effects, of instrumental (radiometric and bolometric) and/or astrophysical (optical, thermal, lensing) origin, or their coupling affecting the measurements. We consider the $C_\ell-$likelihood, the white noise sensitivity of Planck (LFI and HFI) in the three characteristic 70, 100 and 143 GHz channels and also consider a cumulative channel (whose statistical χ^2 is the sum of the χ^2's of the three ones). Lensing effects acts on the CMB tensor B-modes as a contamination by transforming electric E-modes into B-modes. Our study and Monte Carlo Markov Chains (MCMC) simulations show that the lower bounds we find on r are not significantly affected by lensing effects and r has always well defined lower bounds regardless of lensing and/or residuals. We use and test two relevant models: the ΛCDMr model, that is the standard ΛCDM model augmented by the tensor–to–scalar ratio r, and the ΛCDMrT model, that is the ΛCDMr model in which the double–well inflaton potential is included: in this case n_s and r lay on a banana-shaped curve (the upper border of the banana-shaped region Fig. 1 and one novelty in the MCMC analysis with this model is that we include the analytical expressions for n_s and r derived from inflation in it. We take both, ΛCDMr and ΛCDMrT, as fiducial models with the fiducial value $r = 0.0427$ to produce with our MCMC simulations the corresponding skies (mock data) for the CMB anisotropy and polarization multipoles C_l and we obtain the marginalized likelihood distributions for the cosmological parameters in the two test models ΛCDMr and ΛCDMrT.

Our findings are summarized in Figs. 1-2-3. We obtain peaked marginalized distributions for r and a lower bound for r: the progress with respect to our findings with WMAP are notorius (as can be seen by comparing with the previously obtained r distribution curve (OP news April 2008 http://www.obspm.fr/actual/nouvelle/apr08/wmap.en.shtml). Our conclusions including the systematic effects and foreground residuals are as follows:

- The likelihood and probability distributions result almost the same when including the residuals. Only the cosmological parameters sensitive to the B modes appear to be affected by the residuals, namely, the reionization parameters (optical depth τ and redshift z_{re}) and of course the ratio r.

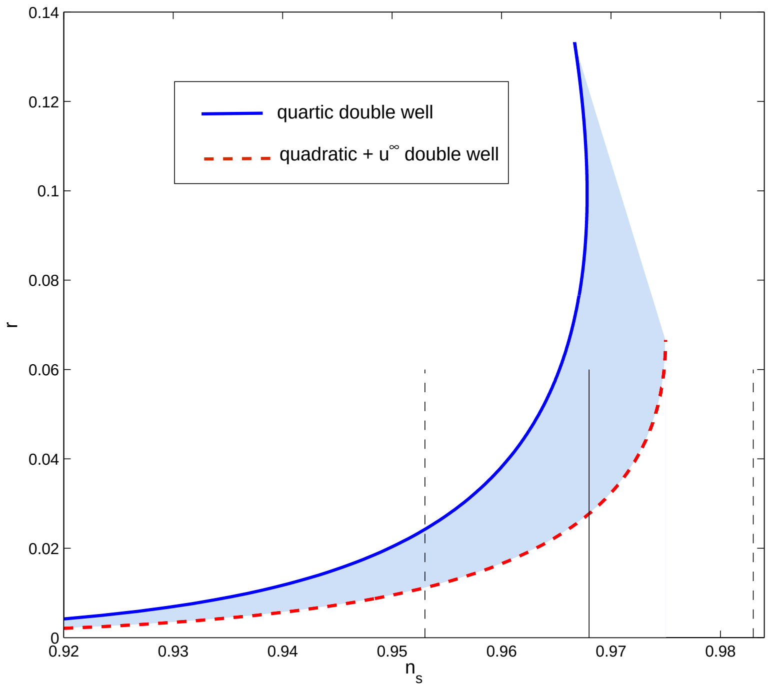

FIG. 1: The "universal banana" region in the (n_s, r)-plane found by the team: All values of the spectral index n_s, and the ratio r of the tensor to scalar fluctuations in the inflation class of double well polynomial potentials fall inside this region. The upper border of the region corresponds to the fourth order double–well potential and provides an excellent fit to the present WMAP+BAO+SN data set. The lower border is described by the limiting high order potential (composed of a quadratic + an infinite barrier double well potential). In between fall the different higher order polynomial potentials. Adding higher orders to the potential does not improve considerably the fits with the present accuracy of the data. The vertical full line is the ΛCDMr value $n_s = 0.968 \pm 0.015$ obtained using the present WMAP+BAO+SN data set. The broken vertical lines delimit the $\pm 1\,\sigma$ region. Planck should considerably constrain the n_s value and r.

- The best value for r in the presence of residuals is about $r \simeq 0.04$ both for the ΛCDMr and the ΛCDMrT models. The ΛCDMrT model turns to be robust, it is very stable (its probability distributions do not change) with respect to the inclusion of residuals. With the ΛCDMrT model we have for r at 95 % CL:
  $0.028 < r < 0.116$ with the best values $r = 0.04, \quad n_s = 0.9608.$
  This shows a substantial progress in the forecasted r values with respect to the

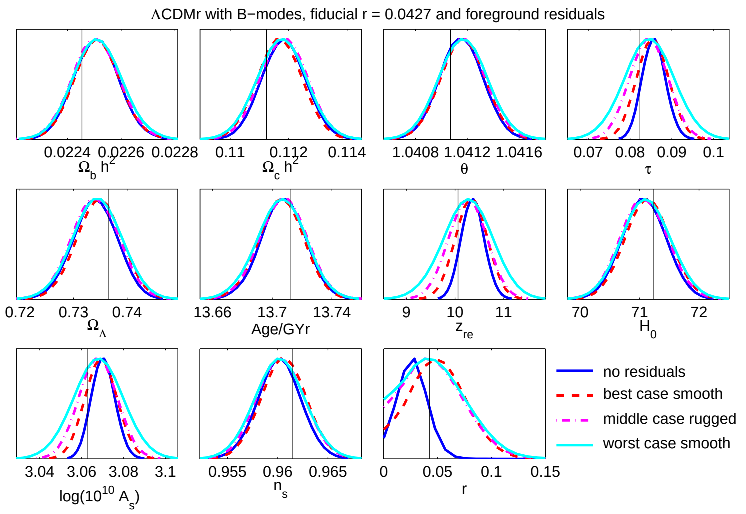

FIG. 2: Cumulative marginalized likelihood distributions from the three channels (70, 100 and 173 GHz) for the cosmological parameters in the ΛCDMr model including B modes and foreground residuals. The fiducial ratio is $r = 0.0427$.We plot the cumulative likelihoods in four cases: (a) without residuals, (b) with 0.3 of the worst case residuals in the TE and E modes and $16\mu K^2$ in the T modes, (c) with the worst case residuals in the TE and E modes and $160\mu K^2$ in the T modes, (d) with 65% of the modelled residuals in the TE and E modes and $88\mu K^2$ in the T modes rugged by Gaussian fluctuations of 30% relative strength.

WMAP+LSS data set for which $r < 0.20$

- Interestingly enough, in the ΛCDMrT model future improvements in the precision on the measured value of n_s alone will immediately give an improvement on the prediction for r as well as for its lower bound. Better measurements for n_s will thus improve the prediction on r from the T, TE and E measurements even if a secure detection of the tensor B modes will be still lacking.

The Planck detection probability for r in its most sensitive HFI-143 channel is border line: We extract hundred thousand skies and compute all the corresponding likelihood profiles for r and their interesting statistical properties (as the most likely value, mean value, standard deviation of most likely distributions, skewness and kurtosis, which measures the departure from a Gaussian likelihood). We finally compute the 99% CL, 95% CL, and 68% CL lower bounds for r and obtain the r detection probabilities. Notice that a real CMB experiment can observe only one sample: the observed sky. So, the possibility of inferring r from one

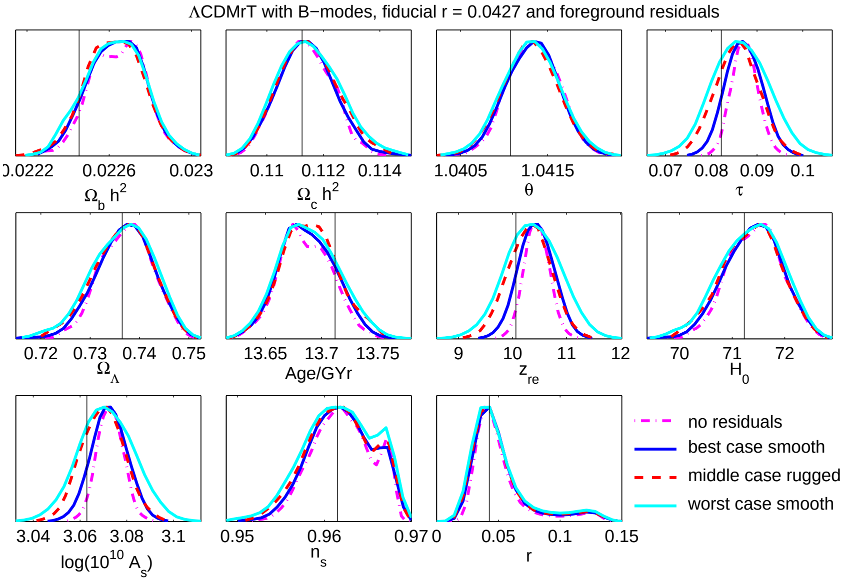

FIG. 3: Cumulative marginalized likelihood distributions from the three channels (70, 100 and 173 GHz) for the cosmological parameters in the ΛCDMrT model including B modes and the foreground residuals. The fiducial ratio is $r = 0.0427$. We plot the cumulative likelihoods in four cases: (a) without residuals, (b) with 0.3 of the worst case residuals in the TE and E modes and $16\mu K^2$ in the T modes, (c) with the worst case residuals in the TE and E modes and $160\mu K^2$ in the T modes, (d) with 65% of the toy model residuals in the TE and E modes and $88\mu K^2$ in the T modes rugged by Gaussian fluctuations of 30% relative strength.

single (albeit very large) sample depends on the sample itself, and therefore, whether r will be or will be not detected depends also of a question on luck. More generally, our results support the quest for B mode polarization in the current CMB data and future B oriented polarization missions under study by both ESA and NASA.

Reference

[1] C. Burigana, C. Destri, H. J. de Vega, A. Gruppuso, N. Mandolesi, P. Natoli, N. G. Sanchez
"Forecast for the Planck precision on the tensor to scalar ratio and other cosmological parameters" arXiv: 1003.6108, The Astrophysical Journal, in press (2010)

Contact Norma G. Sanchez (Directeur de Recherche CNRS, OP LERMA Paris)
Hector J. de Vega (Directeur de Recherche CNRS, UPMC LPTHE Paris)