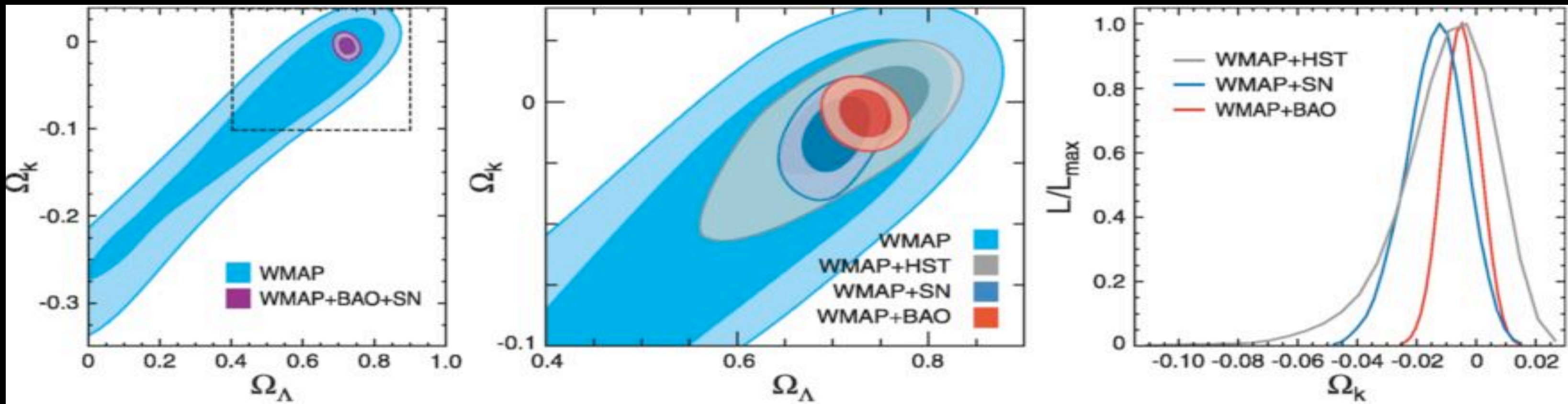
WMAP and Inflation

David Spergel
Princeton

Inflationary Predictions

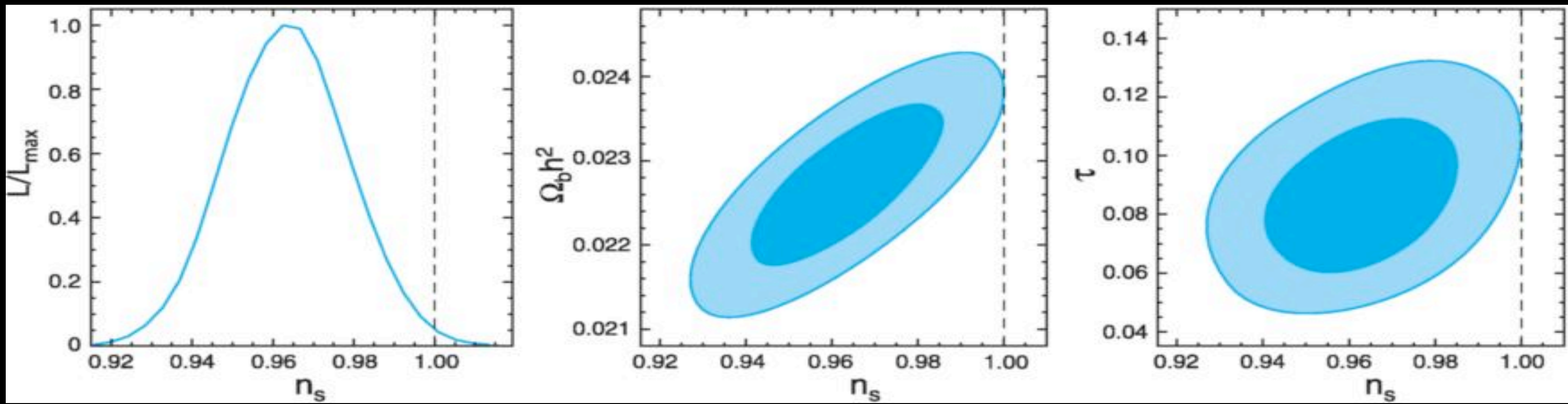
- ✓ Flat Universe
- ✓ Nearly Scale Invariant Fluctuations
- ✓ Gaussian Fluctuations
- ✓ Adiabatic Fluctuations
- ✓ Superhorizon Fluctuations
- ➢ ***Gravity Waves***

Flat Universe



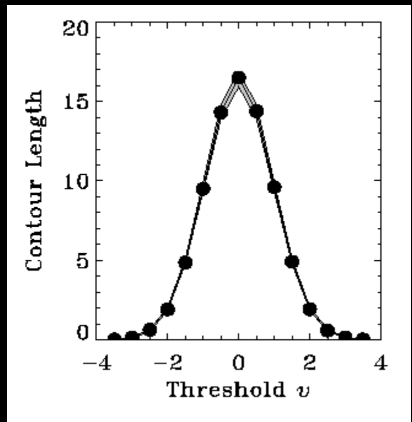
- WMAP measures angular distance to surface of last scatter and matter density
- WMAP + (Lensing, Hubble Constant, SN, or LSS) implies nearly flat universe

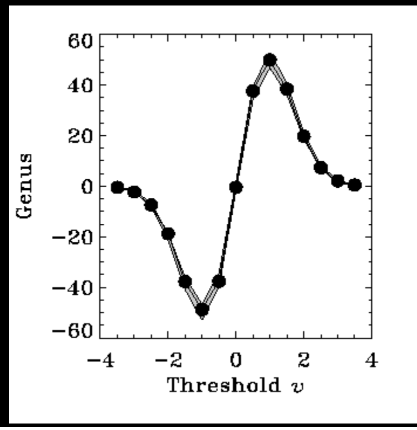
Nearly Scale Invariant Fluctuations

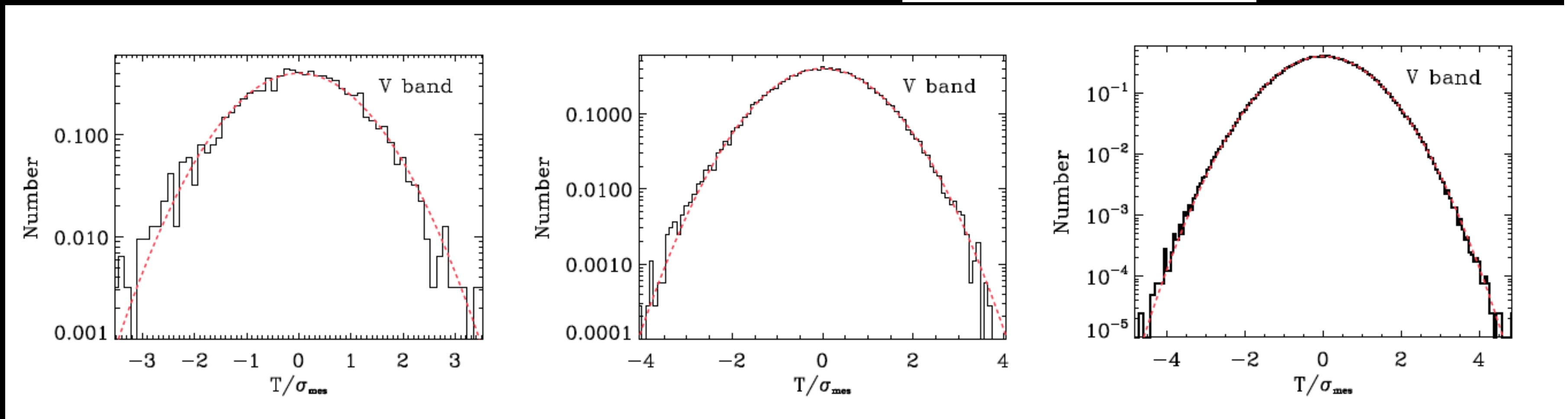


- If fluctuations are purely adiabatic, data favors a slightly red spectrum
- Models with admixtures of adiabatic and isocurvature modes can have flat or slightly blue index

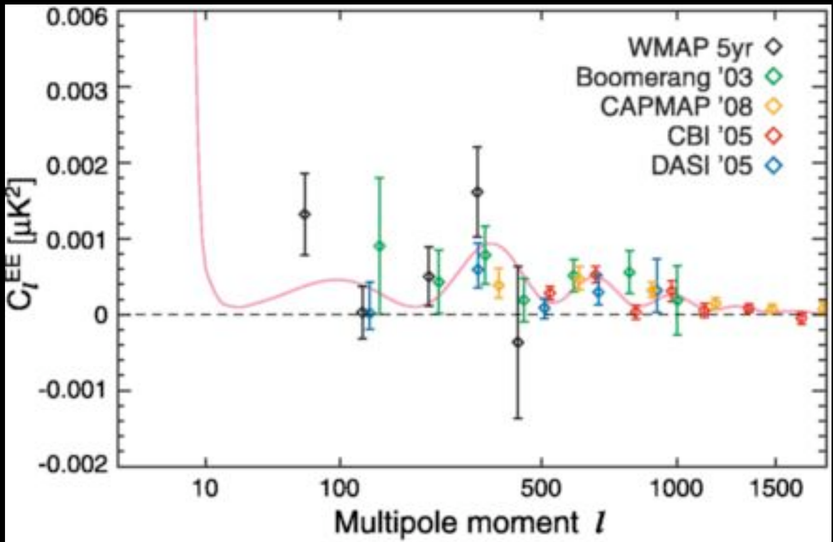
Gaussian Fluctuations

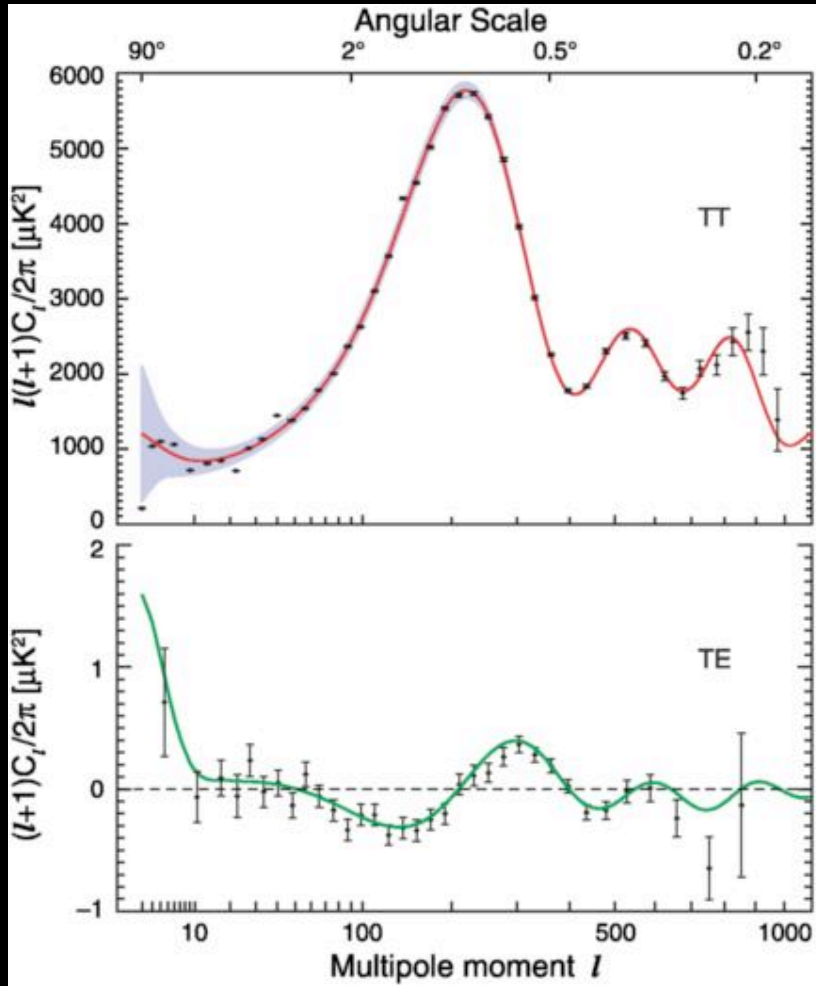






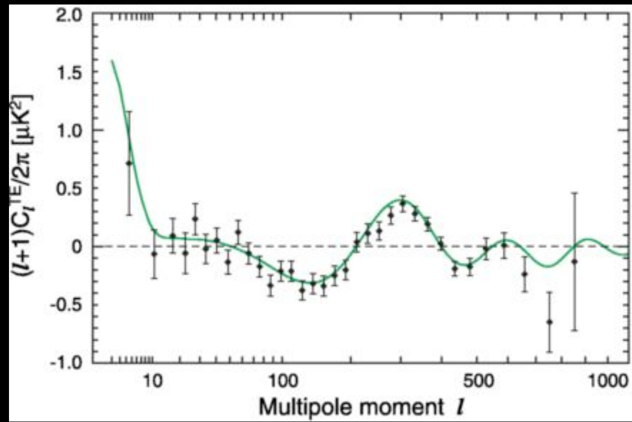
Adiabatic Fluctuations

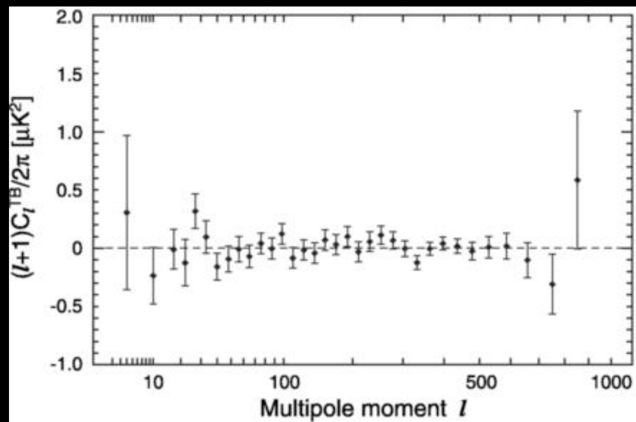




Superhorizon Fluctuations

- While temperature fluctuations can be generated both at the surface of last scatter and along the line of sight, polarization fluctuations are only generated through electron scattering. Small angle TE fluctuations must come from SLS (Zaldarriaga and Spergel 2000)

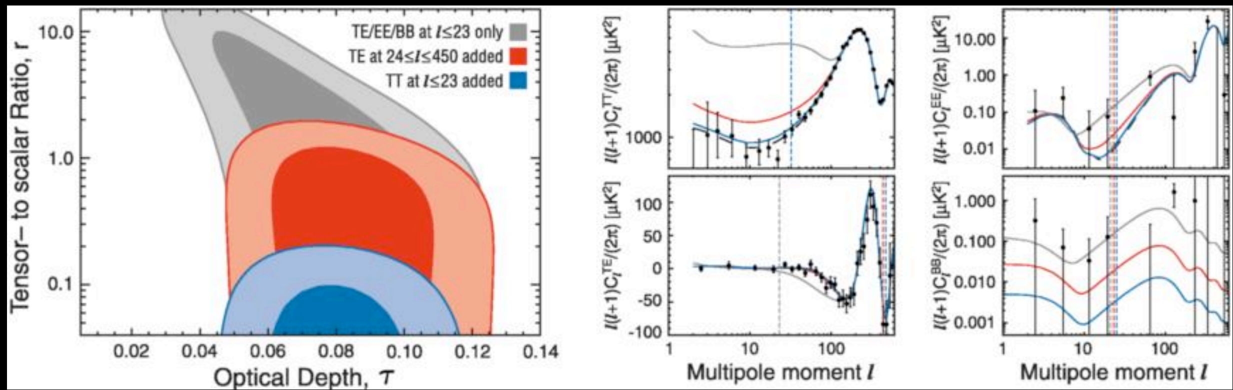


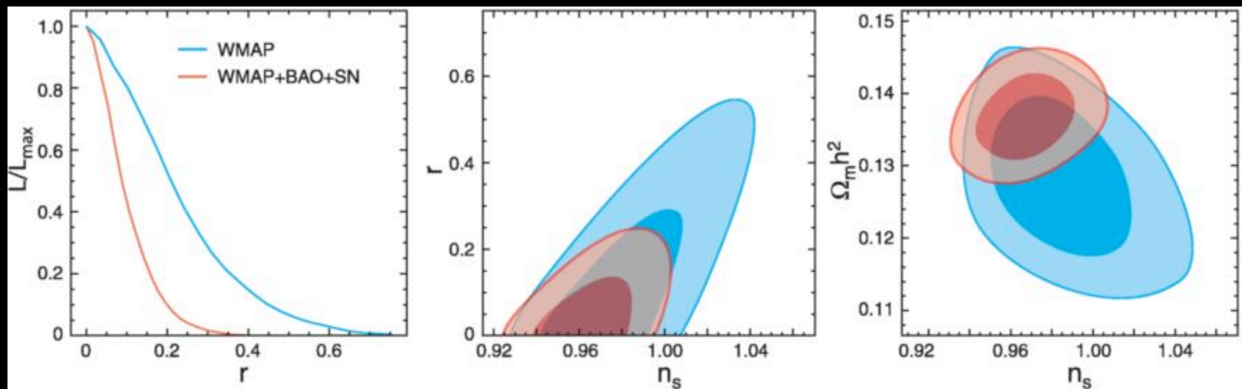


Testing Inflationary Models

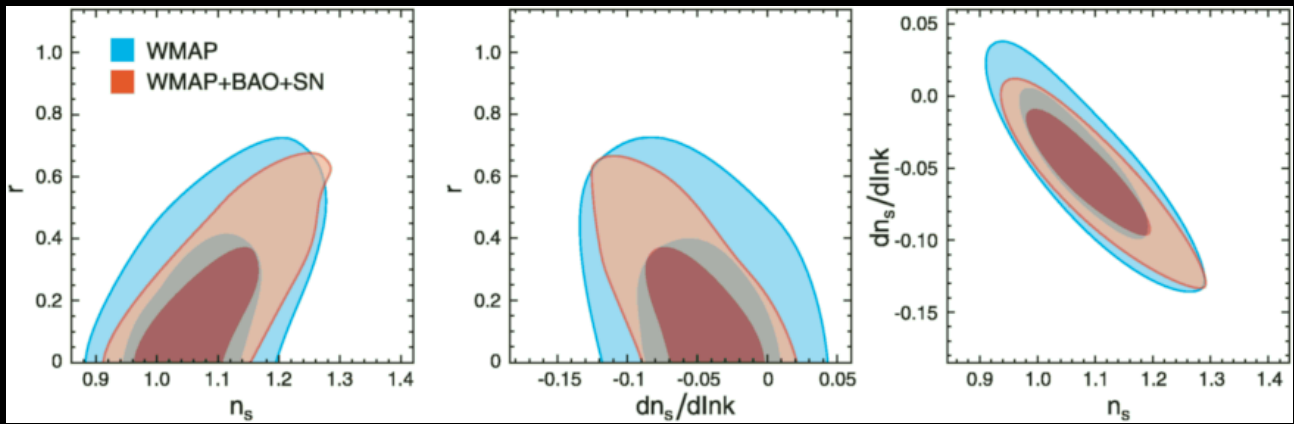
- Spectral Index and Its Scale Dependance
- Spectral Features
- Gravitational Waves
- Non-Gaussianities
- Cosmic Strings
- Isocurvature Modes

Constraints on n and r



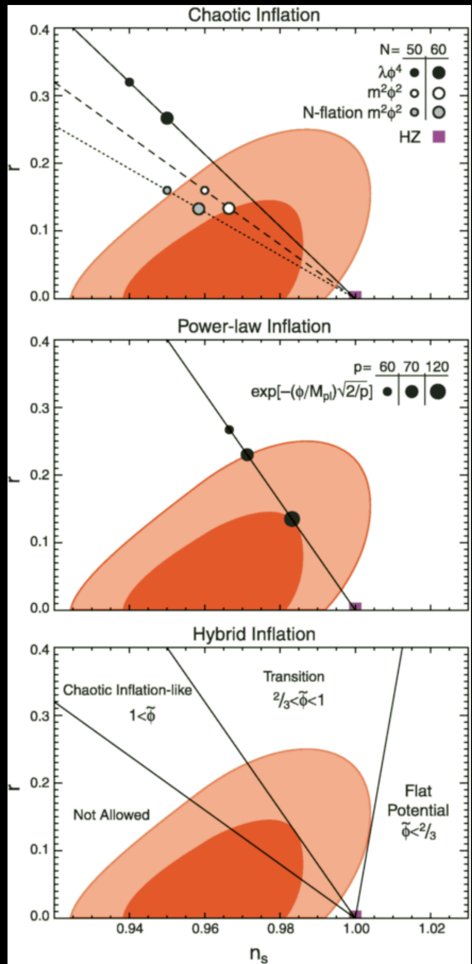


Running Spectral Index



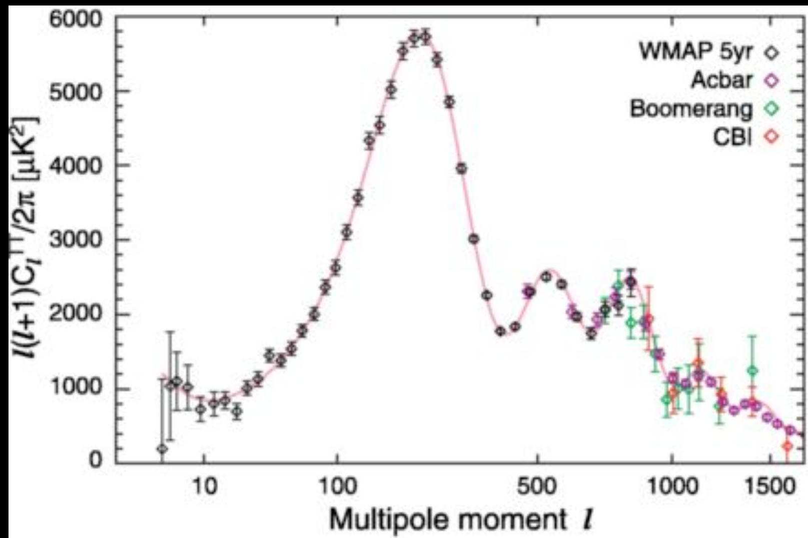
Inflationary Models

- Data disfavors models with flat potential (includes many hybrid models)
- If potential is smooth (few higher order derivatives), then r should be large and in the detectable range for Planck and upcoming ground-based experiments



Spectral Features

- Features in inflaton potential
- Transplanckian physics
- Bumps and wiggles in TT spectrum?
- TE and EE spectrum as test



Primordial Skewness

Spergel and Goldberg 1999

Komatsu and Spergel 2001

$$\Phi(\mathrm{x}) = \Phi_L(\mathrm{x}) + f_{NL}\left(\Phi_L^2(\mathrm{x}) - \langle\Phi_L^2(\mathrm{x})\rangle\right)$$

$$B_{l_1 l_2 l_3}^{m_1 m_2 m_3} = a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}$$

$$B_{l_1 l_2 l_3} = \sum_{m_1, m_2, m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{l_1 l_2 l_3}^{m_1 m_2 m_3}$$

Non-linear Bispectrum Terms

Spergel and Goldberg 1999

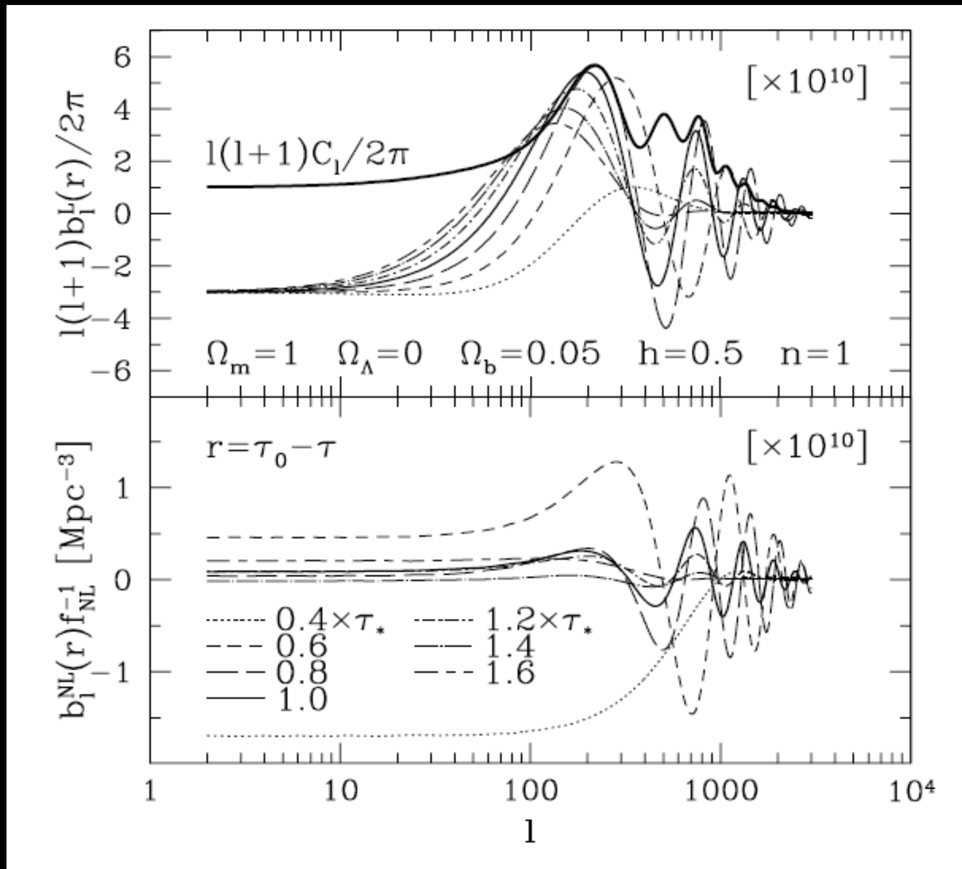
$$T^L(\hat{\mathbf{n}}) = \phi(\tau_r) \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\hat{\mathbf{n}}\tau_r} \Phi_0(\mathbf{k}) g(k),$$

$$a^L_{lm} = \phi(\tau_r) \int \frac{d^3\mathbf{k}}{(2\pi)^3} d\hat{\mathbf{n}} e^{i\mathbf{k}\cdot\hat{\mathbf{n}}\tau} \Phi_0(\mathbf{k}) g(k) Y^*_{lm}(\hat{\mathbf{n}}) \ .$$

$$B^{l_1 l_2 l_3}_{m_1 m_2 m_3} = a^L_{l_1 m_1} a^L_{l_2 m_2} a^{NL*}_{l_3 m_3} + a^L_{l_2 m_2} a^L_{l_3 m_3} a^{NL*}_{l_1 m_1} + a^L_{l_3 m_3} a^L_{l_1 m_1} a^{NL*}_{l_2 m_2}$$

$$\begin{aligned} B^{(1)}_{l_1 l_2 l_3} &= \frac{1}{2\pi^4} \phi^2(\tau_r) \int d\tau \phi^2(\tau) \int dk_1 dk_2 k_1^2 k_2^2 P(k_1) P(k_2) g(k_1) g(k_2) \qquad (14) \\ &\times \sum_{l l' l''} (2l'+1)(2l''+1) Q^{l_1 l_2 l_3}_{l l'' l'} f_l(k_1, k_2, \tau) j_{l'}(k_1\tau) j_{l''}(k_2\tau) j_{l_1}(k_1\tau_r) j_{l_2}(k_2\tau_r) \end{aligned}$$

Bispectrum changes sign…



Bispectrum

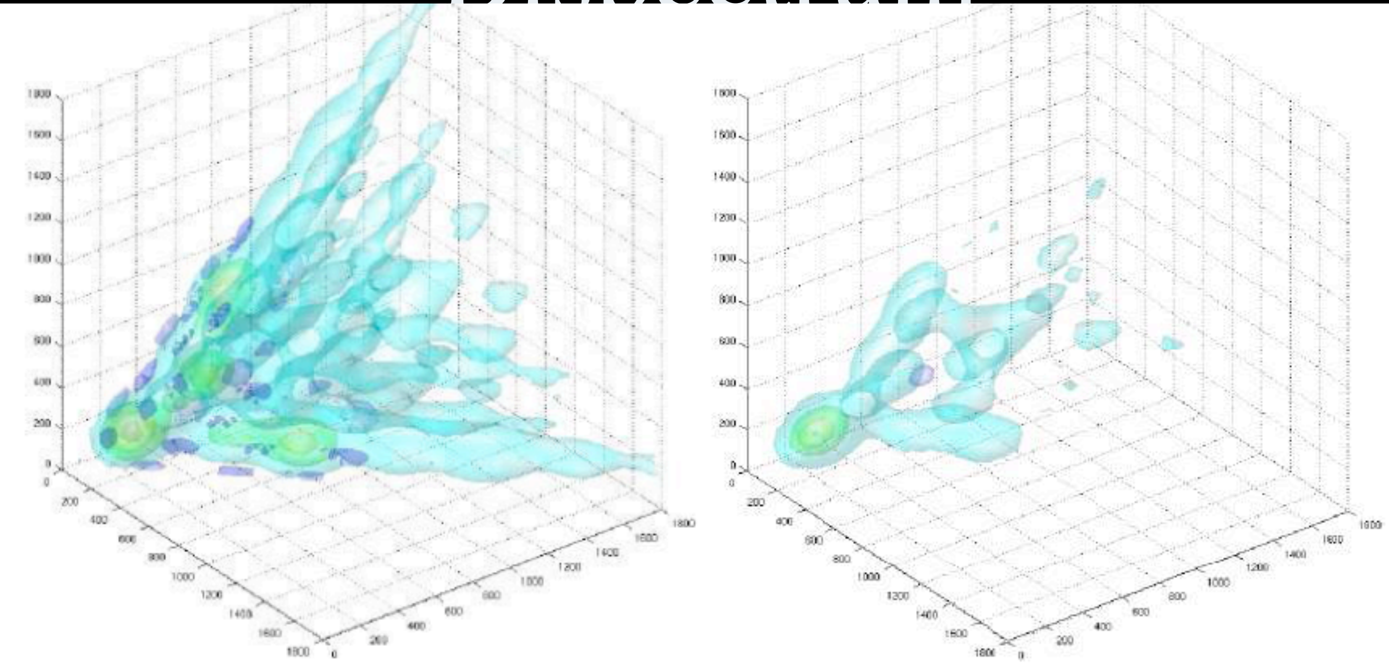


Figure 13: Plots of the bispectrum for the local case (on the left) and for the equilateral case (on the right) for $l < 1800$. Note how in the equilateral case all perturbations off the central axis are suppressed

Fergusson and Shellard 2007

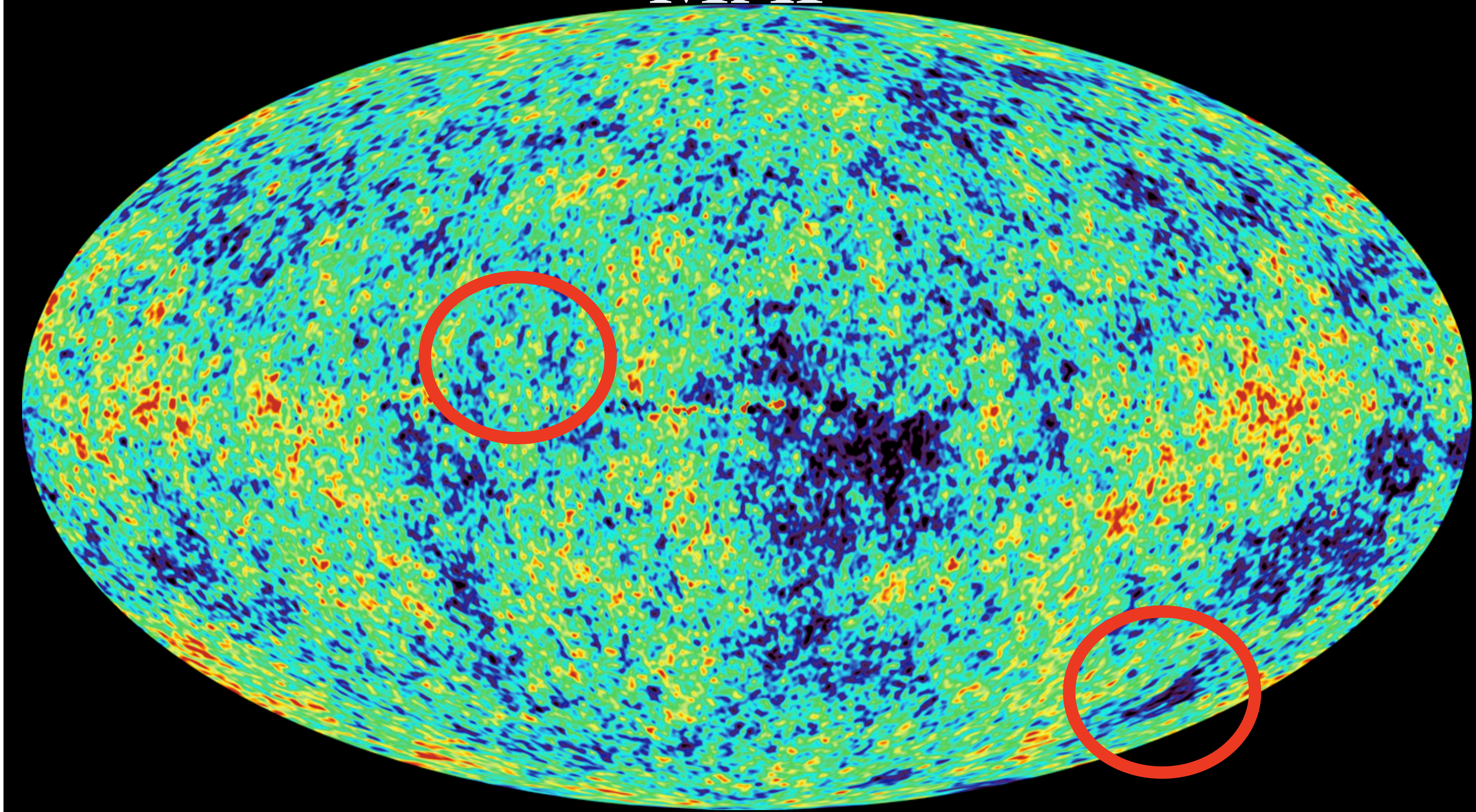
5 year Results

- We do see a positive fNL but its amplitude is only ~2 s
- Amplitude is lower than values claimed by Yadav and Wandelt; however, we see a consistent set of values as a function of sky cut
- Still see contamination effects in Q band
- Need more data to make a convincing case

Band	Mask	$l_{\max}$	f_{NL}^{local}	$\Delta f_{NL}^{\text{local}}$	b_{src}
V+W	*KQ85*	400	50 ± 29	1 ± 2	0.26 ± 1.5
V+W	*KQ85*	500	61 ± 26	2.5 ± 1.5	0.05 ± 0.50
V+W	*KQ85*	600	68 ± 31	3 ± 2	0.53 ± 0.28
V+W	*KQ85*	700	67 ± 31	3.5 ± 2	0.34 ± 0.20
V+W	*Kp0*	500	61 ± 26	2.5 ± 1.5	
V+W	*KQ75p1*[a]	500	53 ± 28	4 ± 2	
V+W	*KQ75*	400	47 ± 32	3 ± 2	-0.50 ± 1.7
V+W	*KQ75*	500	55 ± 30	4 ± 2	0.15 ± 0.51
V+W	*KQ75*	600	61 ± 36	4 ± 2	0.53 ± 0.30
V+W	*KQ75*	700	58 ± 36	5 ± 2	0.38 ± 0.21

Q	Raw	*KQ75p1*[a]	-42 ± 45
V	Raw	*KQ75p1*	38 ± 34
W	Raw	*KQ75p1*	43 ± 33
Q	Raw	*KQ75*	-42 ± 48
V	Raw	*KQ75*	41 ± 35
W	Raw	*KQ75*	46 ± 35
Q	Clean	*KQ75p1*	9 ± 45
V	Clean	*KQ75p1*	47 ± 34
W	Clean	*KQ75p1*	60 ± 33
Q	Clean	*KQ75*	10 ± 48
V	Clean	*KQ75*	50 ± 35
W	Clean	*KQ75*	62 ± 35

FOREGROUND CORRECTED MAP

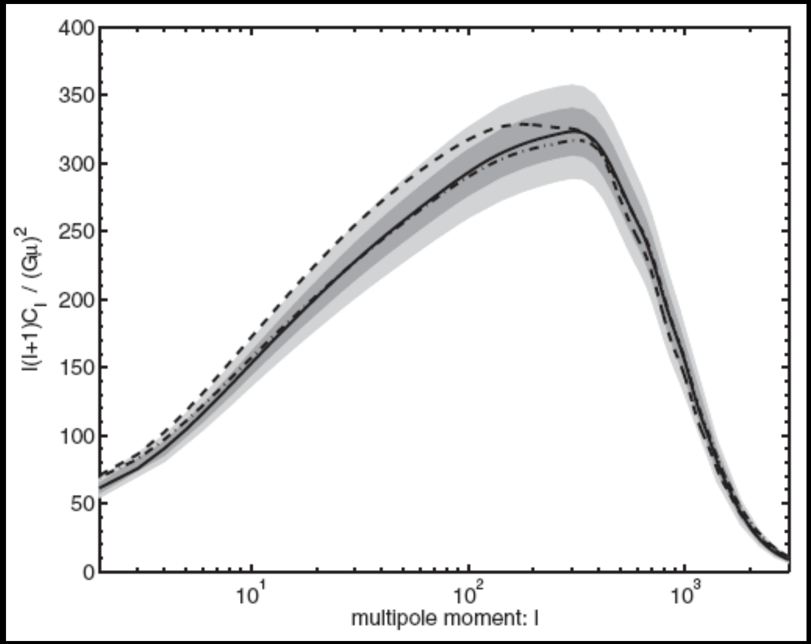


Cold Spot Tests

- Is it a low density region?
  - Minnesota group (Rudnick et al.)
- Is it a texture?
- Key observational tests
  - TE correlation - test if fluctuation is adiabatic fluctuation at SLS
  - Small scale CMB measurements
    - Low density region will produce significant lensing

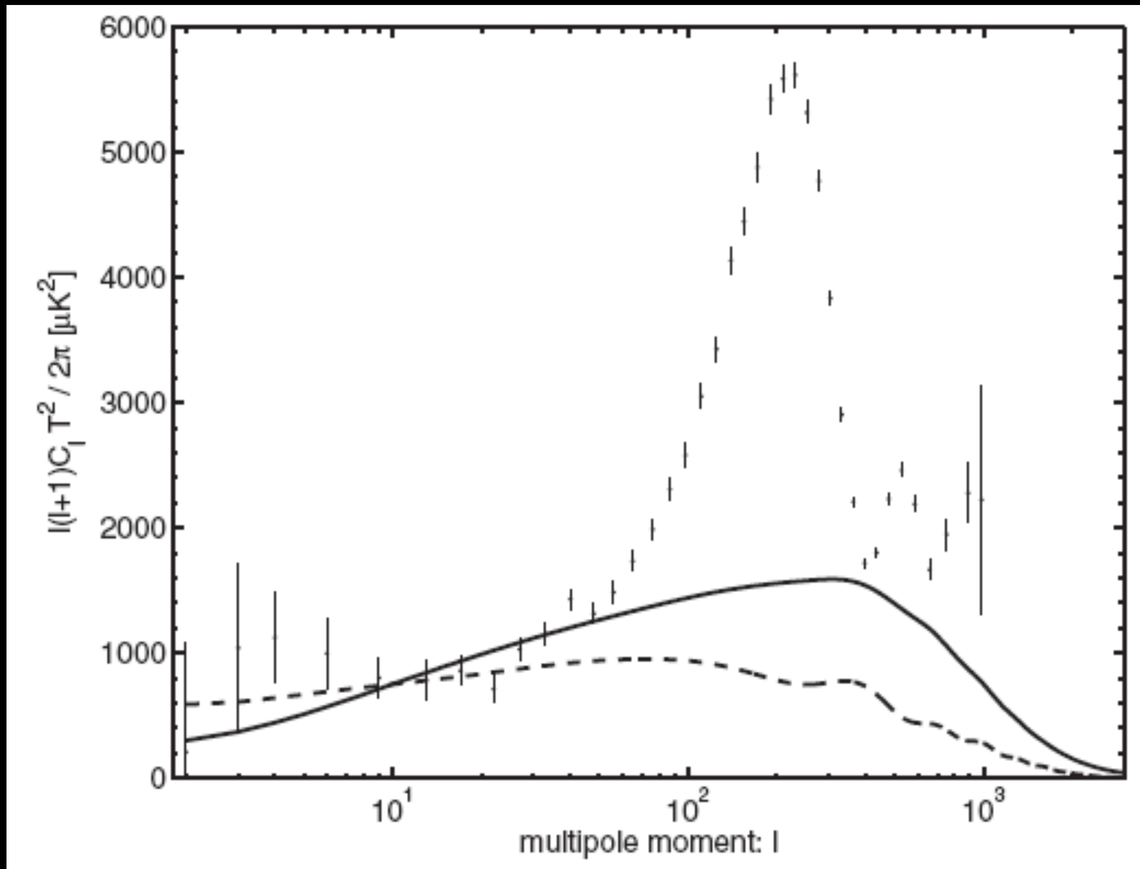
Cosmic Strings

- 1-D topological defects
- Can be produced by a *U(1)* SSB
- Radius ~ 3 trillion times smaller than the radius of H
- Mass ~ 10 miles of string is about the mass of the Earth
- Induce fluctuations in the CMB
- Large-scale power spectrum has a single bump but is otherwise featureless.



Bevis *et al.* PRD **75** 065015 (2007)

WMAP Constraints



Bevis *et al.* PRD **75** 065015 (2007)

End of GUTS?

Would be dangerous for all GUTs…

G -> … -> $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$

If G = SU(8), SU(9), SO(10), SO(14) or E_6(NTL), formation of CS unavoidable

If G = E6(TL), formation of CD in 80% to 98% of acceptable schemes

G = SU(6) or SU(7) incompatible with proton lifetime measurements

(Jeannerot *et al.*, PRD **68** 103514 (2003))

Only show that strings *alone* cannot explain the CMB anisotropies

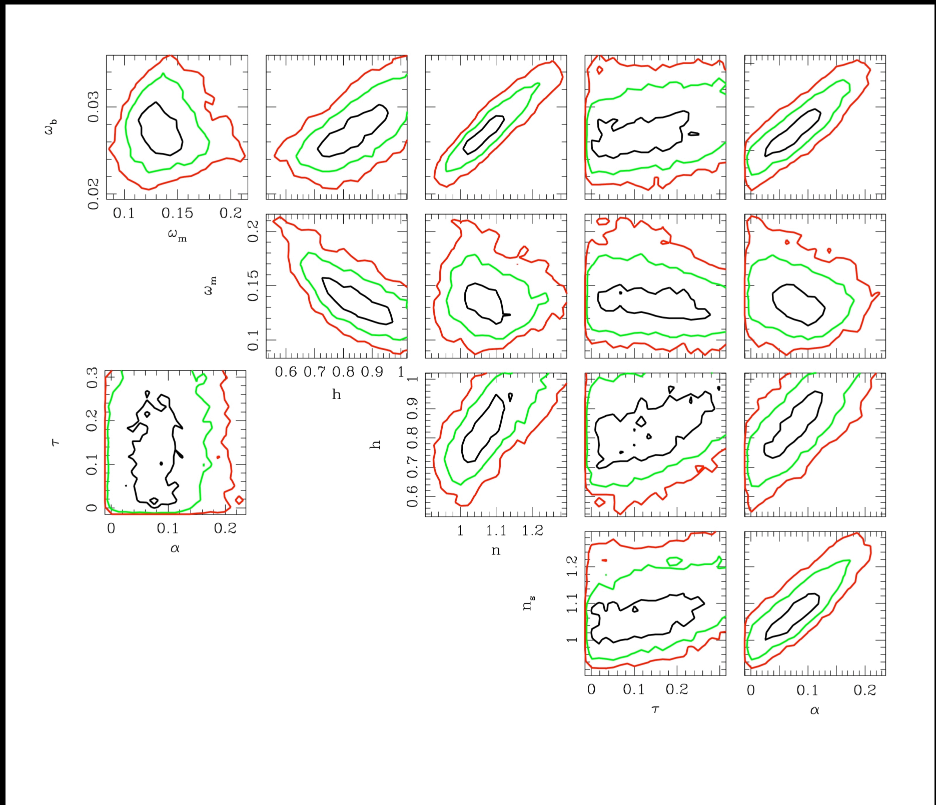
Hybrid Inflation

Inflation and Formation of defects at the end of the inflationary phase

$$C_\ell = (1-\alpha)\, C_\ell^{\Lambda\mathrm{CDM}} + \alpha\, C_\ell^{\mathrm{TD}}$$

Minimal description: α, ω_b, ω_m, h, τ, n_s, A (σ_8)

Complete MCMC necessary



Fraisse
2007

JCAP **0703** 008 (2007)

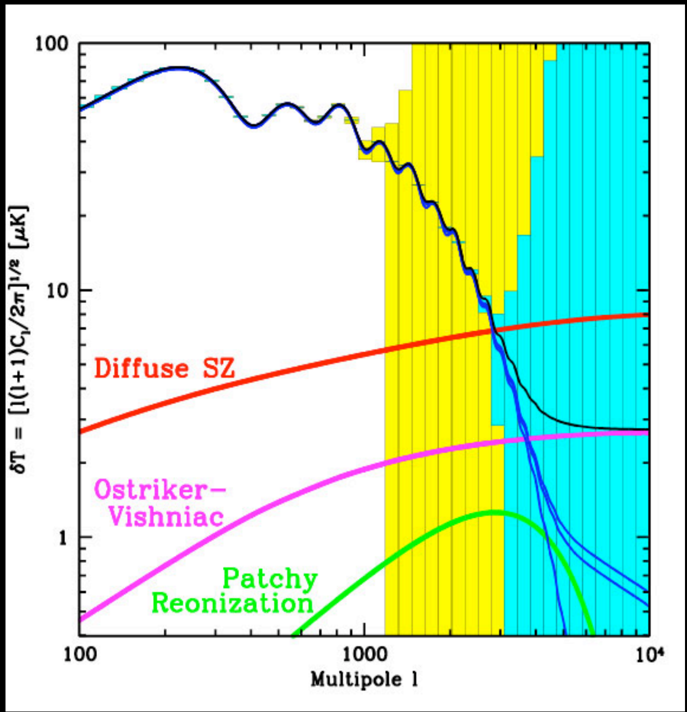
Limits on defects formation and hybrid inflationary models with three-year WMAP observations

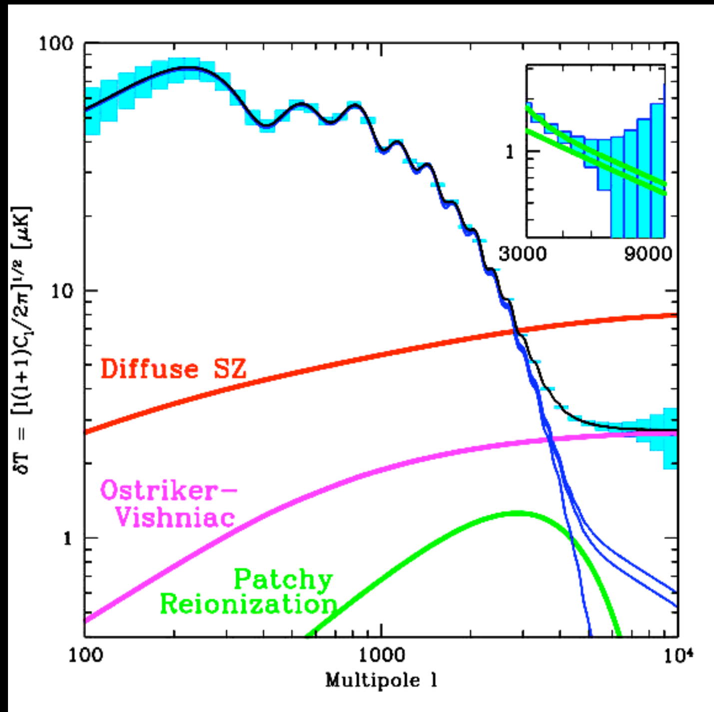
Aurélien A Fraisse

Princeton University Observatory, Peyton Hall, Princeton, NJ 08544, USA

Defects		Upper bound	$G\mu \times 10^7$
Global strings	[21]	13% – 18%	2.4 – 2.8
Local strings	[25]	7% – 11%	2.1 – 2.6
Local strings	[22]	5% – 7%	2.1 – 2.5

Upcoming Experiments..





Need information at small angular scales.

Small-Angle CMB Temperature Anisotropies Induced by Cosmic Strings

Aurélien A. Fraisse[*]

Princeton University Observatory, Peyton Hall, Princeton, NJ 08544, USA

Christophe Ringeval[†]

Theoretical and Mathematical Physics Group, Center for Particle Physics and Phenomenology, Louvain University, 2 Chemin du Cyclotron, 1348 Louvain-la-Neuve, Belgium

David N. Spergel[‡]

Princeton University Observatory, Peyton Hall, Princeton, NJ 08544, USA
Princeton Center for Theoretical Physics, Princeton, NJ 08544, USA

François R. Bouchet[§]

Institut d'Astrophysique de Paris, 98bis boulevard Arago, 75014 Paris, France

(Dated: August 8, 2007)

What we did

High resolution numerical simulations of Nambu-Goto strings.

Use small-angle approximation.

"Integrate" from z = 1089 to z = 0(.3) (take care of loops).

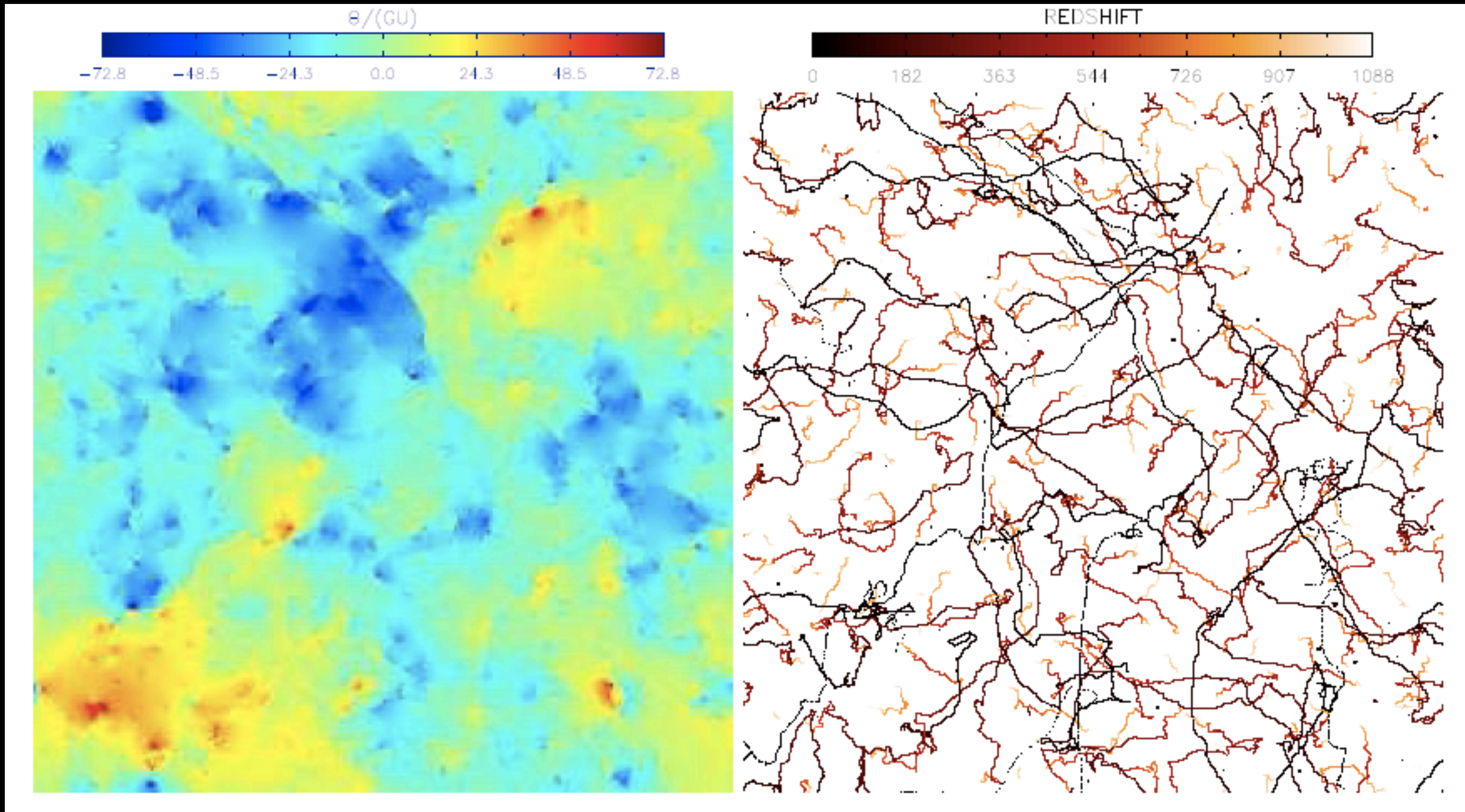
Find **two CPU-years** of computing time.

Produce 84 statistically independent string-induced temperature maps.

$$\Theta_\ell = \int_0^{\eta_0} g(\eta)\, \mathrm{e}^{-k^2/k_\mathrm{D}^2} \left(\bar{\Theta}_\mathrm{o} + \bar{\Phi}\right) j_\ell(k\Delta\eta)\, \mathrm{d}\eta + \int_0^{\eta_0} \eta\, g(\eta)\, \mathrm{e}^{-k^2/k_\mathrm{D}^2}\, \mathrm{i}\, \bar{v}_\mathrm{b}\, j_\ell'(k\Delta\eta)\, \mathrm{d}\eta + \int_0^{\eta_0} \mathrm{e}^{-\tau} \left(\frac{\mathrm{d}\Phi}{\mathrm{d}\eta} + \frac{\mathrm{d}\Psi}{\mathrm{d}\eta}\right) j_\ell(k\Delta\eta)\, \mathrm{d}\eta,$$

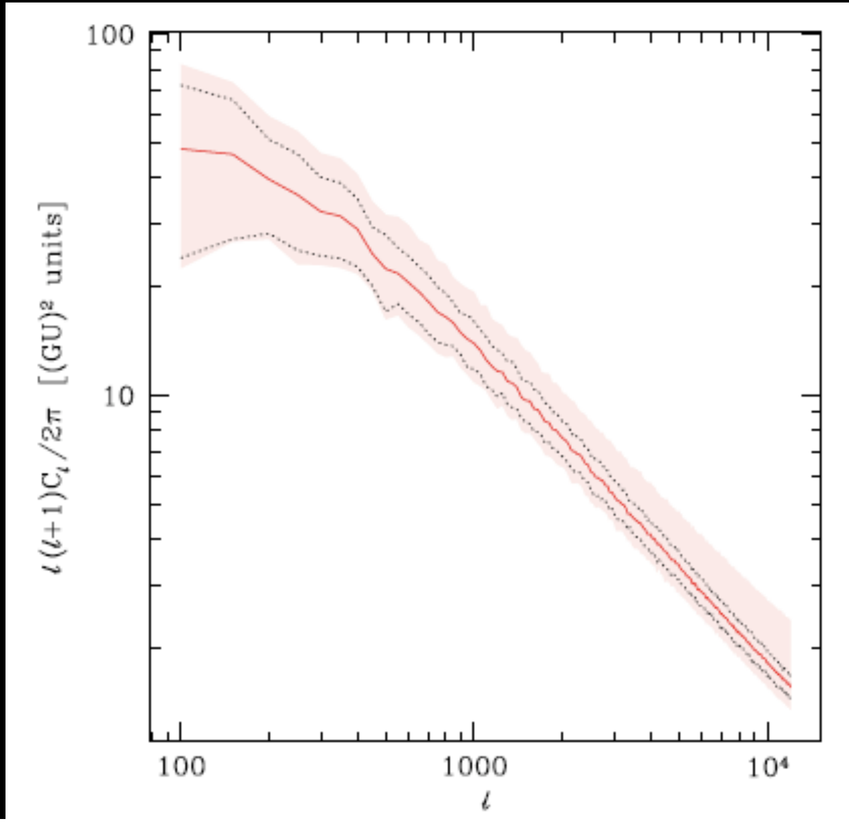
$$T^{\mu\nu} = \alpha \int \mathrm{d}\sigma \left(\epsilon \dot{X}^\mu \dot{X}^\nu - \frac{1}{\epsilon} X'^\mu X'^\nu\right) \delta^3\left(\mathbf{x} - \mathbf{X}\right)$$

String-induced CMB temperature anisotropies



7.2 x 7.2 degree field, 0.42’ angular resolution (1024 pixels)

Angular power spectrum



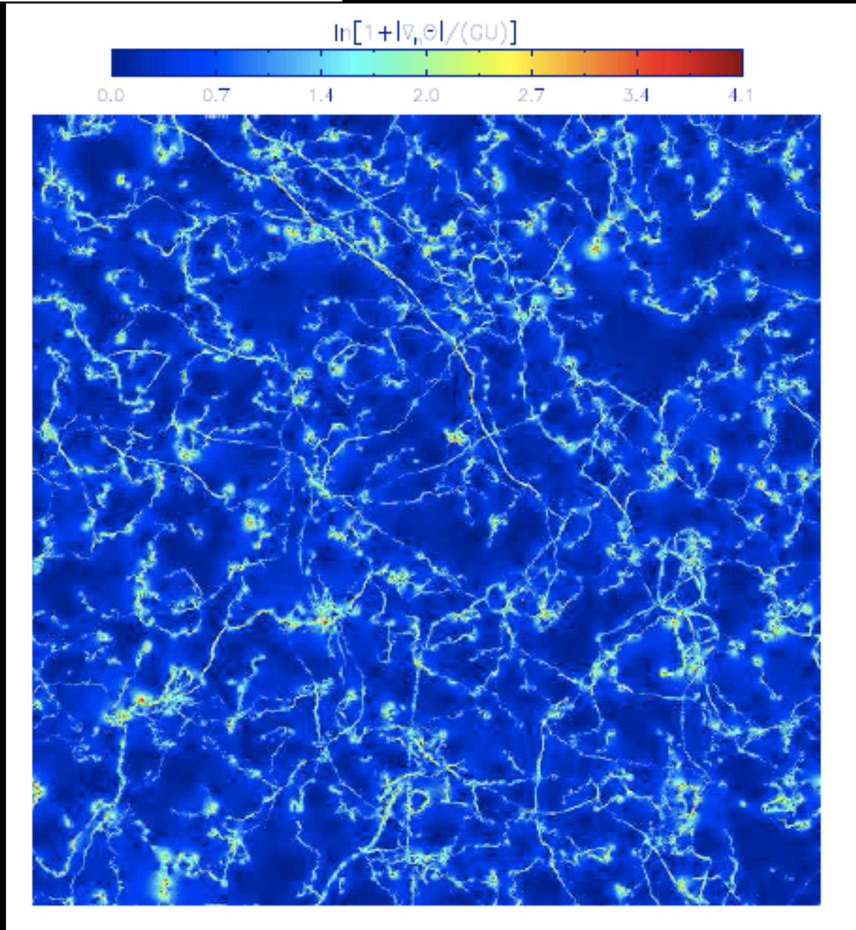
$$\ell(\ell+1)\,C_\ell \underset{\ell\gg 1}{\propto} \ell^{-p} \quad \text{with} \quad p = 0.889^{+0.001}_{-0.090}$$

"Slowly" decaying power-law.

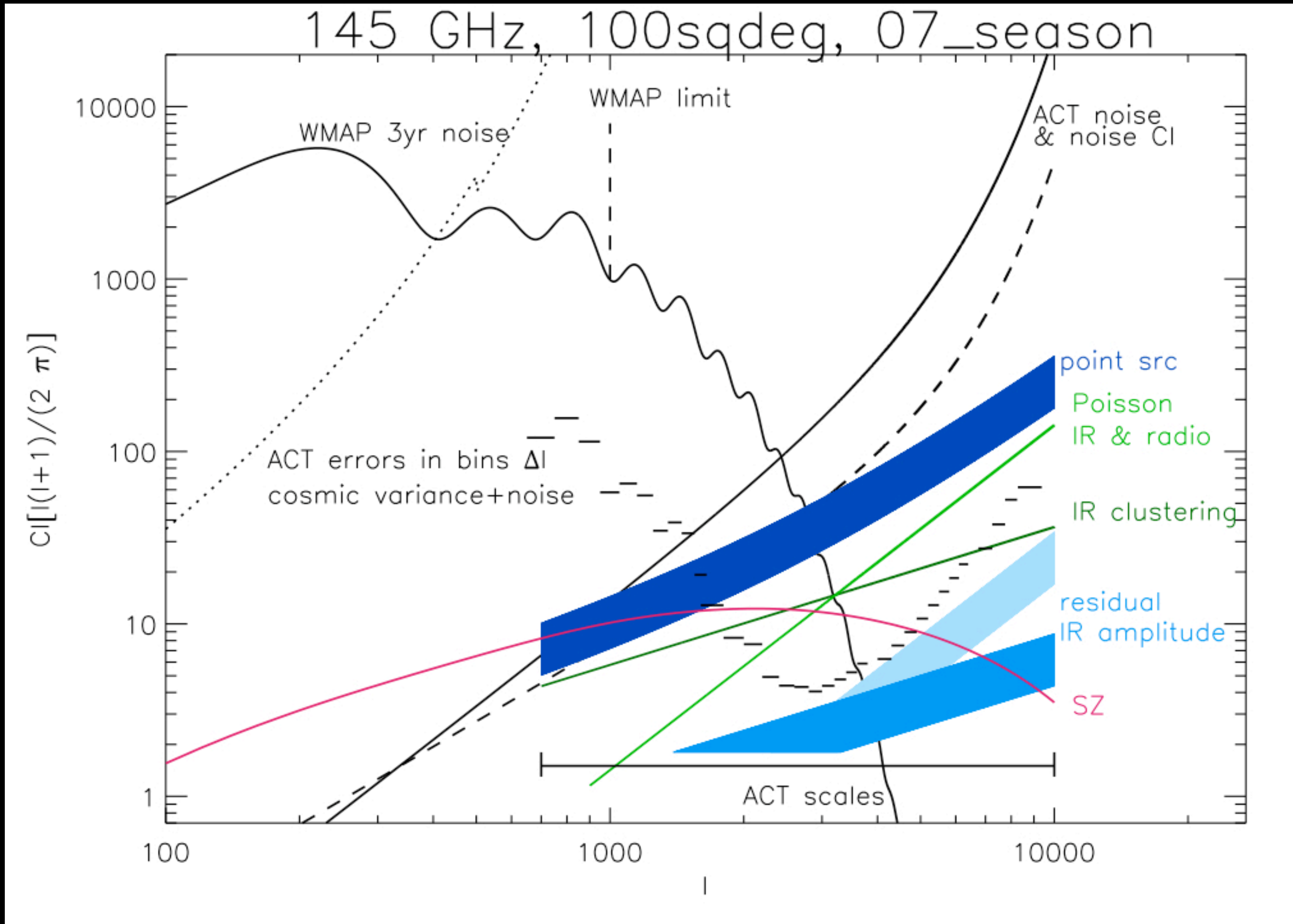
Gradient magnitude map

$$|\nabla\Theta| \equiv \sqrt{\left(\frac{\mathrm{d}\Theta}{\mathrm{d}\alpha}\right)^2 + \left(\frac{\mathrm{d}\Theta}{\mathrm{d}\beta}\right)^2}$$

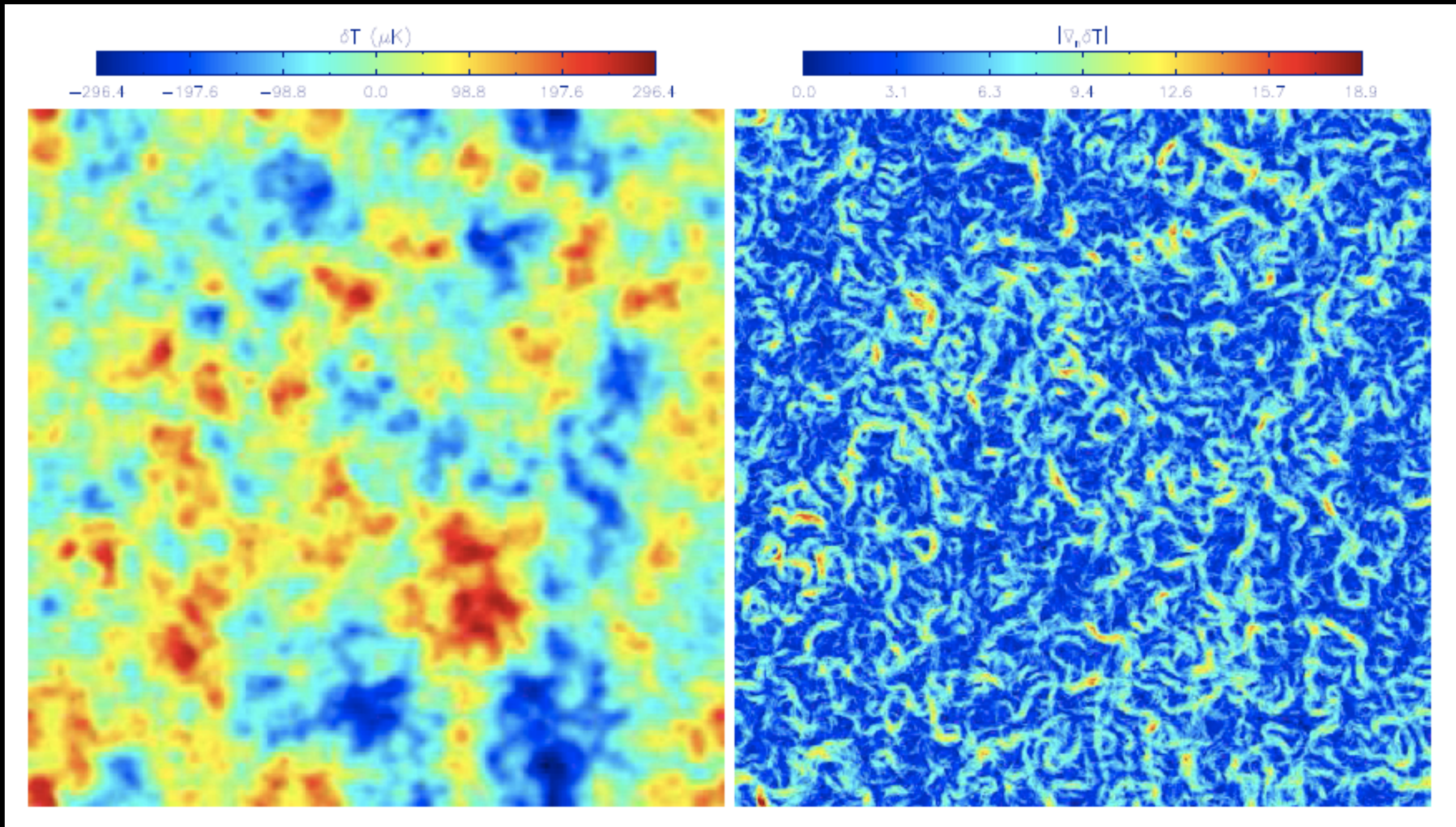
$$|\nabla_{\mathrm{n}}\Theta| \equiv |\nabla\Theta|\,\theta_{\mathrm{res}}$$



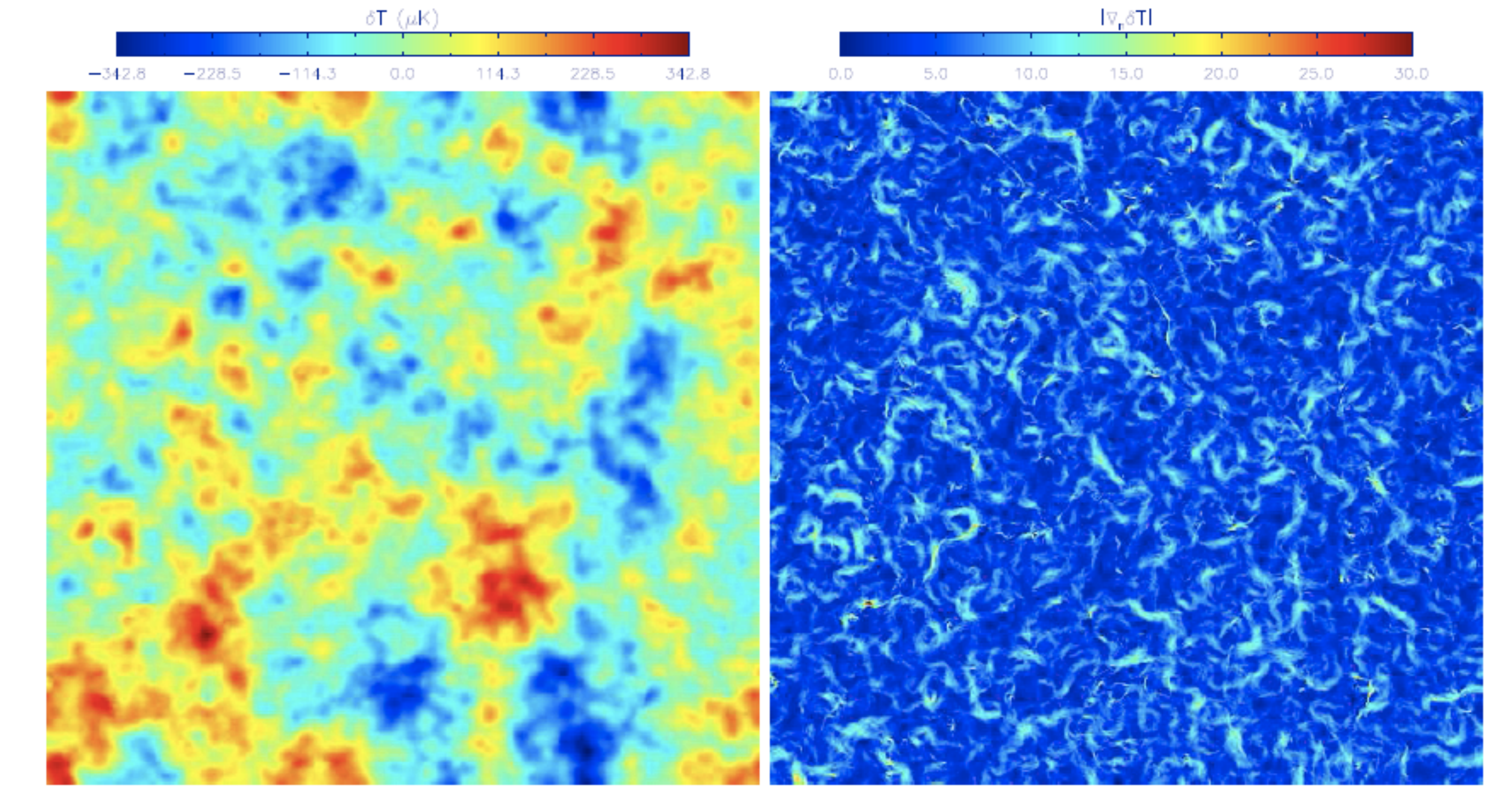
And in real life?



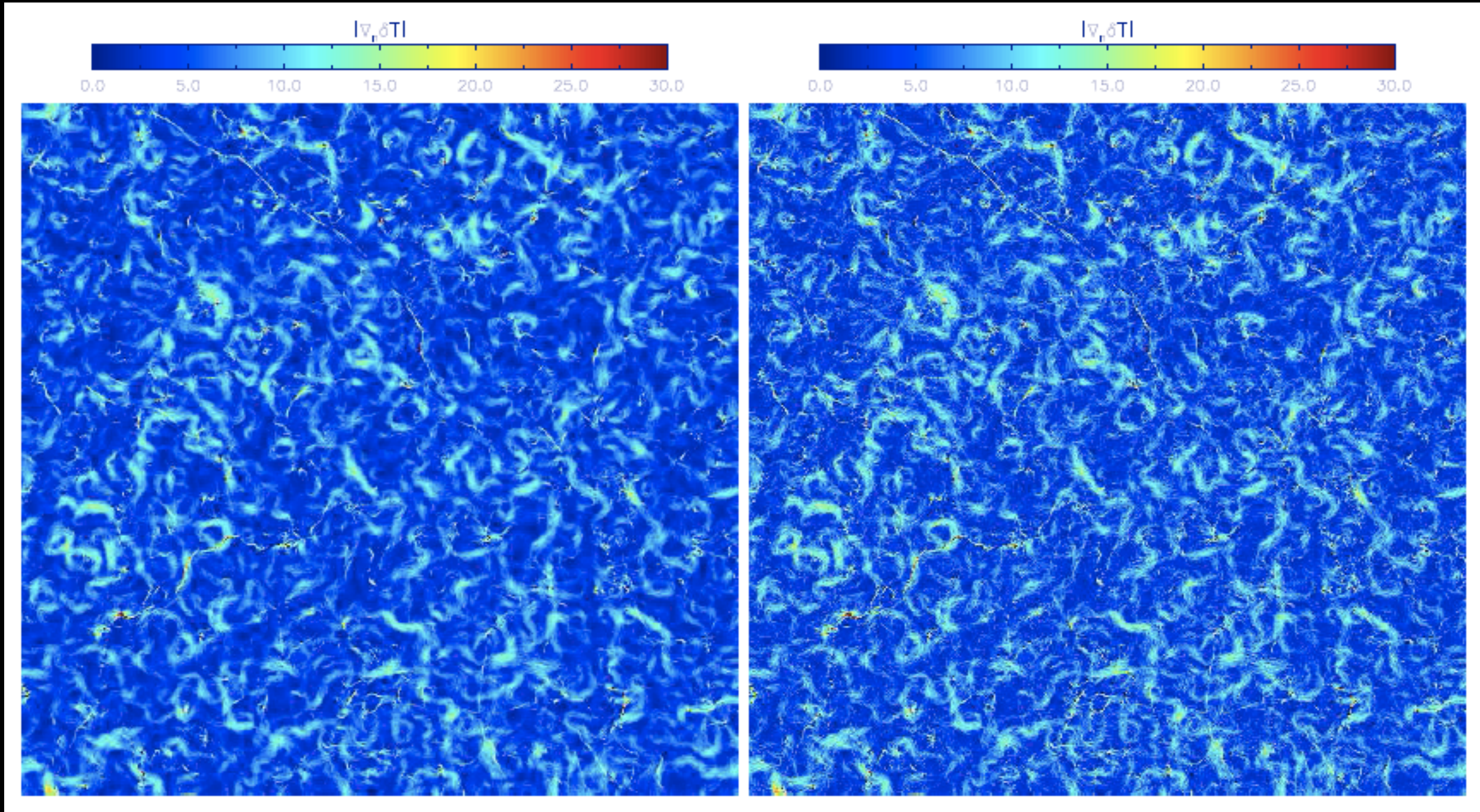
Pure LCDM primary & secondary anisotropies



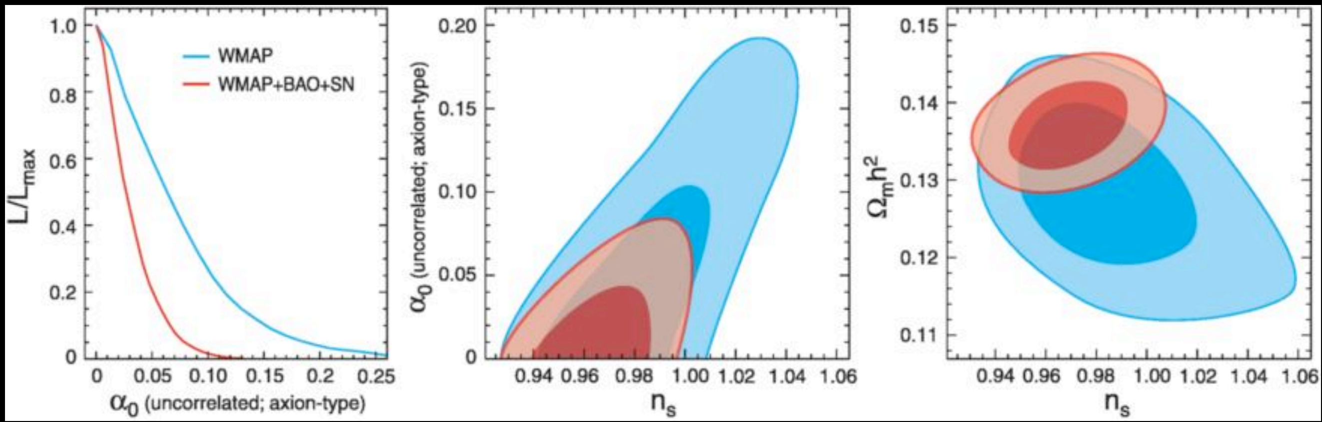
With strings



As a function of frequency



Isocurvature Modes



- Multifield inflationary models *can produce* isocurvature modes
- Correlated or uncorrelated modes possible

Curvaton Type Modes